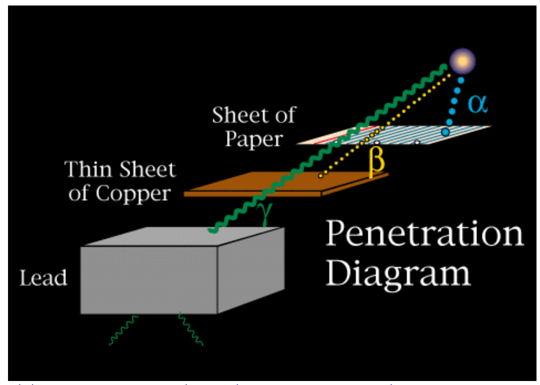
DAY 28

Summary of Primary Topics Covered

Damage to Living Things

The α, β, γ particles emitted in radioactive decay carry energy and act like little bullets that can do damage to the cells of living things. Because they are most massive and carry the most charge, α particles do the most damage if they hit living tissue. On the other hand, because they are so large they are least penetrating, and can be stopped by heavy cardboard in some cases. The relative danger posed by the different types of radiation is quantified by its Relative Biological Effectiveness (RBE). The higher the RBE, the more damaging it is.

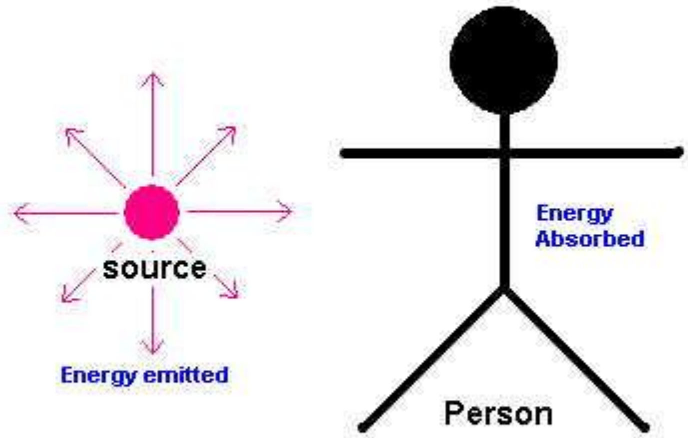


http://www.lbl.gov/abc/experiments/Experiment4.html

	Damage	RBE factor	Penetration
ν	Least damaging. No damage to cells.	0	Most penetrating. Cannot be shielded against. Can pass through the whole Earth unaffected.
γ	Dangerous to living cells. No mass, no charge – pure energy.	1.0	Highly penetrating. Cannot be stopped by a typical metal plate. Requires heavy lead shielding to block.
β	More dangerous to living cells. β particles have some mass (very little, but some) and they have charge. These things make β particles more dangerous than γ-rays.	1.0 – 1.7	Penetrating. May pass through metal foil but typically will be stopped by a metal sheet or plate.
α	Most damaging to cells. Per joule of energy deposited in a given amount of tissue, these are 10x - 20x more damaging than γ-rays.	10 - 20	Least penetrating – can be stopped by thin foil, heavy paper, or a few inches of air.

Measuring Radiation Exposure

When a person is exposed to a source of radiation that person receives energy that has been given off by the decaying nuclei as discussed last class. A common means of measuring exposure to radiation is the radiation absorbed dose or RAD. A RAD is 0.01 kg of energy from the source per kilogram of tissue.



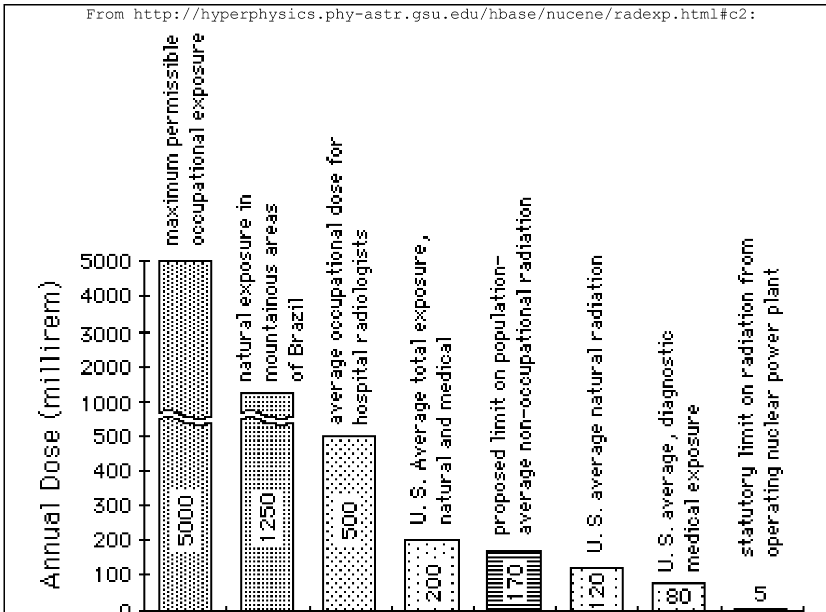
1 RAD = 0.01 J/kg

The RAD unit does not distinguish between energy from α, β, or γ sources. As we just learned, different types of radiation do differing amount of damage, measured by the Relative Biological Effectiveness (RBE). A measurement of exposure that incorporates the RBE factor is the Radiation Equivalent in Mammals or REM.

1 REM = 1 RAD x RBE

X-rays can also be added to the RBE list ($RBE_{X\text{-rays}}$ = 1.0) because exposure to energy from X-rays has similar effects on living tissue as exposure to energy from radioactive sources (energy enters a cell; electrons are stripped from atoms in cells, creating ions and otherwise damaging the cell), and X-ray exposure can also be measured in REMS.

From http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/radexp.html#c2:



Exposure to Radiation

Radiation is part of the environment. Radiation comes from both outer space and from radioisotopes in the Earth. The average American receives a couple of milliREM of exposure per week due to natural sources. It is unavoidable. However, increased exposure can occur due to the use of radiation in medicine or the workplace.

Effects of Low Dose Radiation Exposure (Risk Levels and Risk Comparisons)

Relatively low-level exposure like what is listed here does not cause immediately noticeable effects. Like smoking cigarettes, exposure to low levels of radiation exposure has a cumulative effect and is linked to cancer and other health problems.

However, small, frequent doses of radiation create very limited risks. According the Georgia State University's HyperPhysics web site (http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html), the risk posed by being exposed to one milliREM of radiation dose is a 1 in 8 million risk of dying of cancer or a loss in life expectancy (LLE) of about 1.2 minutes. This is a risk equivalent to crossing the street three times (where you risk getting hit by a car), three puffs on a cigarette (where you risk tobacco-related health effects), or 10 extra Calories for an overweight person (where you risk obesity-related health effects). This all assumes that large dose effects extrapolate linearly through small doses to zero dose – and that assumption is not known for certain.

What is "Loss of Life Expectancy?" LLE is a way of quantifying levels of risk. If exposure to "Z" amount of radiation has an LLE of 1 month, for example, that does not mean that if you are exposed to "Z" then you will die one month earlier than you would have otherwise. It means that people exposed to "Z" will, on average, die one month earlier.

For example, suppose that the hip new "extreme" sport is ramping super-charged All Terrain Vehicles over pits filled with spikes and landing in shark infested waters while wearing a bloody piece of meat tied to one's back. This sport is performed by healthy 20-year-olds who otherwise could expect to live to be 80. Suppose that of every 10 people who try an extreme ramp, 1 dies, but the other 9 emerge basically unhurt.

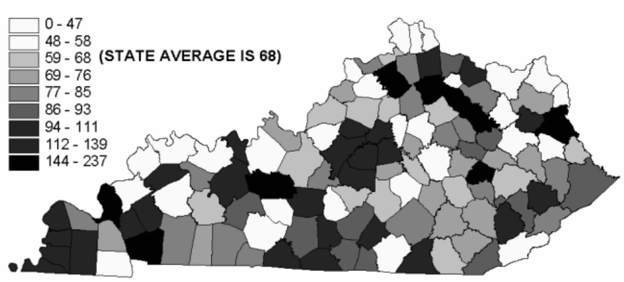
So, if 100 people try extreme ramping, we expect 10 to die. Each dead person represents a loss of life of 60 years by dying at 20 instead of 80. So the total loss of life is 10 x 60 = 600 years among a total of 100 people trying an extreme ramp. Thus the LLE for any person doing an extreme ramp is 600/100 = 6 years.

Note that an LLE of 6 years doesn't mean that if you try an extreme ramp you will live to be 74 instead of 80. Rather, either you die at 20, or you live through it and vow never to be that stupid again. But the risk measurement is an LLE of 6 years.

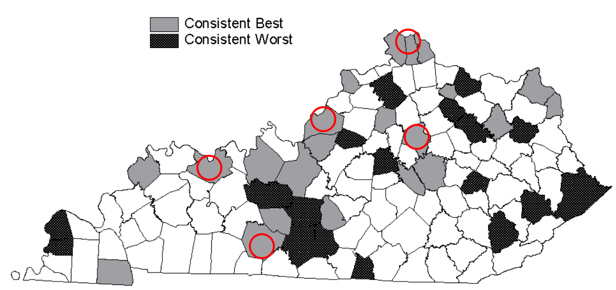
Some risks, such as radiation exposure or cigarette smoking, may not be as "either/or" as this. Some people may be exposed to radiation and still live a long life. Some people may smoke a pack a day and still live a long life. But on average both radiation exposure and cigarette smoking will cause an earlier death – sometimes much earlier – and LLE is a way of measuring that risk. Other risks (such as motor vehicle accidents) are very much "either/or" propositions. LLE is a way of comparing the risks of all different kinds of activities.

When discussing radiation risks we also need to keep in mind that Americans tend to be very worried about some kinds of risk, but not about others. One example of this would be teens and risk. People worry about teens and crime, and tend to associate cities with being dangerous places. However, if you look at Kentucky counties in terms of how dangerous they are for teens, you find that the more urban counties are among the safest counties for teens in Kentucky, whereas the most dangerous counties tend to be rural. Why? The reasons are complex, but a

Teen Violent Death Rate 1992-2001
(per 100,000 teens aged 15-19)
Includes homicide, suicide, accidents, etc.



Teen Violent Death Rate 1982-1991 & 1992-2001



Circles mark Kentucky's largest metro areas (Louisville, Lexington, N. Ky, Owensboro, Bowling Green)

Data from the Kentucky State Data center Kids Count pages: http://ksdc.louisville.edu/1kidscount.htm

quick explanation is that teens are far more likely to be killed by motor vehicles than by criminals, and rural areas require a lot more driving at high speed. The city kid may pass a corner frequented by drug dealers when he walks to school, but the country kid has to pass a dangerous curve when he's driving to school.

> **Marriage vs. Smoking**
> According to "Marital Status and Mortality: The Role of Health" (*Demography*, August 1996) by Lee A. Lillard, Constantijn W. A. Panis, the health benefits of being married have been known since at least the 1850's.
>
> **From *Before It's Too Late: A Scientist's Case For Nuclear Energy*, Bernard L. Cohen 1983:**
> LLE of Smoking (1 pack/day): 4.4 years
> LLE of Being Single: 5.5 years

Another example would be smoking and marriage. We all hear that we shouldn't smoke because of health reasons, and that is true. Smoking has a very high LLE. However, what has an even higher LLE is being single, especially for men. A man who is both single and a smoker could well expect more health benefits from getting and staying married than he would from quitting smoking permanently but staying single. Why? Again, the reasons are complex, but a quick explanation is that married people are, on average, less likely to do stupid things that get them killed, and they have someone else watching out for them and their overall health, talking them into going to a doctor when they need to, and so forth. Americans spend much time and effort debating the health effects of smoking, and how to get people not to smoke, but less time and effort on the health effects of marriage.

The bottom line is that risks due to low-level radiation doses do exist, but need to be kept in perspective.

Effects of High Dose Radiation Exposure

In contrast to low-level radiation doses, large doses of radiation (over 25,000 milliREM) do create <u>immediate</u> effects. The effects in the table below are not long-term effects, they show up right after a person has received an "all-at-once" dose of the amount of radiation shown:

Acute Radiation Exposure -- Effects of Large, Whole-Body Radiation Doses

DOSE (REM)	DOSE (milliREM)	EFFECT
0-25	0 – 25,000	No observable effect
25-100		Slight blood changes
100-200		Significant reduction in blood platelets and white blood cells (temporary)
200-500		Severe blood damage, nausea, hair loss, hemorrhage, death in many cases
>600	> 600,000	Death in less than two months for over 80%

http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/radexp.html#cc1

Another unit often used instead of a REM is a Sievert, which is equal to 100 rems. Thus any dose above 6 Sieverts is generally fatal.

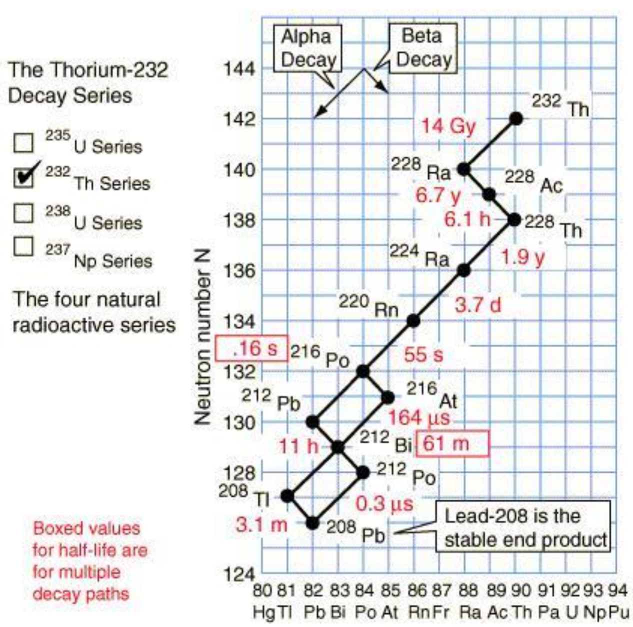
Activity, Power, Time, Distance, Shielding

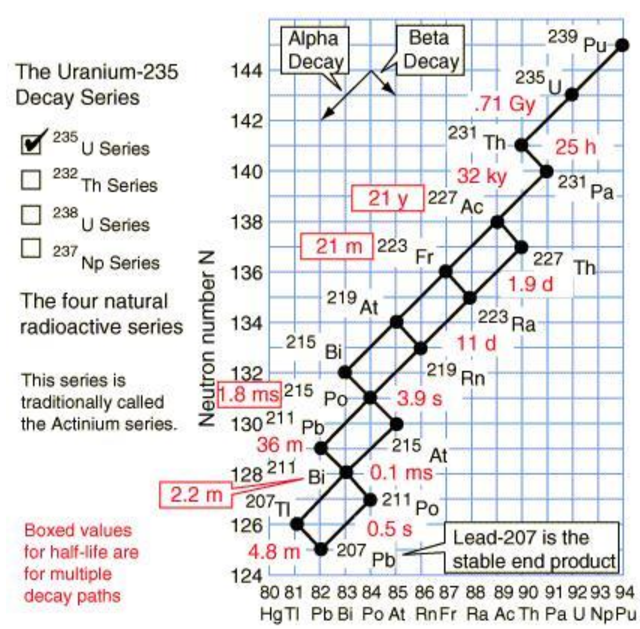
In summary, the danger from radioactive sources depends on a number of factors. First, the amount of unstable nuclei present in a source and the half-life of those nuclei determine the activity level of the source (usually measured in Curies – Ci). The energy released in each decay (which involved the mass defect and $E=mc^2$) determines the rate at which the source puts out energy. For a source of a certain activity level, increasing distance from the source means that the particle flux rate – the intensity of radiation – is decreased. Shielding also decreases intensity of radiation. And limiting the time which one is exposed to a source decreases the amount of milliREMs absorbed.

Decay Series

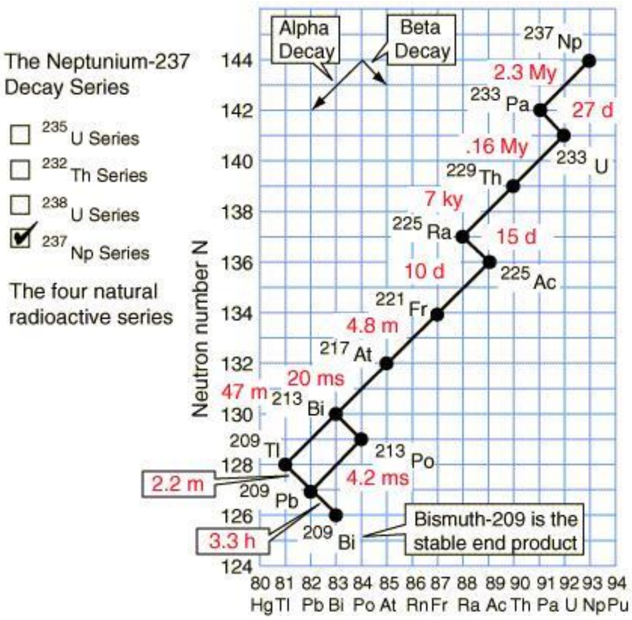
Often the daughter nucleus formed by the decay of an unstable nucleus is itself unstable. This nucleus will in turn decay – and if its daughter is unstable the daughter will decay and so on until a stable nucleus results.

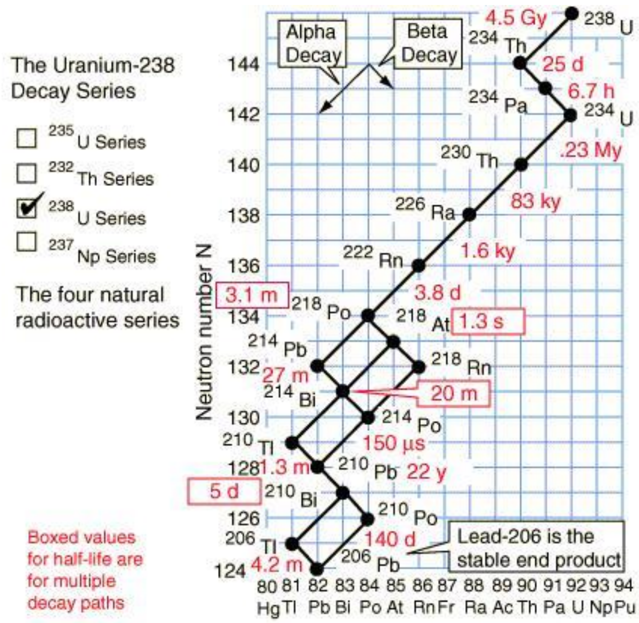
There are four naturally occurring decay series that are responsible for most naturally occurring radioactivity. Man-made radioisotopes can also start a decay series.





Shown here are the four natural radioactive decay series from
http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/radser.html





Example Problem #1:

A 170 lb man is exposed to an α source. He receives energy at a rate of 10 J/min. If the man is exposed to the source for 10 seconds, determine approximately how much exposure he received. Will this produce immediate health effects?

Solution:

Calculate power: $P = 10$ J/min $= 0.167$ J/s

Calculate energy received:

$$P = \frac{E}{t} \quad 0.167\text{ J/s} = \frac{E}{10\text{ s}}$$

E = 1.67 J

Calculate mass of man:

$$170\text{ lb}\,\frac{1\text{ kg}}{2.203\text{ lb}} = 77.1675\text{ kg}$$

The absorbed dose is the energy per kilogram:

$$\frac{1.67\text{ J}}{77.1675\text{ kg}} = .021598\text{ J/kg}$$

$$.021598\text{ J/kg}\frac{1\text{ RAD}}{.01\text{ J/kg}} = 2.1598\text{ RAD}$$

The exposure is found using

REM = RAD × RBE

According to the table RBE for α is 10 – 20 so

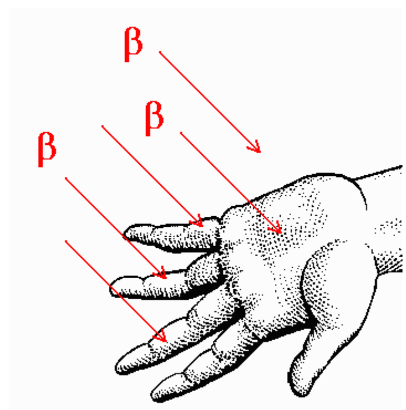
REM = 2.1598 × 10 = 21.598 REM (lower estimate)

REM = 2.1598 × 20 = 43.196 REM (upper estimate)

So the man received between 22 and 43 REM. That's not good. It will probably do damage to his blood.

Example Problem #2:

A person who has mass of 100 kg sticks his hand into a box, not knowing that it contains a sample of Cobalt-60. There are 500 mg of the Co-60 in the box. The person's hand is 20 cm from the Co-60. Someone yells a warning, and the person pulls the hand out after it has been in the box for 2 seconds. The person's hand measures roughly 20 cm long by 9 cm wide (including fingers). Estimate the exposure this person received (in milliREM).



Solution:

Just go at it one step at a time. Find out the power output of 500 mg of Co-60, figure the intensity at the person's hand, and the amount of energy absorbed.

First I go to the periodic table and look up Co-60. The table says it is a β^- decay, and has a half-life of 5.27 years. I'll work out the decay and find out what the daughter is:

$$^{60}_{27}\text{Co} \longrightarrow {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu} + {}^{60}_{28}\text{Ni}$$

Now I look up the mass of Co-60 and Ni-60 and an electron (β^-):

Co-60 mass is 59.933822 u, according to table. But this includes the 27 electrons in a Cobalt atom. The nucleus itself is 59.933822 u – 27(0.0005485 u) = 59.9190125 u

Total mass before decay is
59.9190125 u

Ni-60 mass is 59.9307884 u according to table. But this includes the 28 electrons in a Nickel atom. The nucleus itself is 59.9307884 u - 28(0.0005485 u) = 59.9154304 u

electron mass is 0.0005485 u
neutrino mass is 0

Total mass after decay is

$$
\begin{array}{r} 59.9154304 \text{ u} \\ + \quad 0.0005485 \text{ u} \\ \hline \mathbf{59.9159789 \text{ u}} \end{array}
$$

Now find the mass defect – the amount of mass converted to energy in the decay. I subtract to find how much mass disappeared:

Mass defect = **59.9190125 u** – **59.9159789 u** = 0.0030336 u lost in the decay.

Convert that to kg:

$$0.0030336 \text{ u} \; \frac{1.6605 \times 10^{-27} \text{ kg}}{1 \text{ u}} = 5.03729 \times 10^{-30} \text{ kg}$$

OK, now let's find the energy released in one decay:

$E = m c^2$
$= 5.03729 \times 10^{-30} \text{ kg } (3 \times 10^{8} \text{ m/s})^2$
$E = 4.53356 \times 10^{-13} \text{ J}$

Now to find the power output of 500 mg of Co-60 I need to find out the activity rate (decays per second).

$R = .693(N/T_{1/2})$

OK, let's find the N (number of Co-60 nuclei):

There are 500 mg of Co-60. That's .5 grams of Co-60 or 0.0005 kg of Co-60. Let me get that into u units:

$$0.0005 \text{ kg} \; \frac{1 \text{ u}}{1.6605 \times 10^{-27} \text{ kg}} = 3.01114 \times 10^{23} \text{ u}$$

Now I know the mass of 1 Co-60 nucleus from the information I looked up earlier:

$$3.01114 \times 10^{23} \text{ u} \; \frac{1 \text{ Co-60 nucleus}}{59.933822 \text{ u}} = 5.02411 \times 10^{21} \text{ Co-60 nuclei}$$

So $N = 5.02411 \times 10^{21}$

Now I'll find the $T_{1/2}$ -- have to look that up

$T_{1/2} = 5.27$ years

5.27 yr × (3.156×10^{7} sec/1 yr) = 1.66321×10^{8} sec

So $T_{1/2} = 1.66321 \times 10^{8}$ sec

R = .693(5.02411×10^{21} nuclei/1.66321×10^{8} sec) = 2.09336×10^{13} nuclei/sec

So Co-60 is decaying at a rate of 2.09336×10^{13} nuclei/sec. Each decay releases 4.53356×10^{-13} J of energy. So the power output is

P = (4.53356×10^{-13} J of energy released each time a nucleus decays) X
(2.09336×10^{13} nuclei decaying each second)

P = (4.53356×10^{-13} J/nucleus)(2.09336×10^{13} nuclei/sec)

P = 9.49038 J/sec = 9.49038 W

The intensity at a distance of 20 cm from the Co-60 will be

I = P/A

$A = 4 \pi r^2$
$A = 4(3.1416)(20 \text{ cm})^2$
$A = 5026.55 \text{ cm}^2$

$I = 9.49038 \text{ W} / 5026.55 \text{ cm}^2$
$I = .001888 \text{ W/cm}^2$

The power received by the hand will be the W/cm^2 times the area of the hand:

$P_{\text{on hand}} = .001888 \text{ W/cm}^2 (20 \text{ cm} \times 9 \text{ cm}) = .339849 \text{ W}$

The hand was in the box for 2 seconds, so

E = .339849 W (2 sec)
E = .339849 J/s (2 s) = .679698 J

That's .679698 J in a 100 kg person, so the radiation dose is

.679698 J/100 kg = .00679698 J/kg

$$.00679698 \text{ J/kg} \; \frac{1 \text{ RAD}}{0.01 \text{ J/kg}} = .679698 \text{ RAD}$$

The RBE for b is 1.7 so the exposure is

REM = RBE × RAD

REM = 1.7 × .679698 = 1.15549 REM

Or 1160 milliREM.

That's more than three times the average annual dose received by a hospital radiologist.

Example Problem #3:

Radon-222 ("Radon Gas") does not decay into a stable nucleus. Rather, the decay of Rn-222 is followed by a series of decays. Determine all the decays of the daughters of Rn-222 and determine what stable daughter finally results from this series of decays.

Solution:

I used the Periodic Table with Isotope information from the class web page to look up Rn-222 and its daughters. That told me what type of decay each daughter underwent. I worked out each reaction equation as I went along. As seen in from the work on the next page, the stable product is Lead-206.

$$^{222}_{86}Rn \longrightarrow {}^{4}_{2}\alpha + {}^{218}_{84}Po$$

$$^{218}_{84}Po \longrightarrow {}^{4}_{2}\alpha + {}^{214}_{82}Pb$$

$$^{214}_{82}Pb \longrightarrow {}^{0}_{-1}\beta + {}^{214}_{83}Bi + {}^{0}_{0}\bar{\nu}$$

$$^{214}_{83}Bi \longrightarrow {}^{0}_{-1}\beta + {}^{214}_{84}Po + {}^{0}_{0}\bar{\nu}$$

$$^{214}_{84}Po \longrightarrow {}^{4}_{2}\alpha + {}^{210}_{82}Pb$$

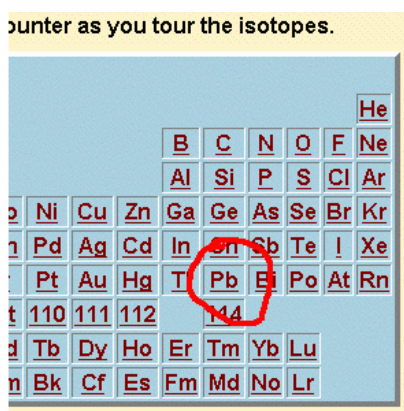
$$^{210}_{82}Pb \longrightarrow {}^{0}_{-1}\beta + {}^{210}_{83}Bi + {}^{0}_{0}\bar{\nu}$$

$$^{210}_{83}Bi \longrightarrow {}^{0}_{-1}\beta + {}^{210}_{84}Po + {}^{0}_{0}\bar{\nu}$$

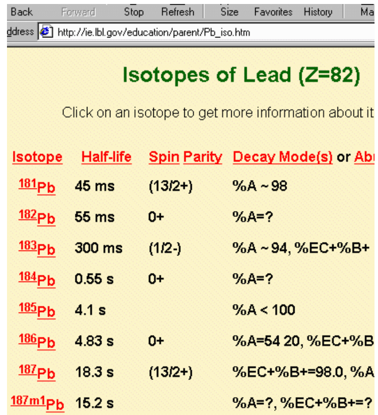
$$^{210}_{84}Po \longrightarrow {}^{4}_{2}\alpha + {}^{206}_{82}Pb$$

Here is a sample of how I did this.

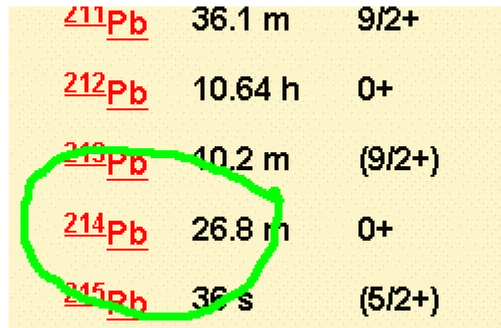
Taking the case of Lead-214 (3rd line), I went to the Period Table with Isotope Information and clicked on Lead (Pb).



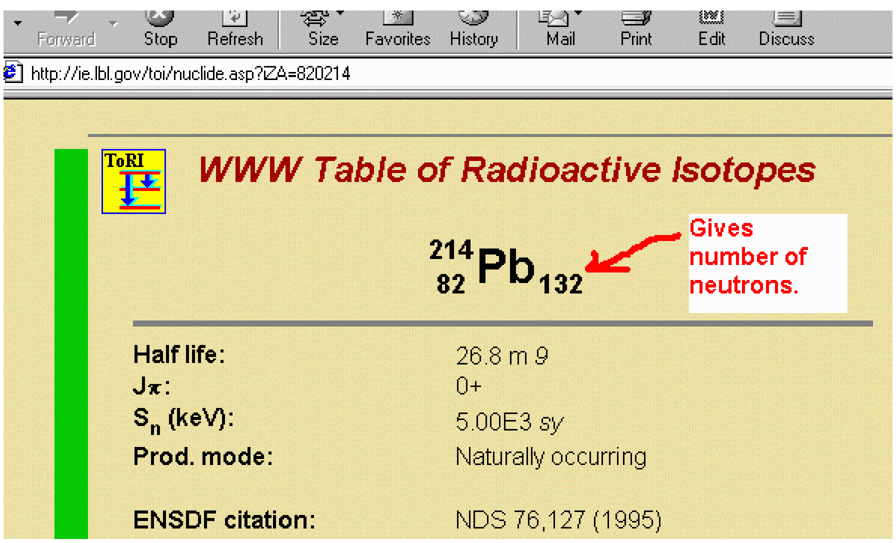
That took me to a page full of information on Lead isotopes.



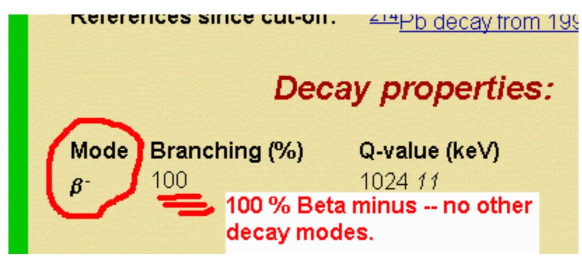
I scrolled down until I found Pb-214, then I clicked on that.



Now I have lots of information on Pb-214.



And I see that Pb-214 undergoes β^- decay 100% of the time. Once I know how it decays I work out the reaction equation. I did this for each decay.



Example Problem #4 (PHY 232 only):

In a decay series, parent nucleus A decays into daughter nucleus B, which decays into "granddaughter" nucleus C.

A has half-life $T_{½A}$ and decay constant λ_A, B has half-life $T_{½B}$ and decay constant λ_B, and C is stable. We start with N_0 nuclei (all A). The number of A's as a function of time is simply the usual exponential

$$N_A = N_0 e^{-\lambda_A t}$$

because whatever happens after the A's decay has no effect on them. But the number of B's and C's is much more complex. The number of B's is given by

$$N_B = N_0 \left(\frac{\lambda_A}{\lambda_B - \lambda_A} \right) \left(e^{-\lambda_A t} - e^{-\lambda_B t} \right)$$

Do the following:

a. Come up with an equation for the rate of change of B's.
b. Show that the above equation solves the rate equation.
c. Derive an equation for the number of C's as a function of time.

Solution:

I'll do part (a) first:

The decay rate of A's is $R_A = -\lambda_A N_A$. Every time an A decays it produces a B, so the rate B's are produced is $\lambda_A N_A$. However, B's also decay at rate $-\lambda_B N_B$. So the rate of change of B's should be

R_B = rate of production minus rate of decay
$R_B = \lambda_A N_A - \lambda_B N_B$

$dN_B/dt = \lambda_A N_A - \lambda_B N_B$ **that's my answer for part (a)**

Now for part (b):

First I'll write down my equation for N_A and N_B...

$$N_A = N_0 e^{-\lambda_A t} \qquad N_B = N_0 \left(\frac{\lambda_A}{\lambda_B - \lambda_A} \right) \left(e^{-\lambda_A t} - e^{-\lambda_B t} \right)$$

Then I'll find dN_B/dt...

$$\frac{dN_B}{dt} = N_o\left(\frac{\lambda_A}{\lambda_B-\lambda_A}\right)\frac{d}{dt}\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$$

$$= N_o\left(\frac{\lambda_A}{\lambda_B-\lambda_A}\right)\left(-\lambda_A e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t}\right)$$

Now I'll sub those into my rate equation **$dN_B/dt = \lambda_A N_A - \lambda_B N_B$** and see if it works out…

$$N_o\left(\frac{\lambda_A}{\lambda_B-\lambda_A}\right)\left(-\lambda_A e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t}\right) = \lambda_A N_o e^{-\lambda_A t} - \lambda_B N_o\left(\frac{\lambda_A}{\lambda_B-\lambda_A}\right)\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$$

$$N_o\left(\frac{\lambda_A}{\lambda_B-\lambda_A}\right)\left(-\lambda_A e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t}\right) = \lambda_A N_o e^{-\lambda_A t} - \lambda_B N_o\left(\frac{\lambda_A}{\lambda_B-\lambda_A}\right)\left(e^{-\lambda_A t} - e^{-\lambda_B t}\right)$$

cancel out N_o's and multiply through parentheses…

$$-\frac{\lambda_A\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_A t} + \frac{\lambda_A\lambda_B}{\lambda_B-\lambda_A}e^{-\lambda_B t} = \lambda_A e^{-\lambda_A t} - \frac{\lambda_B\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_A t} + \frac{\lambda_B\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_B t}$$

$$-\frac{\lambda_A\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_A t} + \frac{\lambda_A\lambda_B}{\lambda_B-\lambda_A}e^{-\lambda_B t} = \lambda_A e^{-\lambda_A t} - \frac{\lambda_B\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_A t} + \frac{\lambda_B\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_B t}$$

cancel terms & multiply this by $\frac{\lambda_B-\lambda_A}{\lambda_B-\lambda_A}$

$$-\frac{\lambda_A\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_A t} = \lambda_A\frac{\lambda_B-\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_A t} - \frac{\lambda_B\lambda_A}{\lambda_B-\lambda_A}e^{-\lambda_A t}$$

$$-\frac{\lambda_A \lambda_A}{\lambda_B - \lambda_A} e^{-\lambda_A t} = \lambda_A \frac{\lambda_B - \lambda_A}{\lambda_B - \lambda_A} e^{-\lambda_A t} - \frac{\lambda_B \lambda_A}{\lambda_B - \lambda_A} e^{-\lambda_A t}$$

cancel terms

$$-\lambda_A \lambda_A = \lambda_A(\lambda_B - \lambda_A) - \lambda_B \lambda_A$$

$$-\lambda_A \lambda_A = \lambda_A \lambda_B - \lambda_A \lambda_A - \lambda_B \lambda_A$$

$$0 = 0$$ ✓ The solution works

That takes car of part (b).

Part c should be pretty easy -- the number of C's is just N_0 minus the A's and B's.

$$N_C = N_0 - N_A - N_B$$

$$N_C = N_0 - N_0 e^{-\lambda_A t} - N_0 \left(\frac{\lambda_A}{\lambda_B - \lambda_A} \right) \left(e^{-\lambda_A t} - e^{-\lambda_B t} \right)$$

$$N_C = N_0 \left[1 - e^{-\lambda_A t} - \left(\frac{\lambda_A}{\lambda_B - \lambda_A} \right) \left(e^{-\lambda_A t} - e^{-\lambda_B t} \right) \right]$$

That takes car of part (c).

Example Problem #5 (PHY 232 only):

In the above problem, make a graph of the number of A's, B's, and C's vs. time if N_0 = 100 Trillion nuclei and

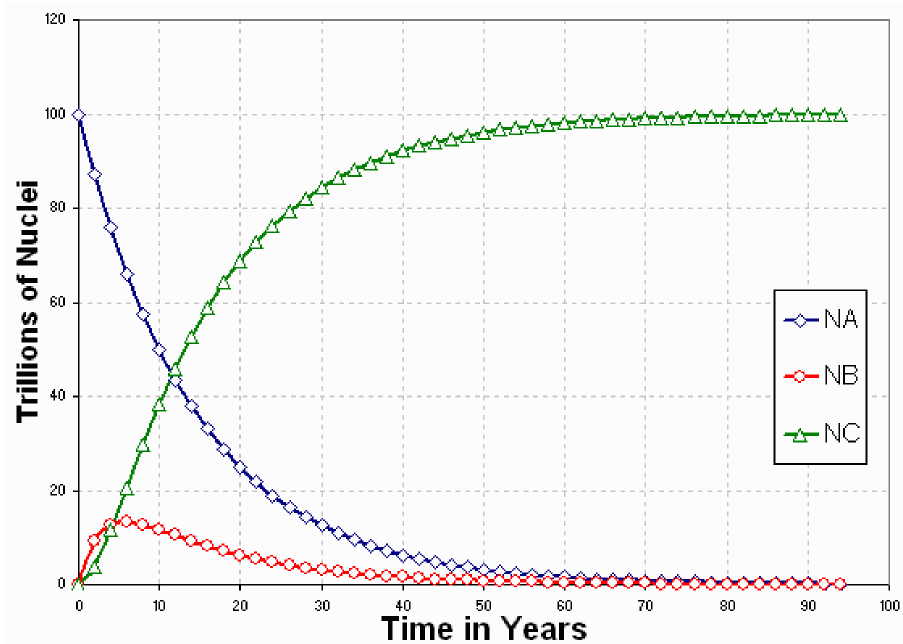
a. $T_{½A}$ = 10 years and $T_{½B}$ = 2 years.
b. $T_{½A}$ = 2 years and $T_{½B}$ = 10 years.
c. Comment on your results.

Solution:

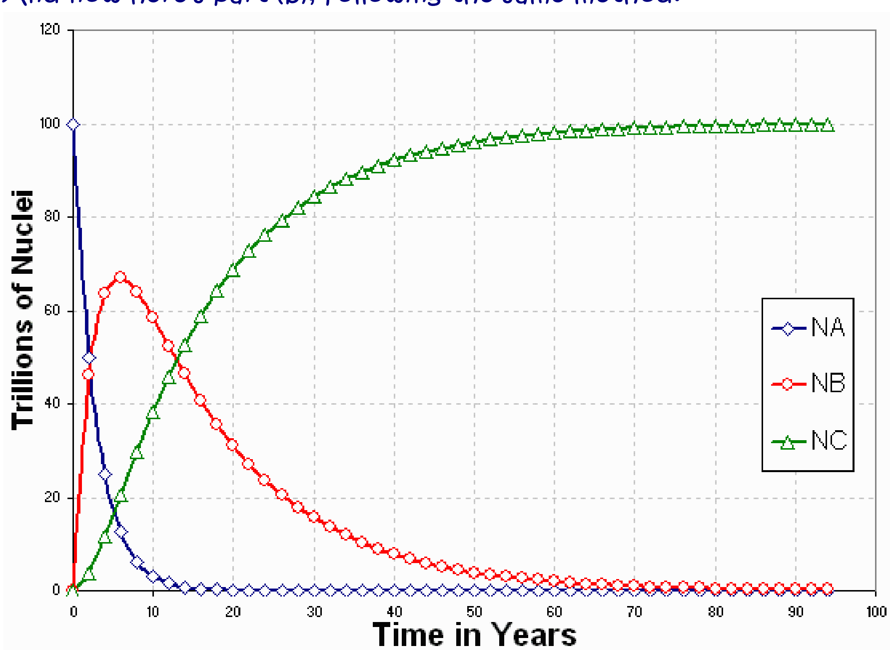
$\lambda_A = 0.693/T_{½A}$ = .693/10 years = .0693 1/years.
$\lambda_B = 0.693/T_{½B}$ = .693/2 years = .3465 1/years.

N_o = 100 trillion

I'll plug in points and plot the equations for N_A, N_B, and N_C:



And now here's part (b), following the same method:



My comments -- it appears that the C's numbers were the same in both cases -- or pretty darn close. In the second case you got a lot more B's.