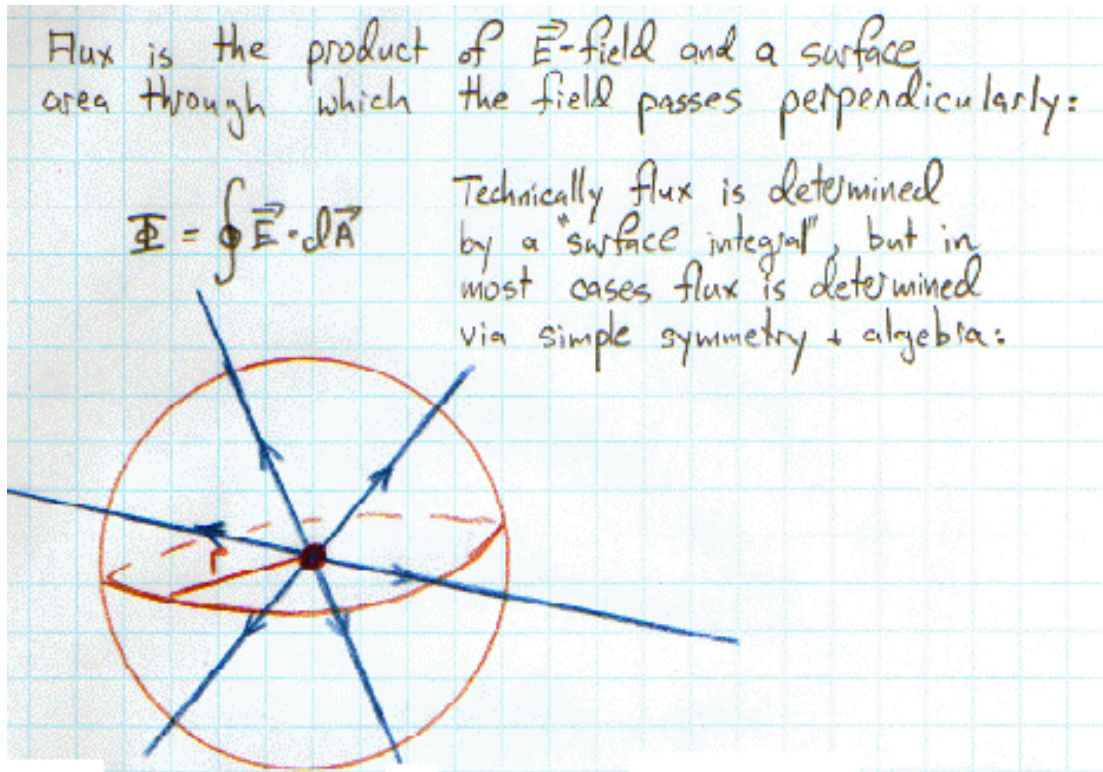


## DAY 9

### Summary of Topics Covered in Today's Lecture

#### Flux and Gauss's Law



This is a spherically symmetric field. The electric flux here is

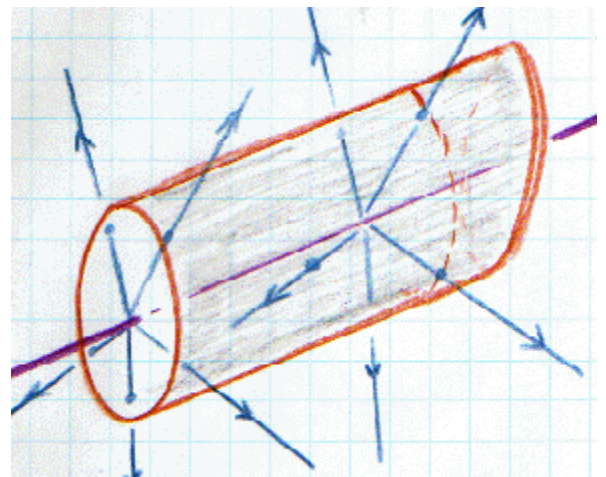
$$\Phi = \oint \vec{E} \cdot d\vec{A} = "EA_{\perp}" = E(4\pi r^2)$$

Here the surface that is always perpendicular to the field is a sphere and the surface area of a sphere is

$$A = 4\pi r^2$$

In the case of an E-field that is radially symmetric, the surface that is always perpendicular to the field is the walls of a cylinder:

$$\Phi = "EA_{\perp}" = E(2\pi rl)$$



The area of the walls of a cylinder is

$$A = 2 \pi r l$$

for a cylinder of length  $l$  and radius  $r$ . The ends of the cylinder are unimportant because the field does not pass through them - there is no field perpendicular to the surface on the ends so there is no flux at the ends.

Gauss's Law can be used to determine the charge that creates a flux. Gauss's Law says

$$\Phi = 4\pi k q_{\text{enclosed}}$$

where  $q_{\text{enclosed}}$  is the charged enclosed by the "Gaussian Surface" (the sphere, cylinder, etc.) and  $k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$ , the Coulomb Constant we've used before.

Gausses Law can also be written as

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Here  $\epsilon_0 = 1/(4\pi k)$ . It is sometimes referred to as the "Permittivity of Free Space".

### **Voltaic Cells & Batteries (and Frankenstein)**

Our modern world owes much of what it is to the physics of electricity and magnetism. Without that physics we would have no electric lights, no electric motors, no computers, no phones - nothing that operates on electricity. While people had known about electrical phenomena (such as "static cling") since ancient times, putting electricity to practical use required that it be possible to generate a reliable flow of charge - not just a build-up of charge. The first device to be able to generate charge this way was the "Voltaic cell" - which in modern form is your typical "Energizer" battery.



Voltaic cell technology can be traced to Luigi Galvani, an Italian professor of anatomy at the

University of Bologna in the late 1700s.\* One day in 1786, Galvani was researching a dissected frog in his laboratory as an electrical storm raged outside. To his surprise, the frog's leg muscle twitched whenever his scissors touched a nerve. This made him wonder if the lightning could have exerted some influence on the frog's nerves and muscles. That same year, during the course of another experiment, one of Galvani's assistants casually touched the lumbar nerve of a dissected frog with a scalpel. The frog's legs kicked. On this occasion there was no electric storm, but Galvani's wife pointed out that an electrostatic generator was turned on in another part of the laboratory. Like the lightning, could the electrostatic generator have affected the frog through the air?

Intrigued by these coincidences, Galvani embarked on a new series of experiments, coming to an astounding conclusion: electrical energy was intrinsic to biological matter. He argued that the metal scissors and scalpel served as conductors that moved electricity from the nerve to the muscle. Realizing the momentous import of any conclusion that electricity was the "vital force" of life, Galvani was cautious about releasing his results without conducting more research. In 1791, he published *De viribus electricitatis in motu musculari commentarius*, announcing to the world that electricity was an innate force of life. Some pictures from *De viribus* are shown on the next page.

Europe's intellectuals heralded Galvani's work as a great achievement. They were abuzz with excitement at his theory on the relationship of electricity to life.

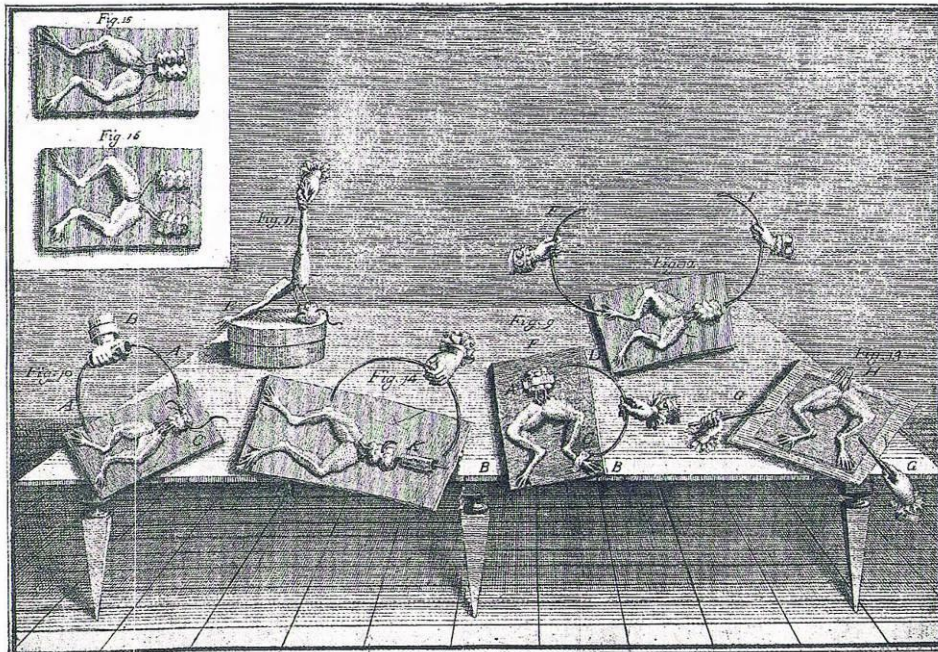
Alessandro Volta, Galvani's contemporary and a physics professor at the University of Pavia, believed that Galvani's work "contain[ed] one of the most beautiful and most surprising discoveries," but his own experimentation led him to a different interpretation of Galvani's experiments. For Volta, the electrical phenomenon that Galvani observed arose from the action of dissimilar metals, not an internal property of life. In the end, Volta's view prevailed and opened the science and technology of electrolytic action and batteries, but Galvani was not completely wrong; just think of the electrical activity of

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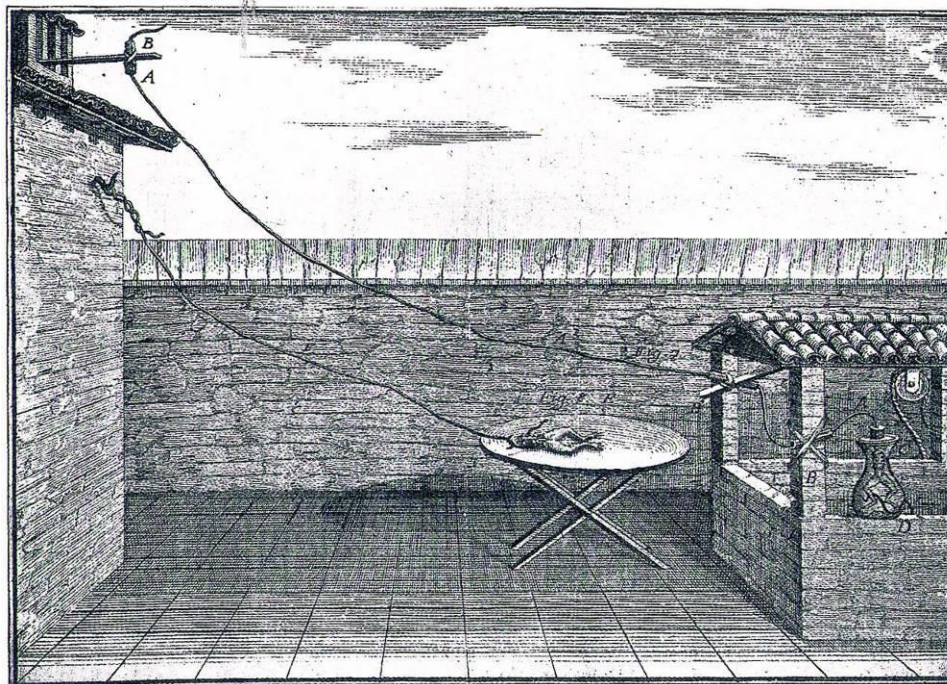
\* Material on Galvani & Volta is taken, with some modifications, from IEEE-USA's "Engineering Online --Engineering and Pop Culture: Galvani and the Story of Frankenstein" (<http://www.todayseengineer.org/2004/Apr/history.asp>) and the "Exhibition for the Italian Institutes of Culture Abroad -- The Legacy of Galvani and Volta in contemporary science" (<http://www.bo.infn.it/galvani/cultura-estero/latin-america/pannelli/a4.html>)



the heart and brain and of course the fact that muscles do respond to electrical impulses in general.

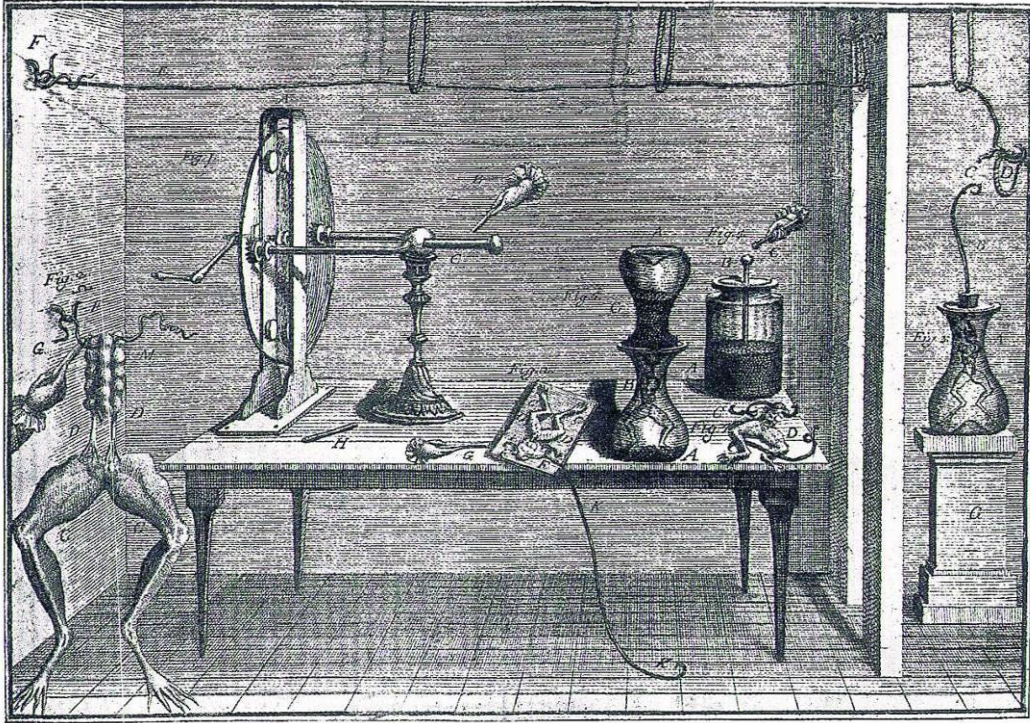


Details of frogs "prepared in the usual way" for the experiments, and of the metallic arcs used to produce contractions.

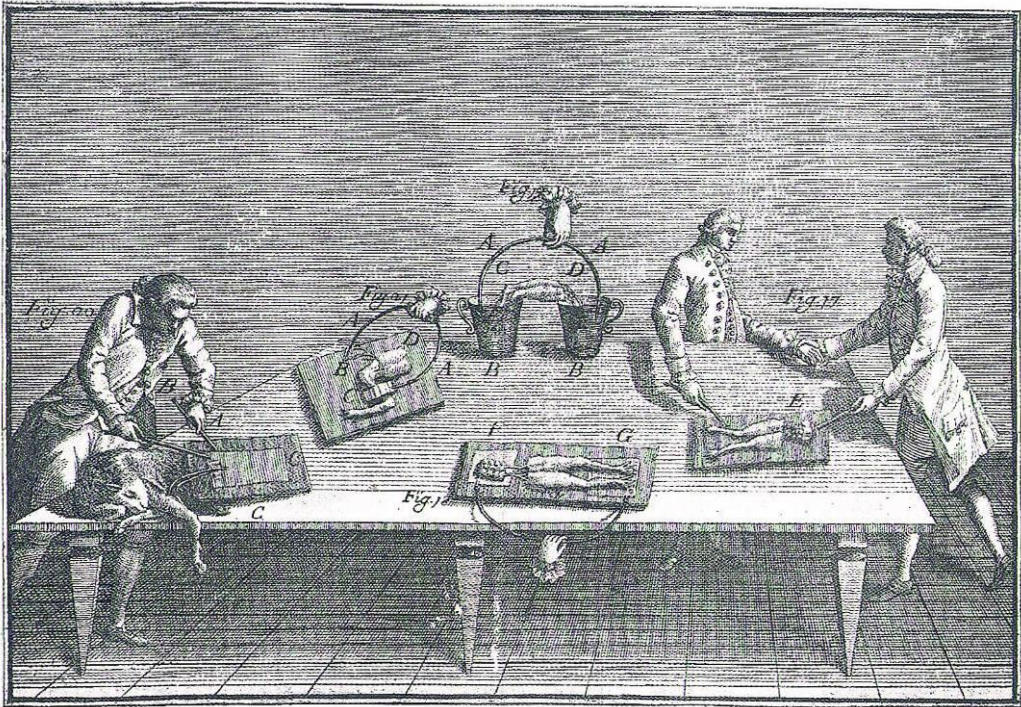


Galvani's "atmospheric discharge detector": the half frog, with the nerves connected to the lightning-conductor and the muscles connected to the water of the well, acts as a "coherer antelitteram", a device that detects electrical action.





Frog "prepared in the usual way" for experiments with electrical equipment.

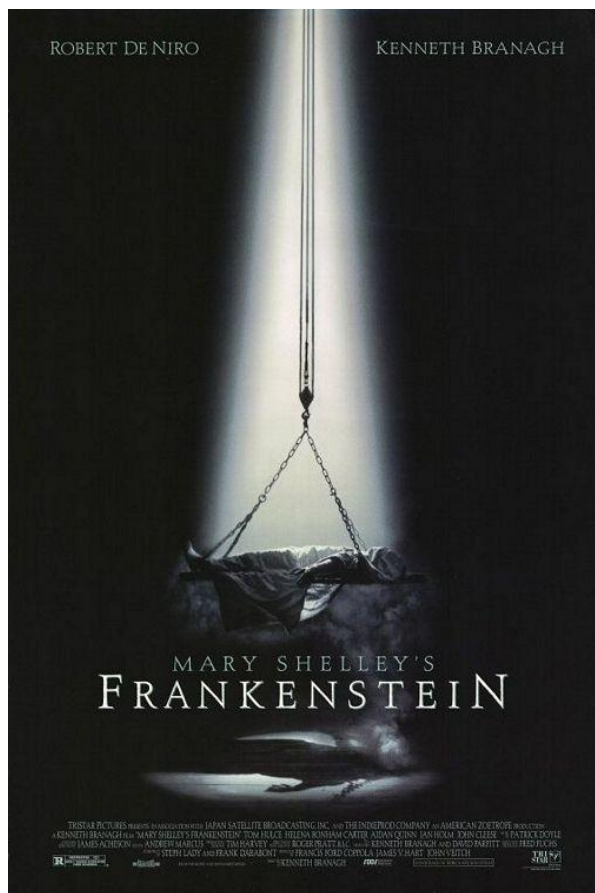


In Galvani's laboratory.



Although Volta's ideas supplanted Galvani's theory of animal electricity within the scientific community, the notion of electricity and reanimation still lingered in the air when Mary Shelley penned her novel *Frankenstein*. Discussions between Shelley's husband and Lord Byron on Luigi Galvani's experiments and how Galvanism suggested the possibility of reanimating the dead led Shelly to explore, in fiction, the moral and personal responsibilities and the dilemmas of scientific advance.

Because Voltaic cells and batteries are essentially a chemical phenomenon - the conversion of chemical energy to electrical energy - I'm really out of my league in discussing them. Below are some selections from web sources on the topics of batteries to which I will refer you, as you probably know as much chemistry as I do.



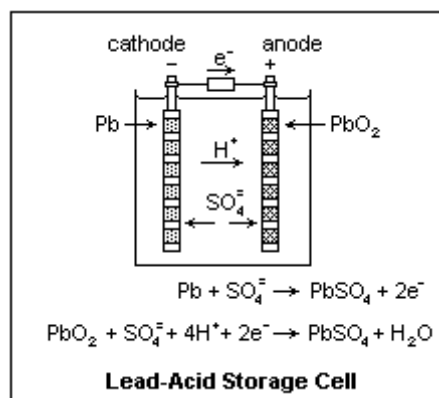
**The Lead-Acid Battery (from the web pages of Dr. J. Calvert, University of Denver -- <http://www.du.edu/~jcalvert/> )**

Gaston Planté (1834-1889) was experimenting with electrolysis in the 1850's. This was before the introduction of efficient dynamos, so all direct current electricity came from primary cells. He put two lead plates in dilute sulphuric acid, and passed a current between them. At one plate, the one connected with the positive pole of his battery, oxygen was evolved, and at the other plate hydrogen was evolved. This was no surprise, since water had been electrolyzed for over half a century at the time. What was a surprise was that when he had allowed the current to pass for some time and then disconnected the battery, the lead plates acted like a battery on their own, something that had not been observed before. The plate that had been connected to the (+) pole of the battery, the *anode*, was at a higher potential than the plate that had been connected to the (-) pole of the battery, the *cathode*, and now current flowed in the opposite direction. The lead plates were observed to collect a layer of a white substance, lead sulphate, as this happened. When the current finally stopped, the cycle could be repeated by reconnecting the power source. Planté had discovered a *storage battery*. He announced his discovery in 1859, but it attracted little interest, since there was no demand for such a device.

By 1880, there was a demand. Edison and Swan had devised practical incandescent lamps, and Edison was promoting a complete system, from dynamos to lamps, based on direct current distribution. If there were a practical storage battery, then the battery could be floated across the lines to supply power when the dynamos were shut down, at times of low demand or for repairs. Power could be produced when there was excess generating capacity, and used to bridge over times of unusual demand. Camille Faure looked at the Planté battery, and overcame its principal faults--the lack of capacity, high internal impedance and the long "forming" process necessary to prepare the plates. The cathode was made of spongy lead for lower internal resistance, and the positive plate was consisted of pockets filled with red lead, which was converted more easily to the active ingredient,  $\text{PbO}_2$ , in the forming process. This battery was much more practical, and soon commanded a good market.

Edison tried to improve the Faure battery, extending its life and reducing its weight, looking toward applications in electric vehicles. The result was the Edison alkaline cell, in which the life was extended, and the battery rendered much more rugged. However, the weight problem was not solved, and is not solved to the present day. The lead-acid battery is still the best secondary battery in most heavy-duty applications. Electric street railways operated by battery cars were actually put into service. The batteries were to be charged overnight, when the cars were not in service, and their power used during the day, obviating the need for overhead conductors. There were also battery cars and trucks for road service. The expense and weight of the batteries made such use uneconomic, especially after the invention of the self-starter made internal combustion engines available to all. Of course, at the same time this led to the use of a lead-acid battery in each vehicle for starting and for maintaining the electricity supply with the engine idling or stopped.

The present lead-acid cell consists, in a state of full charge, of a negative plate, or cathode, of spongy lead in a grid of hard lead, a positive plate, or anode, of  $\text{PbO}_2$  paste in a grid of hard lead, and an electrolyte of dilute sulphuric acid of specific gravity 1.28. This is a 37% solution, with 472.5 g/l of  $\text{H}_2\text{SO}_4$ . At full discharge, the electrolyte is of specific gravity 1.05, an 8% solution containing 84.18 g/l of acid. Both plates are coated with  $\text{PbSO}_4$ . Approximately 4 moles of acid are used per litre, which corresponds to 213 A-h of charge (an ampere-hour is a current of one ampere flowing for one hour, or 3600 coulomb). Assuming that 4 moles of Pb are reacted at the cathode, and 4 moles of  $\text{PbO}_2$  at the anode, the total weight of active materials is about 3 kg. This gives a weight-to-capacity ratio of 14 g/A-h. Of course, this is much lower than is required for a practical battery, with case, electrode grids and other necessities. However, a limit of perhaps 25 g/A-h represents the maximum that can be expected of a lead-acid battery, and a limit of about 200 A-h per litre of electrolyte volume.



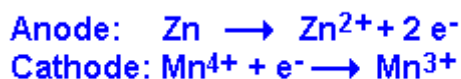
The cathode reaction is  $\text{Pb} + \text{SO}_4^{2-} \rightarrow \text{PbSO}_4 + 2\text{e}^-$ . For each atom of lead, two electrons pass through the external circuit when the cell is delivering current. At the anode, the reaction is  $\text{PbO}_2 + 4\text{H}^+ + 2\text{SO}_4^{2-} + 2\text{e}^- \rightarrow \text{PbSO}_4 + 2\text{H}_2\text{O}$ . This reaction uses the two electrons sent by the cathode through the external circuit. For each two electrons, two molecules of acid are turned into two molecules of water and two molecules of lead sulphate. The electrode potential of the cathode reaction is -0.355V, and the electrode potential of the anode reaction is 1.685V, at standard concentrations. The net potential difference is  $1.685 - (-0.355) = 2.040\text{V}$ . At the concentrations in a fully charged battery, the potential difference is closer to 2.2V, decreasing to 2.0V for a fully discharged battery.

It is easy to measure the specific gravity of the electrolyte with a hydrometer, and this gives an accurate estimate of the state of charge of the battery. This is one of the great advantages of the lead-acid cell. Note that the electrode reactions do not show any evolution of gases. With open cells, there is in fact some emission of H<sub>2</sub> and O<sub>2</sub>, so the water lost in this way must be replenished regularly. This was once a regular duty in servicing a car, but modern batteries require very little care, and some are sealed, venting gas only when necessary. Also, ventilation was necessary to prevent the hydrogen from becoming an explosion hazard. When ordinary car batteries are charged rapidly, water is electrolyzed.

The lead in batteries is easily reclaimed, and secondary lead is an important source of the metal. There is even a special blast furnace that accepts a charge of scrap batteries, from which metallic lead can then be tapped. It is necessary to handle the impurities involved, coming from the battery cases and the wooden separators between the plates. These components may be burned to provide the fuel for the process. The contrast with tires, that just pile up in unattractive mountains with little value or use, and sometimes smokily catch fire, is notable.

**Dry Cells, Lead Acid Cells, and Fuel Cells (from the web site of PHYSCHEM Training Corp. -- <http://www.physchem.co.za/>)**

The *dry cell*, invented in 1867 by the French engineer Georges Leclanché (1839 - 1889), is widely used as a source of electric energy in electric torches and small appliances such as transistor radios. It makes use of the two reactions, which (in a simplified form) may be described as



The manganese is supplied as manganese dioxide, and the actual cathode reaction taking place is



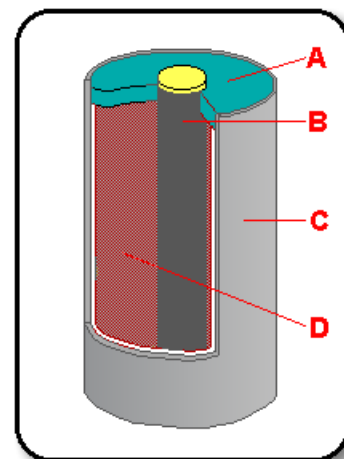
The H<sup>+</sup> ions are in turn provided by ammonium ions NH<sub>4</sub><sup>+</sup>, through the reaction



The dry cell is an example of a primary cell, as once it is discharged it cannot be recharged, and must be discarded.

A typical dry cell is shown below, with a section cut away in order to expose the interior parts.

The top of the battery is closed with a non-conducting sealing material (A). The cathode consists of a graphite (carbon) rod (B) (tipped with a



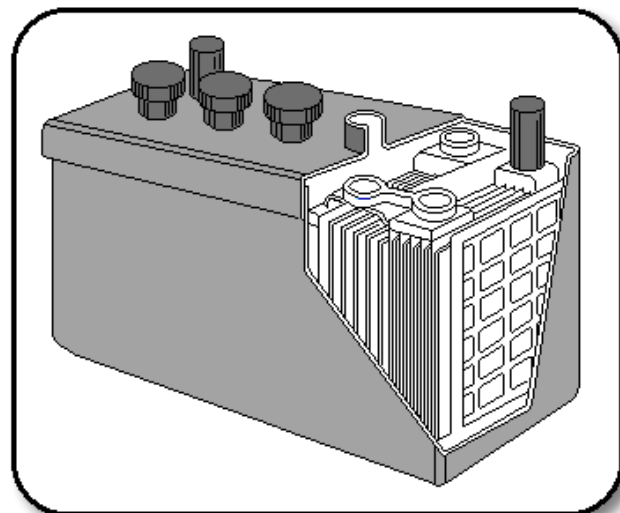
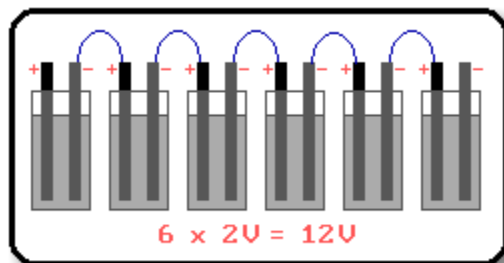


metal contact), which serves as the positive pole of the battery. The anode is a cylindrical zinc casing (C) (the bottom of the battery is normally exposed and serves as the negative pole). The battery is filled with a mixture of manganese dioxide ( $\text{MnO}_2$ ) as oxidant, ammonium chloride ( $\text{NH}_4\text{Cl}$ ) as a source of  $\text{H}^+$  ions, and zinc chloride ( $\text{ZnCl}_2$ ) (D). These two salts serve as electrolytes.

The above mixture is separated from the zinc walls of the battery by a porous material soaked in a solution of the two salts. The cell produces a potential of about 1.5 V.

In 1859, the French physicist Gaston Planté (1834 - 1889) invented a device called the *lead-acid accumulator*, which, with minor design changes, is still used today as the motor car battery. It is an example of a secondary electrochemical cell, since it can be recharged.

The battery consists of 6 cells connected in series, each cell producing a potential of about 2 V, giving 12 V as the potential difference between the terminals of the battery.



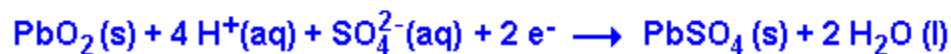
The general appearance of the battery is shown on the right, with part of its casing removed to show the individual cells.

When the battery is discharging, i.e., when it is supplying a current, the reactions are shown below.

At the anode:



At the cathode:

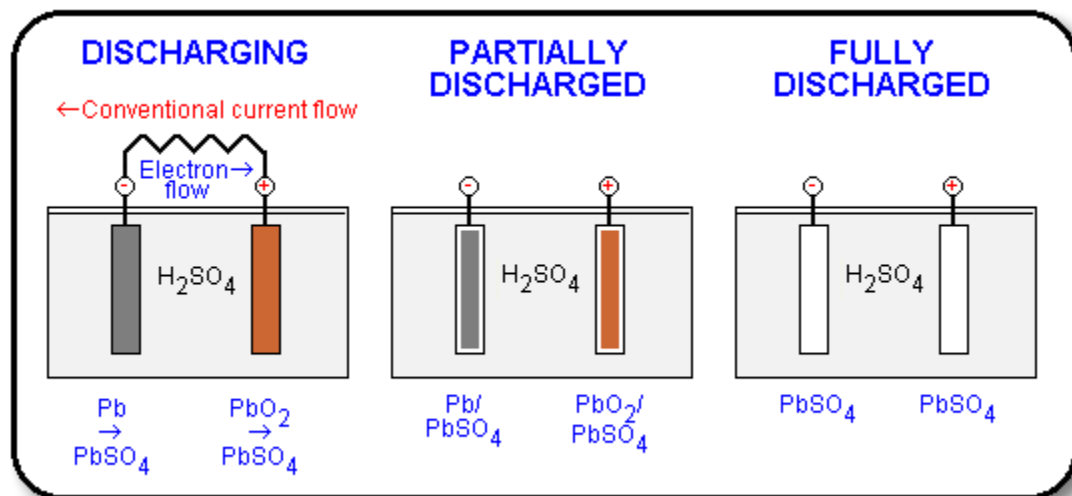


The overall reaction (during discharge) is therefore

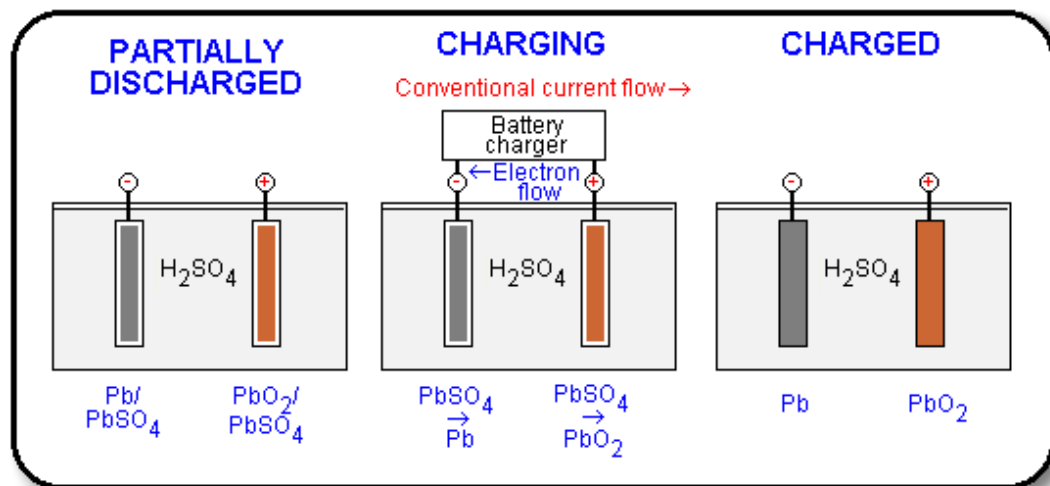


Note that the electrolyte is sulfuric acid (H<sub>2</sub>SO<sub>4</sub>), which is gradually used up. Both electrodes become coated with an insoluble layer of lead sulfate, which, being an insulator, would eventually ruin the battery.

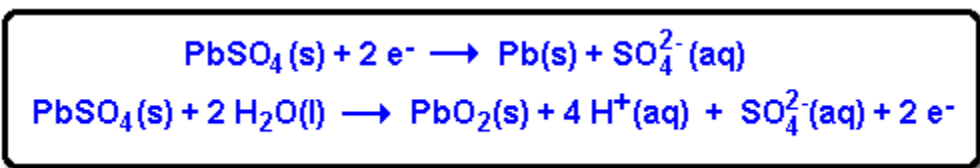
As the battery provides energy, it is discharged and this leads to the gradual formation of lead sulfate at the electrodes, and a steady decrease in the concentration of the sulfuric acid:



When the battery is being charged, (using a battery charger or the motor car's alternator), electrons are supplied to the anode of the battery and this converts the lead sulfate to lead. At the same time, the lead sulfate at the anode is converted to lead dioxide.



The reactions are:



The *fuel cell* converts the energy of combustion directly into electrical energy. For example, one type of such cells, used in spacecraft and advanced non-nuclear submarines, makes use of the reactions



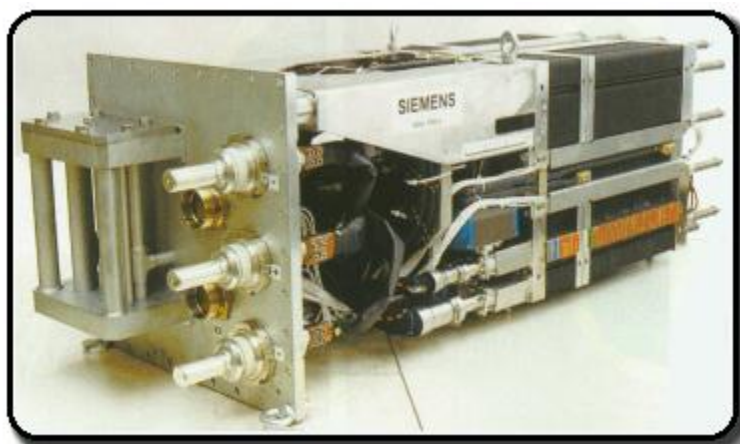
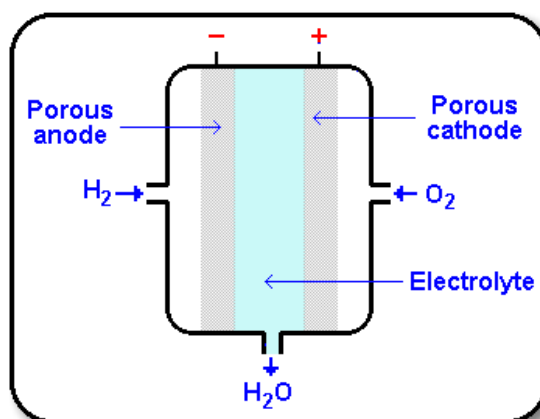
giving the overall reaction



with a theoretical potential of 1.23V. Platinum is used as a catalyst, and the only by-product, water, is utilized by the spacecraft or submarine crews.

Note that this fuel cell is not a primary cell, since neither reactants nor products are stored, or a secondary cell which can be recharged. Rather, the reactants have to be fed in continuously and the product removed.

The diagram shows the basic design of a fuel cell. In practice, it is a very complex piece of equipment, as shown in the picture below.





However, the basics for this class are as follows:

- ❑ A Voltaic cell consists of two dissimilar conductors and an electrolyte (a moist conducting solution). A copper penny, galvanized (zinc-coated) nail, and some lemon juice will make a Voltaic cell and will generate voltage.
- ❑ The amount of voltage produced by the cell depends on the composition of the cell (i.e. what the two conductors are and what the electrolyte is).
- ❑ Voltaic cells can be connected positive-to-negative (in series) to create a battery with voltage that is the sum of the cell voltages.
- ❑ The rate at which a cell of a given type can generate charge depends on things like the surface area of contact between reagents in the cell.
- ❑ How much charge a cell of a given type can generate depends on things like the amount of reagents present in the cell; a larger cell can generate more charge than a small cell. This means a D-cell will last longer in any given application than a AA-cell, even though both produce the same voltage.

### Example Problem #1

Calculate what the E-field is 1 cm from a wire that is 10 m long and has +10  $\mu\text{C}$  of charge uniformly distributed along it. Is this answer valid at any point along the wire? Explain. Can this much charge be put on a wire in air?

#### Solution:

This is a radially symmetric problem, one around which I can draw a cylinder as a Gaussian surface.

Wire length =  $L = 10\text{ m}$       Wire charge =  $Q = +10\ \mu\text{C}$   
 $r = 1\text{ cm}$

$E$

$l$       Gaussian Cylinder

$r$       WIRE

$E = 4\pi k q_{\text{enc}}$  Gauss's Law

$E(2\pi r l) = 4\pi k q_{\text{enc}}$  (I used flux for a radially symmetric field through a cylinder).

$q_{\text{enclosed}}$  is just the part of the wire's charge that is inside the Gaussian surface.

Total  $\frac{Q}{L} = \frac{q_{\text{enclosed}}}{l}$       Since the charge is uniformly spread over the wire, the charge per length should be the same everywhere

Now we have

$$E(2\pi r l) = 4\pi k (Q/L) l$$

The small 'l' and the 'π' cancels on each side...

$$E(2r) = 4k (Q/L)$$

$$E = 2kQ / (rL) = 2(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(10 \times 10^{-6} \text{ C}) / ((.01\text{ m})(10\text{ m}))$$

$$= 180 \times 10^3 \text{ Nm}^2/\text{C}/(.1\text{m}^2) = 1.8 \times 10^6 \text{ N/C}$$

This answer is valid at any point on the wire – nothing in the answer depends on the location along the wire.

This much charge could not be put on a wire in air, because air breaks down and becomes conductive for E-field strengths of about 10,000 V/cm, or 1,000,000 V/m. This E-field strength is 1,800,000 V/m one centimeter away from the wire (and it will get stronger closer to the wire).

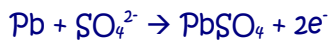
### Example Problem #2

Describe how you might estimate the amount of time a lead-acid cell could produce charge?

**Solution:**

I would need to know the mass of the lead plate. If I knew the mass of the plate, I could look up the mass of a lead atom, and calculate how many lead atoms were in the plate.

Then I know that each lead atom will yield two electrons:



So the cell can produce twice as many electrons as it has lead atoms. So I find out how many electrons does the cell have to produce each second, and figure how long it will take to use up those electrons. That is the absolute maximum time the cell can last. Probably the cell can't use up every last bit of its plates, so the actual time will be less than that.