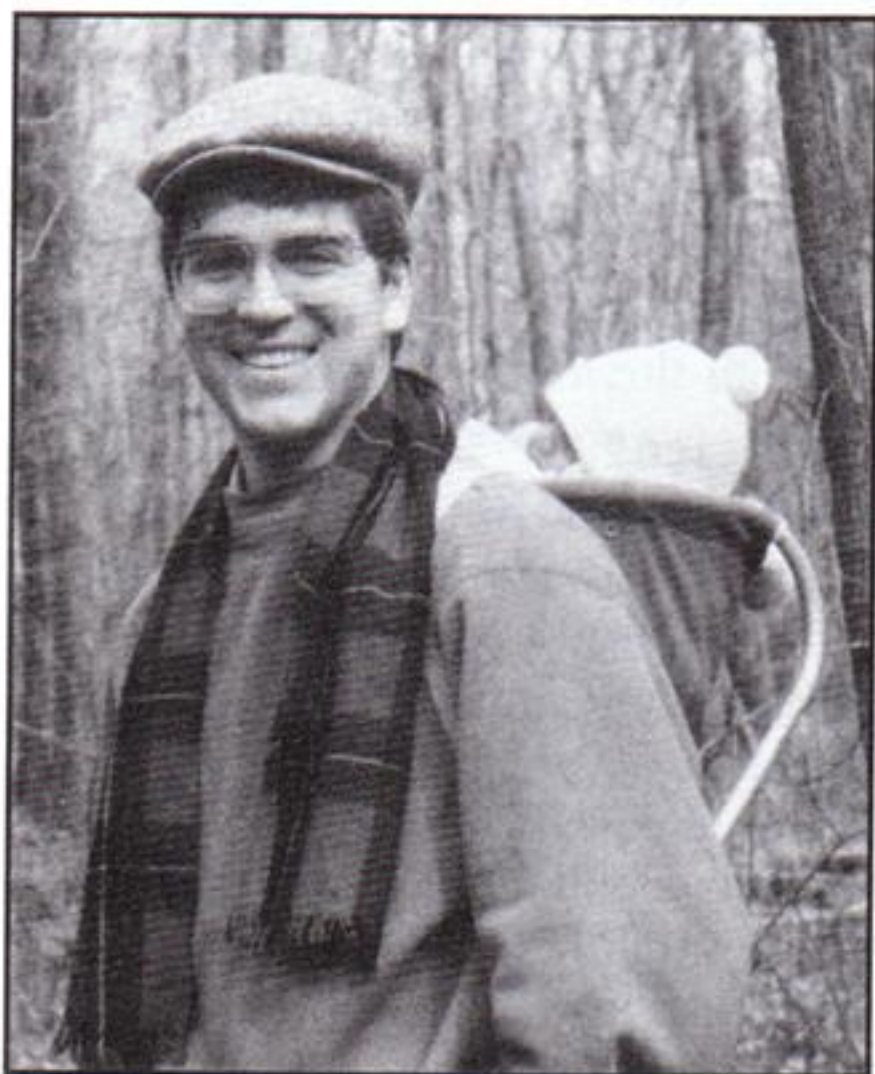


Taking a Swat at Physics with a Ping-Pong Paddle

By Chris M. Graney



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At the end of the Fall '92 semester a Ping-Pong table appeared in the Student Center here at Jefferson Community College. The table rapidly became a gathering spot for students and faculty (including myself) who enjoyed a good game of "Pong." However, it didn't take long for me to discover that my skills were not up to the level of most of my opponents. What was most humiliating was that I was regularly being soundly beaten by some of my own students! I was unable to handle the "spin game." Opponents put so much spin on the ball that it would just leap off my paddle in a manner that seemed largely beyond my control. Since I could not seem to overcome this problem by skill, I decided to attempt to surmount it with science.

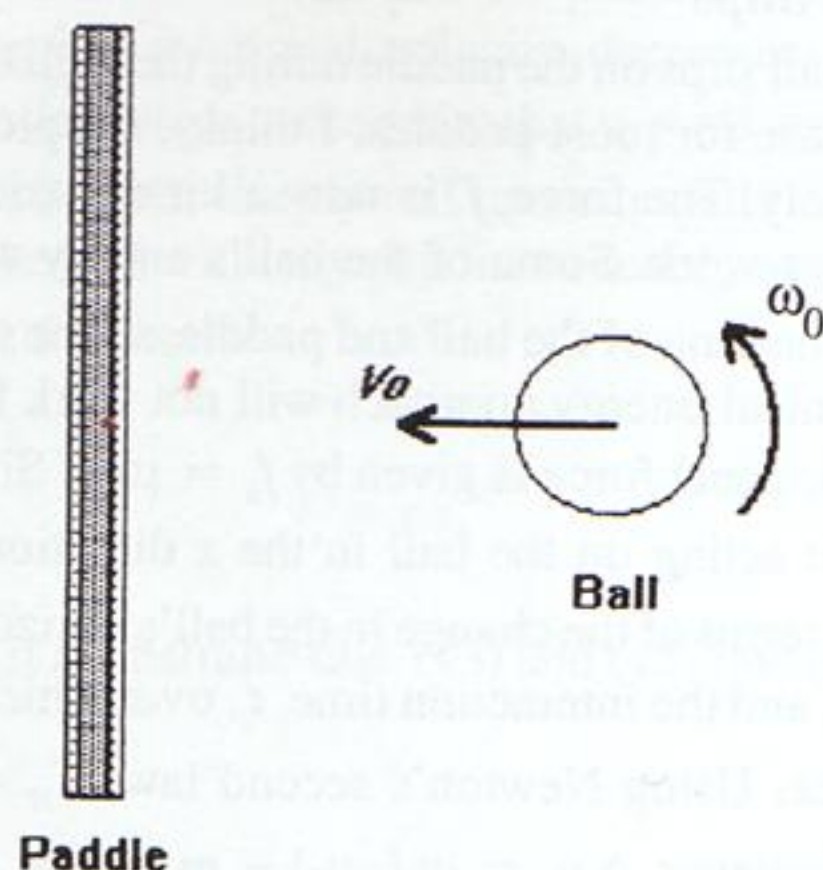
A number of questions came to mind while watching balls careen off my paddle. What was the physics behind this phenomenon? Could I use some basic physics to help me build a paddle that would be insensitive to a spinning ball? Certainly a frictionless paddle would eliminate any interaction between ball and paddle due to spin, but could I really build a paddle with low enough friction to be effectively frictionless? What could players and students learn from seeing physics applied in such a tangible way?

Using basic physics to analyze a simple ball-paddle collision problems, I came up with some interesting answers to these questions. The analysis was done at a level that my students could comprehend and includes some simplifications, especially in treating certain parts of the ball-paddle collision as perfectly elastic. However, the simplifications are not too much more sweeping than the ones typically made in introductory calculus physics laboratories and classes. I'm sure my answers are not perfect, but I believe they are accurate enough to give both me and my students some insight into the physics of Ping-Pong balls and paddles. Furthermore, as you will see at the end of this article, with the help of physics I have devised a paddle that improves my game!

The Ball-Paddle Collision

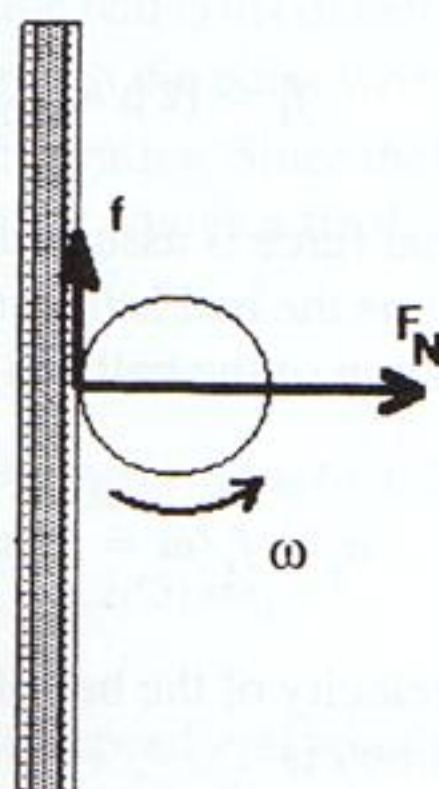
I considered what would happen if a ball rotating counterclockwise with angular velocity ω_0 and moving horizontally to the left with speed v_0 approached a rigidly held perpendicular paddle (see Fig. 1). This is pretty much what occurs when a player tries to simply "block back" a shot.

The ball collides with the paddle and the two are in contact and interact for a time t_i . During that interaction time only two forces act on the ball. The first is the normal force the paddle exerts on the ball, F_N . The second is the frictional force between ball and paddle, f (Fig. 2). The two forces are perpendicular to one another.



Paddle

Fig. 1. Ball approaching a fixed paddle. The ball is moving perpendicular to the paddle with velocity v_0 and angular velocity ω_0 .



Paddle

Fig. 2. When the rotating ball is in contact with the paddle, two forces act on it: the normal force F_N and the force of friction f .

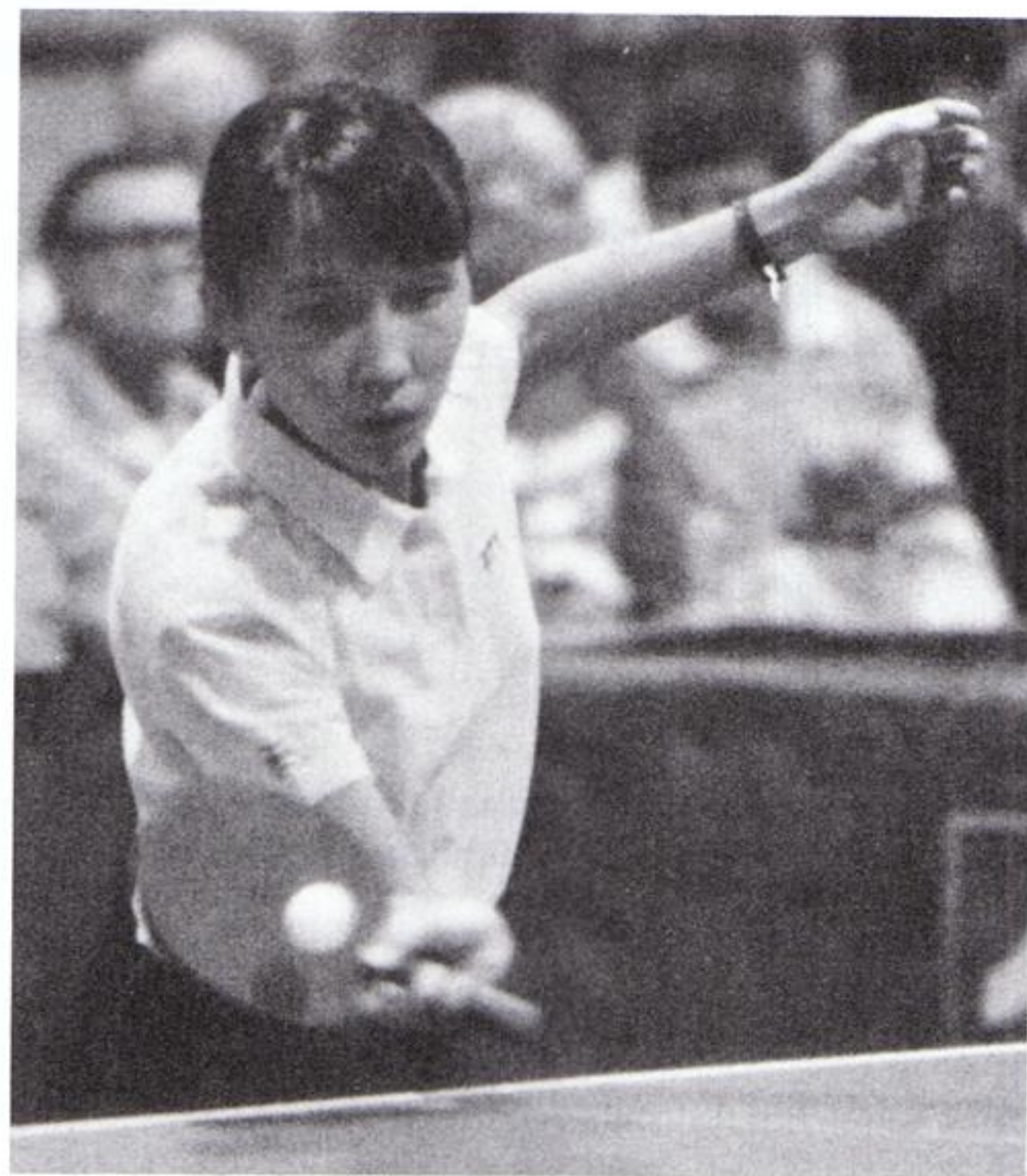
F_N is an elastic force brought about via deformation of the paddle and ball. The ball's motion in the x direction should then correspond to an elastic collision with a very massive object (the firmly held paddle). The ball's horizontal velocity component after the collision v_{fx} is then

$$v_{fx} = -v_0 \quad (1)$$

However, f will impart a y -component of velocity v_{fy} to the ball after the collision, causing the ball to rebound back at an angle θ and with a final velocity v_f that is greater than v_0 (Fig. 3). It is just this θ and v_f that makes the ball so hard to control when it comes across the net with a spin on it.

No Slipping

Upon coming into contact with the paddle, the ball can either slip against the paddle (thus losing energy to work done



Wang Wei playing in the finals at the U.S. National Championship for Table Tennis in Las Vegas, Nevada in December 1993. Photo courtesy of Robert Compton.

by friction) or grip on the paddle and not slip (and not lose energy).

The nonslipping case (if it actually exists) corresponds to a very tacky paddle. If the ball does not slip, then f is a static frictional force f_s and does no work. The forces that act to slow the ball's rotation and accelerate it upward are assumed to be elastic in nature, so mechanical energy is conserved in the ball-paddle interaction: $E_i = E_f$.

The ball will not move vertically much while in contact with the paddle, so I will ignore gravitational potential energy and only consider the ball's translational and rotational kinetic energies:

$$\frac{1}{2} m v_0^2 + \frac{1}{2} I \omega_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 \quad (2)$$

$$m v_0^2 + I \omega_0^2 = m (v_{fx}^2 + v_{fy}^2) + I \omega_f^2$$

$$m v_0^2 + I \omega_0^2 = m (-v_0)^2 + m v_{fy}^2 + I \omega_f^2$$

$$I \omega_0^2 = m v_{fy}^2 + I \omega_f^2$$

The moment of inertia about a central axis of a spherical shell (a Ping-Pong ball) of radius r and mass m is

$$I = \frac{2}{3} m r^2 \quad (3)$$

and if there is no slipping then

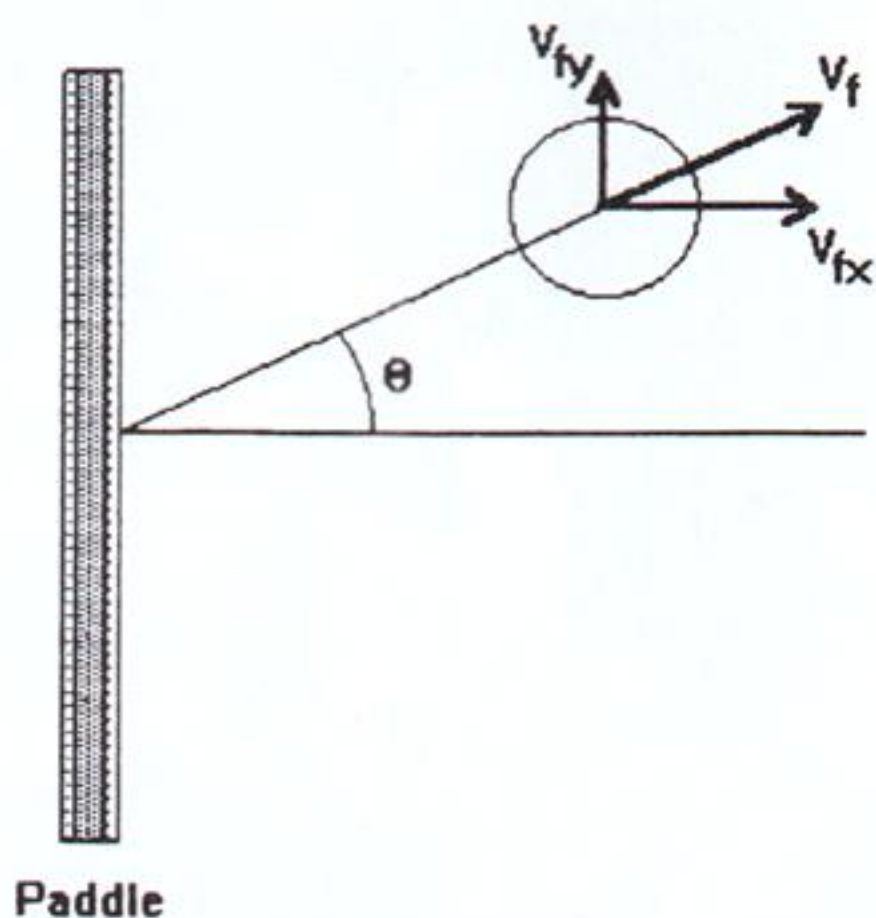


Fig. 3. The ball leaves the paddle with velocity v_f (shown here with components v_{fx} and v_{fy}) at an angle θ from its original path.

$$v_{fy} = r\omega_f \quad (4)$$

Substituting Eqs. (3) and (4) into (2):

$$(2/3 mr^2) \omega_0^2 = mv_{fy}^2 + (2/3 mr^2) (v_{fy}/r)^2 \quad (5)$$

$$v_{fy} = \sqrt{(2/5)} r\omega_0$$

From Fig. 3:

$$\tan(\theta) = v_{fy}/v_{fx} \quad (6)$$

$$v_f = \sqrt{(v_{fx}^2 + v_{fy}^2)} \quad (7)$$

so

$$\theta = \arctan[\sqrt{(2/5)} (r\omega_0)/v_0] \quad (8)$$

$$v_f = \sqrt{(v_0^2 + (2/5) r^2 \omega_0^2)} \quad (9)$$

From Eq. (8) it is clear that the more spin the ball has the greater the deflection of the ball, and the more speed it has the smaller its deflection. A slow-moving, fast-spinning ball will come off the paddle at the greatest angle.

What's more, Eq. (9) says that the ball comes off the paddle with increased speed since the collision converts some of the ball's rotational kinetic energy into translational kinetic energy. The greatest fractional change in velocity will occur for, again, a slow-moving, fast-spinning ball.

My experience using paddles with "good" tacky rubber bears these results out; slow-moving, fast-spinning balls seem to literally jump off the paddle and are very hard to control.

The Ball Slips

If the ball slips on the paddle during the collision (the more realistic case for most paddles, I think), the picture changes considerably. The force, f , is now a kinetic frictional force, f_k , and does work. Some of the ball's energy will be lost to frictional heating of the ball and paddle, so the conservation-of-mechanical-energy approach will not work for this case.

The frictional force is given by $f_k = \mu F_N$. Since F_N is the only force acting on the ball in the x direction, F_N can be written in terms of the change in the ball's horizontal momentum, Δp_x , and the interaction time, t_i , over which this change takes place. Using Newton's second law: $F_N = \Delta p_x / \Delta t = \Delta p_x / t_i$. However, $\Delta p_x = m(-v_0) - m(v_0) = -2mv_0$ so

$$|F_N| = (2mv_0)/t_i \quad (10)$$

The average frictional force can now be written

$$f_k = (2\mu mv_0)/t_i \quad (11)$$

If the frictional force is assumed to be equal to this value over the entire time the ball is in contact with the paddle, the vertical acceleration of the ball can be written

$$a_y = f_k/m = (2\mu v_0)/t_i \quad (12)$$

so the vertical velocity of the ball after being in contact with the paddle for time t is

$$v_y = a_y t = (2\mu v_0 t)/t_i \quad (13)$$

The friction also exerts a torque to slow the ball's rotation via $\tau = I\alpha$. Since $\tau = -f_k r$ (negative since the torque acts to slow the ball's rotation) and I is known from Eq. (3), I can solve for the ball's angular acceleration α :

$$-f_k r = (2/3 mr^2) \alpha \quad (14)$$

$$-2\mu mv_0 r/t_i = (2/3 mr^2) \alpha$$

$$\alpha = -3\mu v_0/(r t_i)$$

Therefore, the angular velocity ω of the ball after time t will be

$$\omega = \omega_0 + \alpha t \quad (15)$$

$$\omega = \omega_0 - (3\mu v_0 t)/(r t_i)$$

From the moment the ball hits the paddle, it accelerates upward while its rotational velocity decreases. If v_y increases and ω decreases to the point that $v_y = r\omega$, the ball will stop slipping and will simply roll up the paddle at constant speed. Thus the ball will only slip for a time, t_s , which can be determined as follows:

$$v_y = r\omega \quad (16)$$

when $t = t_s$ so if I substitute Eqs. (13) and (15) into (16) using $t = t_s$ I have

$$(2\mu v_0 t_s)/t_i = r[\omega_0 - (3\mu v_0 t_s)/(r t_i)] \quad (17)$$

$$t_s = r\omega_0 t_i / (5\mu v_0)$$

At this point I see that if $v_0 > r\omega_0/5\mu$, then the slip time will be less than the time the ball is in contact with the paddle ($t_s < t_i$), and the ball will reach the point where it rolls up the paddle with no further acceleration. Since the ball has upward acceleration a_y for time t_s , it attains a final upward velocity of

$$v_{fy} = a_y t_s = (2\mu v_0/t_i) \cdot (r\omega_0 t_i)/(5\mu v_0) \quad (18)$$

$$v_{fy} = (2/5) r\omega_0$$

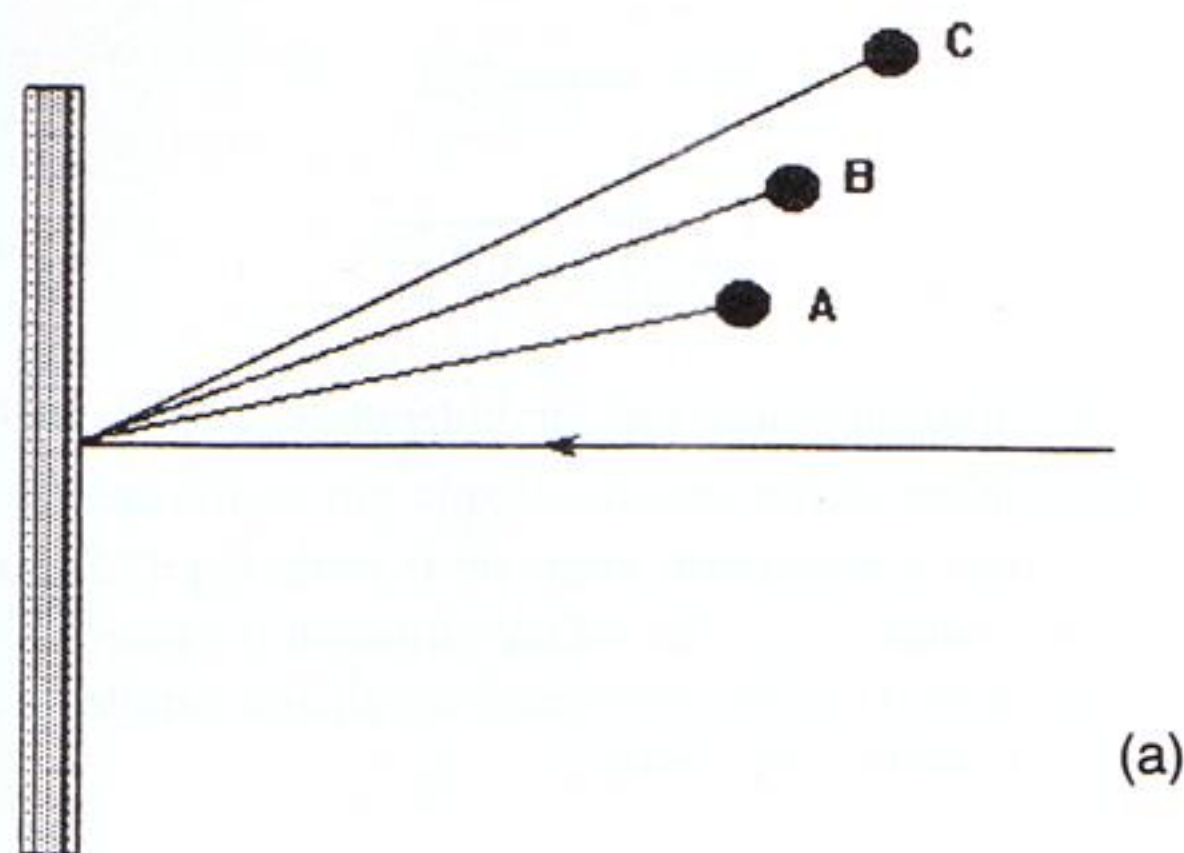
Once again, the angle θ and speed v_f at which the ball comes off the paddle are found from Eqs. (6) and (7) giving

$$\theta = \arctan [(2/5) r\omega_0/v_0] \quad (19)$$

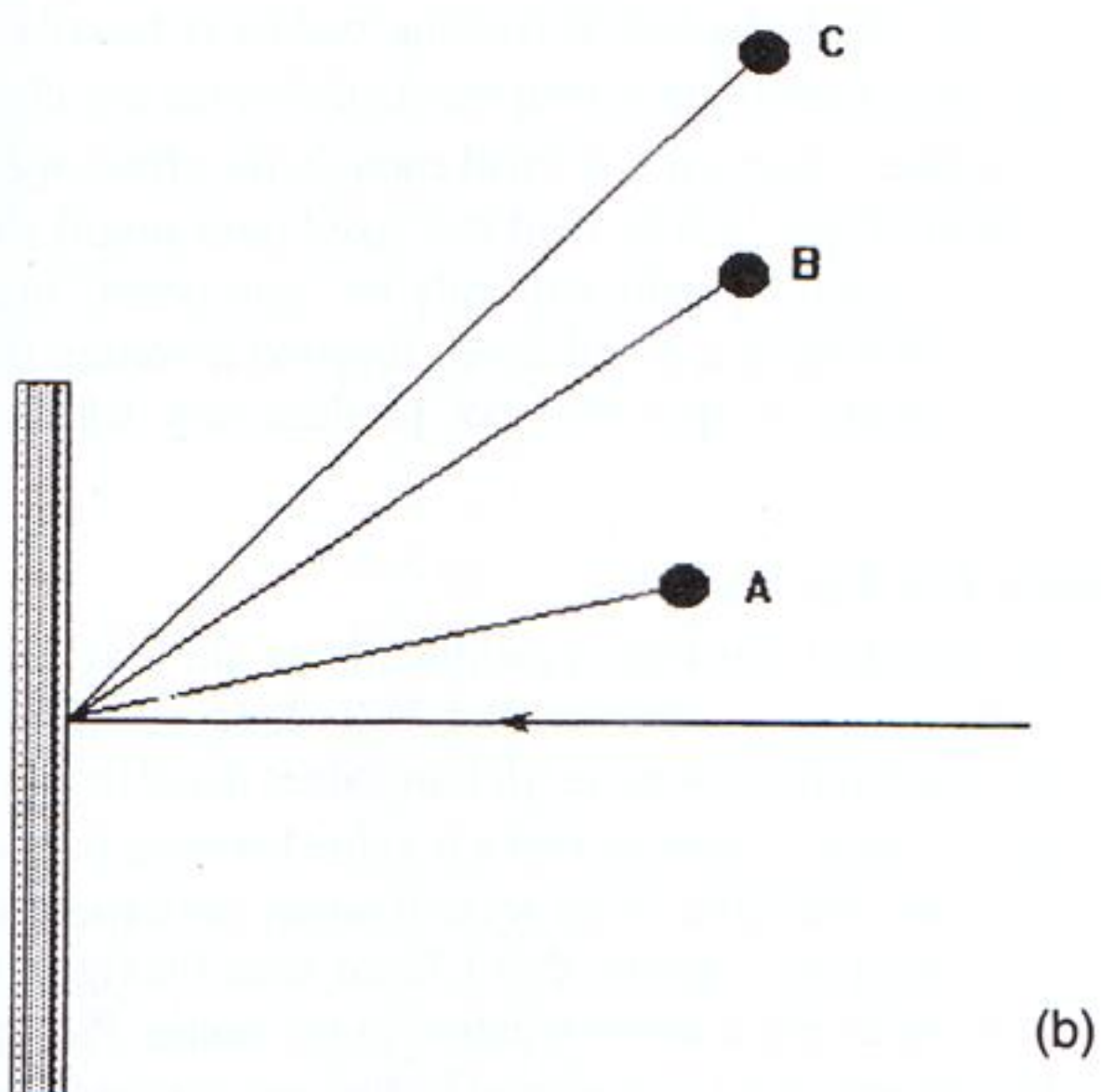
$$v_f = \sqrt{v_0^2 + (4/25) r^2 \omega_0^2} \quad (20)$$

Clearly, since the $\sqrt{(2/5)} \approx 0.632$ term present in the “no slip” case is replaced by $2/5 = 0.400$ in this case, the ball will come off the paddle at a less radical angle and with less speed if the ball slips on the paddle than if it does not for any combination of speed and spin. If the assumption that the frictional force is equal to its average value is abandoned, Eq. (17) becomes questionable but Eqs. (18) through (20) can still be obtained. For certain values of v_0 and ω_0 the slip time will be less than the time the ball is in contact with the paddle ($t_s < t_i$), and the ball will reach the point where it rolls up the paddle with no further acceleration.

However, what is more interesting is to consider the case where this never occurs [$v_0 \leq r\omega_0/5\mu$ using Eq. (17)]. This is the case of a ball that is moving relatively slowly and spinning relatively rapidly—the very ball that I found to be



(a)



(b)

Fig. 4. A ball with a given speed and spin will leave different paddles at different angles and with different speeds. Paddle A is a low-friction paddle. Paddle B is a standard rubber paddle. Paddle C is a tacky (no-slip) paddle. A ball moving slowly with significant spin will be deflected less by A than by either B or C since its maximum deflection for that particular combination of speed and spin is quickly reached (Fig. 4a). Furthermore, if still more spin is placed on the ball while its speed remains the same, there will be no further deflection off paddle A, whereas the ball will deflect even further off B (up to its own maximum deflection—a much greater value) and C (deflection limited only by the amount of spin an opponent can place on the ball; see Fig. 4b).

the most difficult to control. In this case $t_s \geq t_i$ so the ball slips (and therefore accelerates upward) the entire time it is in contact with the paddle. The ball's final upward velocity in this case is

$$v_{fy} = a_y t_i = (2\mu v_0/t_i) \cdot t_i \quad (21)$$

$$v_{fy} = 2\mu v_0$$

So I see that in this case θ and v_f are given by

$$\theta = \arctan(2\mu) \quad (22)$$

$$v_f = \sqrt{v_0^2 + 4\mu^2 v_0^2} \quad (23)$$

These are limiting values that are independent of ω_0 and thus are independent of the amount of spin put on the ball. Equation (22) puts a maximum value on θ while Eq. (23) puts a maximum value on v_f . The whole situation is controlled by the coefficient of friction between the ball and paddle, something that is within my control.

What It Means

From this analysis it seems that the greatest advantage of a slippery paddle is that the maximum deflection angle and speed with which the ball leaves the paddle is heavily dependent on μ , something within the control of the one choosing the paddle. Clearly if μ is small enough, the effect of even a great deal of spin will be held to a fixed (and small) value (see Fig. 4). A slick paddle will truly be "spin-proof" in that for a ball moving at a given speed, beyond a certain point greater amounts of spin will not produce any additional effect.

Putting It All to Work

I proceeded to construct a paddle that would have a very low coefficient of friction with a ball. Judging from the various coefficients of kinetic friction values listed in numerous physics texts, it seemed that a μ value between 0.05 and 0.1 should be attainable. After some thought and experimentation, I settled on a paddle that I faced with Plexiglas left over from replacing a storm window in my home. Plexiglas is pretty slippery, but I was able to further reduce the friction between the Plexiglas and the ball by rubbing the acrylic plastic surface with car polish followed by plastic protectorant. A few table-top experiments have convinced me that the coefficient of friction between the paddle face and the ball is now roughly 0.09. The coefficient of friction between a regular rubber paddle and a ball is probably an order of

magnitude greater than this. The maximum angle at which a ball that struck this paddle perpendicularly could be deflected would be significantly less than that of a ball being deflected off a rubber paddle. I was confident the paddle would be effective.

The paddle is *very* effective and its overall behavior is as predicted. The effect of spin on the paddle is minimal. Balls I was hardly able to keep on the table I can now return with accuracy limited primarily by the tendency of spinning balls to curve. An unexpected side benefit is that since the paddle is "spin-proof," any spin an opponent puts on the ball is returned to him, and often in a manner that is contrary to the usual dynamics of the game. The typical pattern is now that if an opponent serves up a slow, high-spin serve, I "dink" the ball back so that the ball comes back slowly and with a lot of the spin he put on it still there, and my opponent, confounded by his own spin, puts the ball in the net! The dependence on the coefficient of friction is also very apparent. As the polished surface degrades under use, the paddle becomes noticeably less effective. I have to wax the paddle every time I play.

The Students' Reaction

When I told those who play "Pong" what I was doing, quite a few—including several of my calculus physics students—proclaimed it would not work, even though they were able to follow the physics of this problem. They were convinced only when their efforts to overcome my paddle by simply putting still more spin on the ball produced no results.

The success of the Plexiglas paddle has been a clear demonstration that physics can be applied to real-world situations. I have made the point that this is nothing but "sports physics"—no different than designing a golf club or a running shoe. The paddle has made a lasting impression and I hope that this "out-of-class demonstration" will remind them what physics is all about for some time to come.

As for my Ping-Pong game, it has been helped to the point that I at least am no longer humiliated by my own students. I am hoping to create a "next generation" paddle using a surface that is even more slippery than polished Plexiglas, but so far I haven't found one.

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