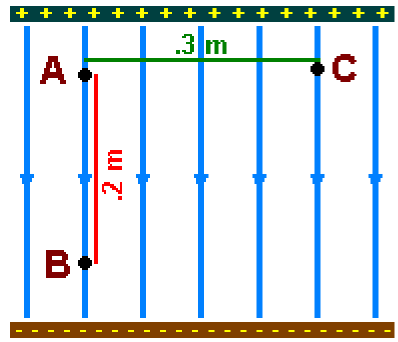
DAY 3

Summary of Topics Covered in Today's Lecture

The Gradient



$\Delta U_g = -\mathbf{g}\cdot\mathbf{r}$ and $\Delta U_E = -\mathbf{E}\cdot\mathbf{r}$. Since these equations will give us change in potential if we know field strength and distance, couldn't we calculate field strength if we knew potential?

Yes, in certain simple cases. Suppose an electric field is created between two plates of positive and negative charge, as shown in the figure. Two points in the field, A and B, are separated by 0.2 m as shown. The potential difference between A and B is measured to be 100 V (with A having the higher potential). Well, because A & B fall along a field line, it's not hard to calculate the field strength:

$\Delta U_{AB} = U_B - U_A = -100\ \text{V}$

(why negative? Because going from A to B means dropping in voltage from 100 V to 0.)

$\Delta U_{AB} = -\mathbf{E}\cdot\mathbf{r} = -E\ r\ \cos(\theta)$

$\Delta U_{AB} = -100\ \text{V}$
$r = 0.2\ \text{m}$
$\theta = 0°$ (A → B is moving in same direction as **E** field)

$-100\ \text{V} = -E\ 0.2\ \text{m}\ \cos(0)$
$-100\ \text{V} = -E\ (0.2\ \text{m})$
$E = (100\ \text{V})/(0.2\ \text{m}) = 500\ \text{V/m}$

On the other hand, if we measured the potential difference between A and C we would get 0 V.

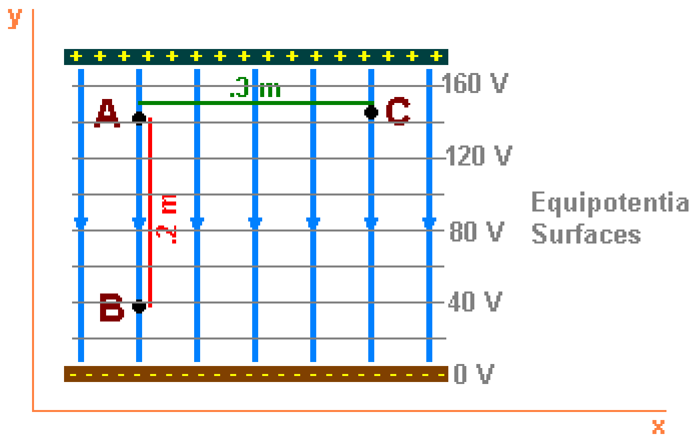
$\Delta U_{AC} = U_C - U_A = 0\ \text{V}$
$\Delta U_{AB} = -\mathbf{E}\cdot\mathbf{r} = -E\ r\ \cos(\theta)$

$\Delta U_{AC} = 0\ \text{V}$
$r = 0.3\ \text{m}$
$q = 90°$ (A → C is moving across **E** field)

$0\ \text{V} = -E\ 0.3\ \text{m}\ \cos(90)$
$0\ \text{V} = -E\ (0.3\ \text{m})(0)$
$0\ \text{V} = -E\ (0)$

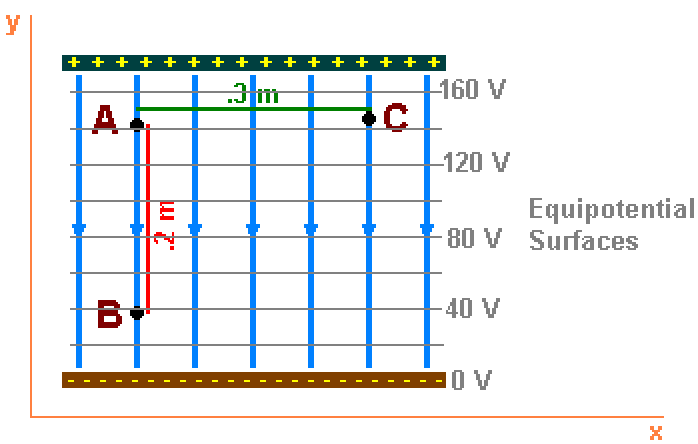
$\text{Cos}(90) = 0$
This tells us nothing. Anything times zero is zero, so E could be anything.

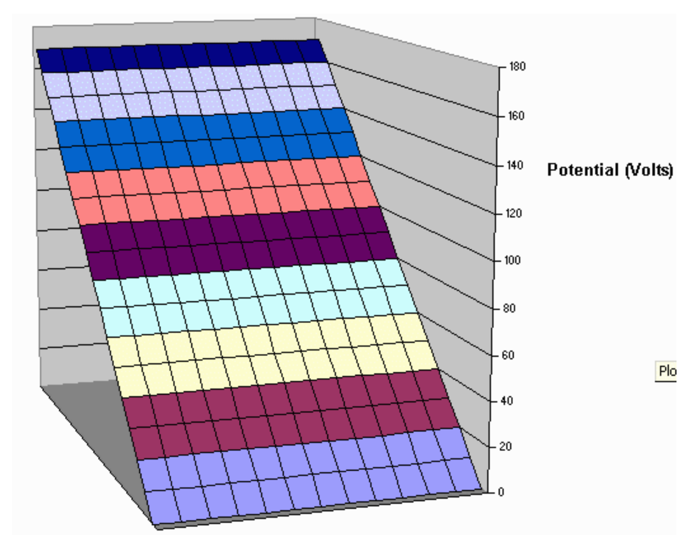
So we see that direction plays a role in calculating field strength from potential. Now let's draw in some equipotential surfaces in our figure:



The negative charges have a potential of zero in this example. Note that while A is at 140 V and B is at 40 V, the potential difference between A and B is 100 V, with A being higher than B, just as we said earlier.

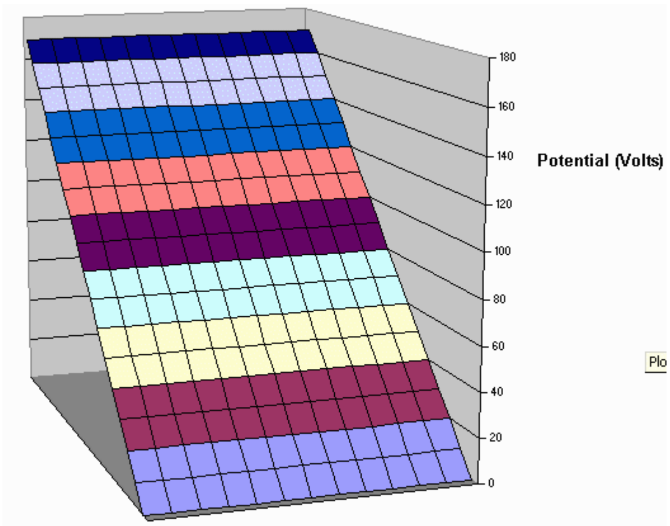
Note that we've added x and y axes. The potential increases with y, but is unchanging with x. Because we know from Day 1 that potential is analogous to height, we'll make a plot that shows potential as height. This is called a **surface plot**.



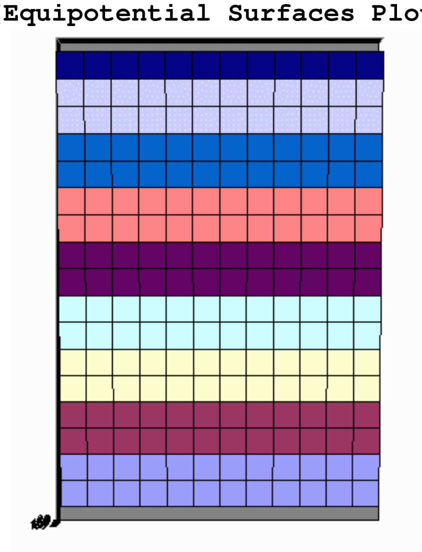


Looking "down" on the surface plot produces an equipotential surfaces plot – also known as a **contour plot**. (Yes, the equipotential surfaces plot is not the same as a surface plot, and a contour plot shows equipotential surfaces. Go figure.)

Surface Plot

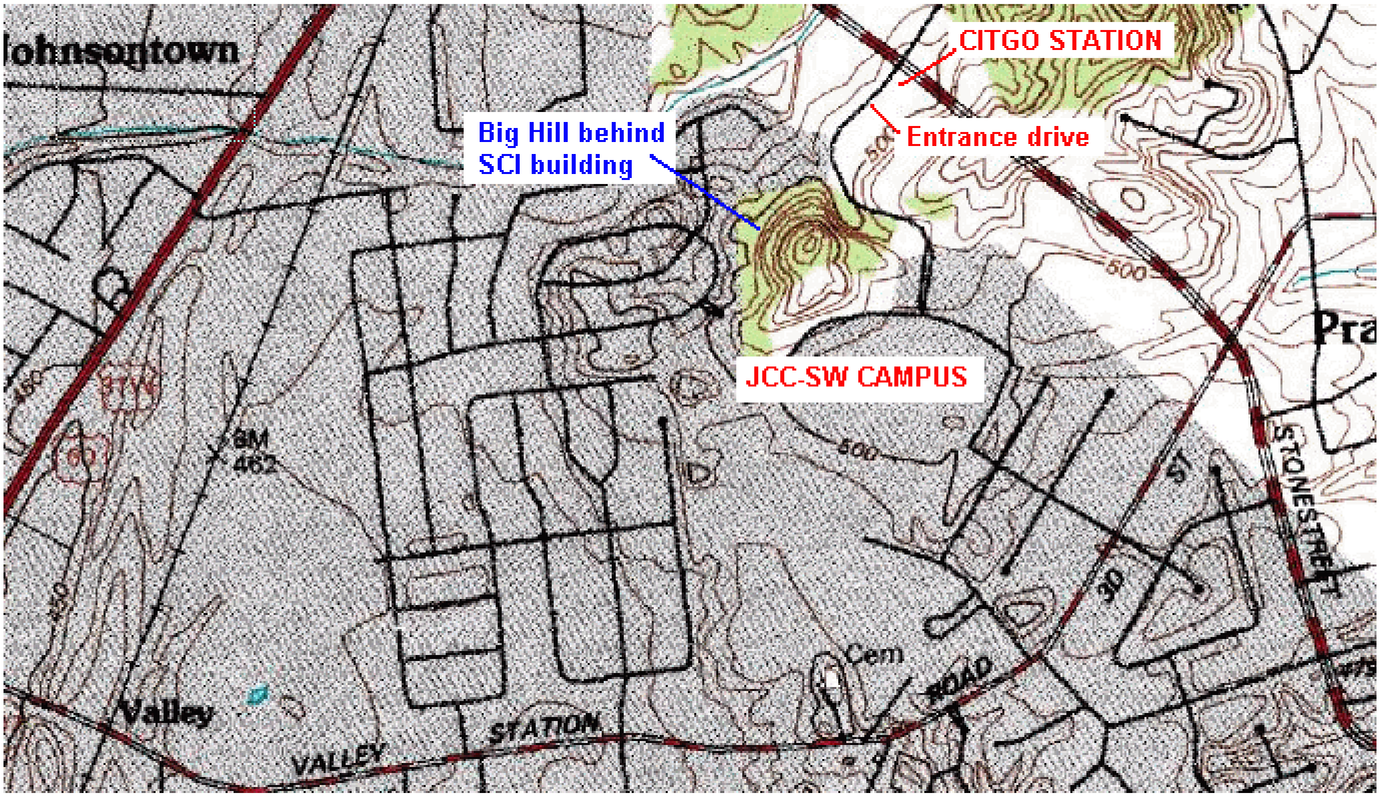


Contour Plot
(Equipotential Surfaces Plot)



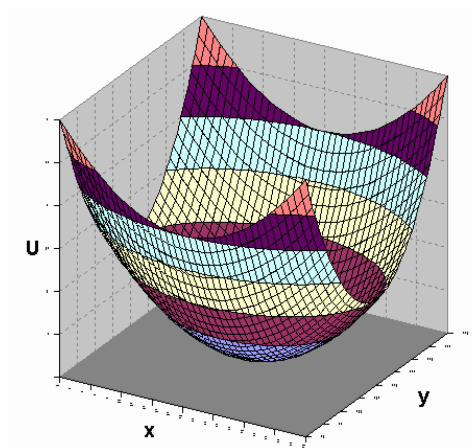
Note that the **E** field points in the “down hill” direction on these plots. This is a general rule for fields and potentials – the field is pointing “down hill” on a surface plot. Also, if the field was stronger the potential difference would be greater between A and B – meaning the surface plot would be steeper, and the lines on the contour plot would be closer together. This relationship between field and potential is mathematically called a **gradien**t. The field is the gradient of potential. If the potential is “steep”, changing rapidly, the field is strong, and equipotential surfaces (contours) are closely spaced. If the potential is “shallow”, changing slowly, the field is week, and equipotential surfaces (contours) are widely spaced.

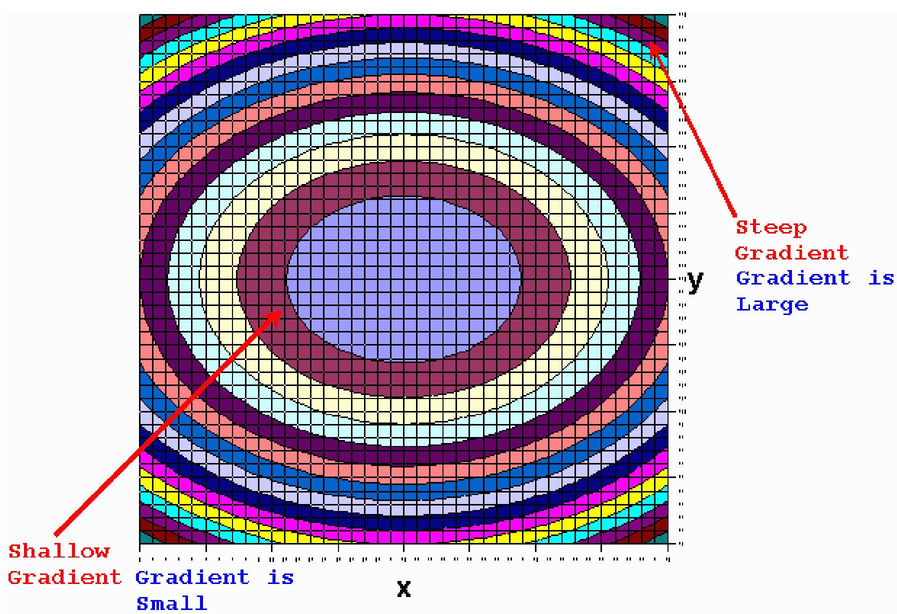
The gradient concept is most easily visualized using landforms. Consider the image below – a map of the area around JCC-SW. Note that where the land is hilly (rising and falling rapidly; steep gradient) contour lines are closely spaced. Where the land is more flat (shallow gradients), the contour lines are widely spaced.



Now let's think about a complex potential, like the one whose surface plot is shown at right.

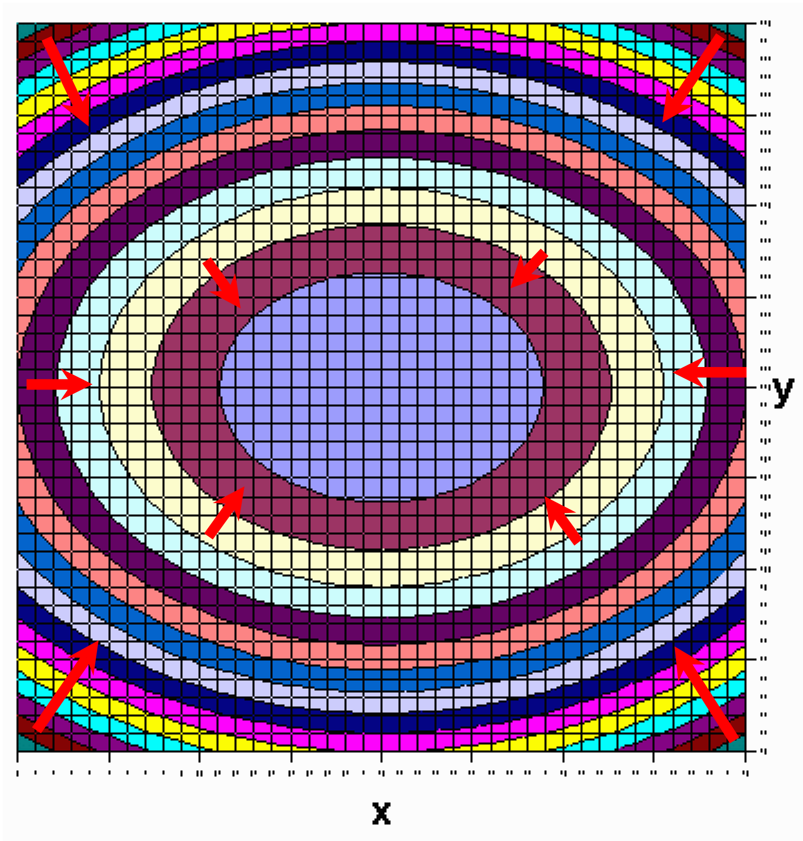
Looking at this in terms of equipotential surfaces we get a plot like the one below.





Note that far from the "bottom" of the plot, where the gradient is steep, equipotential surfaces are closely spaced. Near the "bottom" of the plot, where the gradient is shallow, equipotential

surfaces are widely spaced.
So let’s draw in some field vectors that point downhill, perpendicular to the equipotential surfaces, and are larger where the surfaces are closely spaced and smaller where the surfaces are widely spaced:

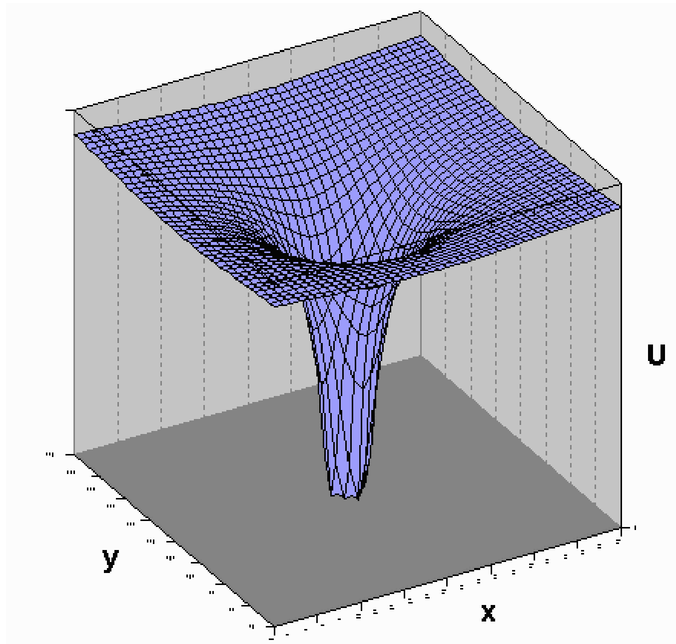


Now we are really determining the field, based on the potential. We can do it almost by looking at it.

The gradient concept shows up in many places: a gradient in electric potential indicates an electric field; a gradient in temperature in a thermal conductor indicates a flow of heat; a gradient in pressure in the atmosphere indicates wind; a gradient in land forms indicates direction of water drainage; and so forth.

Example Problem #1

Shown here is the plot of the gravitational potential for a very small, very massive object such as a black hole or neutron star. Sketch the equipotential surfaces and field vectors.



Solution:

This appears to be radially symmetric. So my surfaces will be circles centered on the "hole". Furthermore, the gradient of the surface is steepest near the center, so my equipotential surfaces will be closest together near the center, and more widely spaced further out. The field vectors will point "downhill", will be perpendicular to the equipotential surfaces, and will be larger where the surfaces are close together.