DAY 27

Summary of Primary Topics Covered

Activity

The *activity* of a sample of radioactive material is the number of decays per second taking place in the sample. If you have more nuclei you would expect to have greater activity simply because there are more nuclei to decay. Likewise, if you have a short half-life you would expect greater activity because each nucleus in the same is more likely to decay. Activity is determine by the equation

$$R = \frac{0.693N}{T_{1/2}} = \lambda N$$

PHY 232 Only

The decay rate is the rate of change of the number of nuclei over time. In other words, it is the time derivative of N.

$$R = \frac{dN}{dt} = \frac{d}{dt}\left(N_0 e^{-\lambda t}\right)$$

$$R = N_0 \frac{d}{dt}\left(e^{-\lambda t}\right) = N_0(-\lambda)e^{-\lambda t}$$

$$R = -\lambda N_0 e^{-\lambda t}$$

Since $N_0e^{-\lambda t} = N$ we can sub N into this:

$$R = -\lambda N$$

The minus sign simply indicates that the number of nuclei is decreasing, and is frequently dropped from the equation.

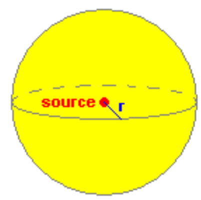
Here R is the activity, N is the number of nuclei present, $T_{1/2}$ is the half-life of the nuclei, and λ is the decay constant. The SI unit for activity is the Bequerel (Bq). *1 Bq = 1 decay per second*. Another common unit is the Curie (Ci). *1 Ci = 37 billion Bq*.

Particle Flux Rate

If a radioactive source radiates particles isotropically (same in all directions), then you would expect that the further you were from the source the less intense the radiation from that source would be, even if there was nothing to shield you from the source.

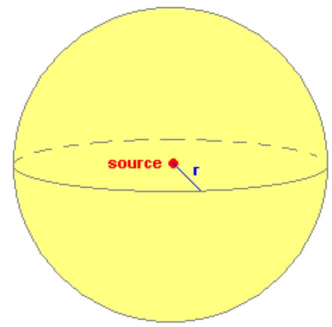
For example, suppose the source in the figure at right is emitting particles at a rate of 1000 Bq. The particles leave the source in all directions, so as they move away from the source they spread out – are less concentrated together. The further away from the source you go, the less concentrated the flow of particles will be.

A



Particles spread out over the area of a sphere with r = 6 cm.

B



Particles spread out over the area of a sphere with r = 10 cm. Flux rate is lower than A.

The measure of the concentration of the flow of particles is called the *flux rate* (ϕ):

$$\phi = \frac{\text{Rate at which particles are generated}}{\text{Area over which particles are dispersed}}$$

For isotropic radiation, the area over which the particles are dispersed in the area of a sphere ($A = 4\pi r^2$). The rate at which particles are generated is the same as the decay rate, since all the decays we've studied generate one of each type of particle per decay. So our flux rate equation can be written

$$\phi = \frac{R}{4\pi r^2}$$

Mass-Energy Equivalence, the Mass Defect, and Energy in Radiation

One of the more famous equations in physics is Einstein's equation for the equivalence of mass and energy, which we discussed earlier in the class:

$E = m c^2$

This says that energy is stored in matter, and the amount of energy in matter is the product of the mass in kg (m) and the speed of light (c = 300,000,000 m/s) squared. There is an enormous amount of energy stored in matter. If humans could develop the technology to easily extract that energy, all of humankind's energy needs would be satisfied.

This mass-energy equivalence is the source of the energy released in radioactive decay. When a parent nucleus decays into a daughter nucleus + particles, the sum of the masses of the daughter nucleus + particles is less than the mass of the original parent nucleus.

The difference between the mass of a parent nucleus before a decay and its products after a decay is called the *mass defect*.

   Total mass of parent before a decay
\- Mass of daughter + other particles left after decay
   Mass defect

This mass defect is turned into energy via $E = m c^2$. In other words, the energy from radioactive decay comes from turning some of the mass of the parent nucleus into energy.

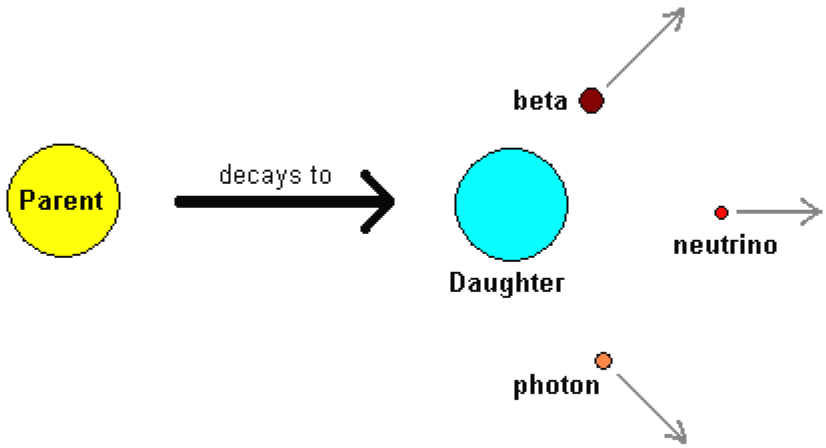
The rate at which energy is released -- in other words the power output -- depends on the energy released per decay and the number of decays per second (the activity rate).

Power = Energy per decay x Decays per second
= Energy per decay x Activity

$P = \text{mass defect} \times c^2 \times R$

Radioactive materials, if they are highly radioactive, can release so much power that they literally are hot!

What forms does the energy released in radioactive decay come in? Well, the particles released in the decay have Kinetic energy. And the photon carries its own "radiant" energy (it is photons that carry the energy in heat transfer by radiation).



Mass of daughter + mass of beta is less than mass of parent.
(Photon has zero mass. Neutrino has insignificant mass.)

Useful Data

Precise masses of some basic particles are as follows:

Particle	**Symbol**	**Mass in u**
electron	β^-	0.0005486
positron	β^+	0.0005486
proton	p	1.0072765
neutron	n	1.0086649
alpha (He-4)	α	4.0015060
neutrino	ν	0.00 (approx.)
photon	γ	0.00

Masses of different isotopes have to be looked up in the INTERACTIVE periodic table or other source of information.

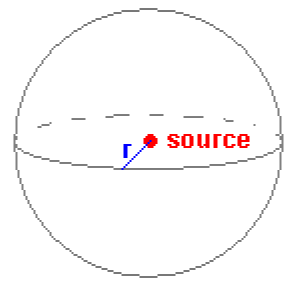
NOTE -- usually the masses given for various isotopes include the electrons in a neutral atom (one with as many positive charges as negative charges). For instance, a neutral Helium atom will have two electrons because it has two protons (Z=2); a neutral Carbon atom will have six electrons because it has six protons (Z=6). To get the mass of a nucleus by itself you must subtract the mass of Z electrons.

Intensity of Energy from Radiation

As energy radiated from a source moves away from the source, it spreads out. We define the intensity of radiation in terms of the power output of the source and the area over which the power is dispersed.

$$I = P/A$$

I is usually measured in Watts per square meter. For isotropic radiation of energy, the area is the surface area of a sphere, and $A = 4 \pi r^2$.

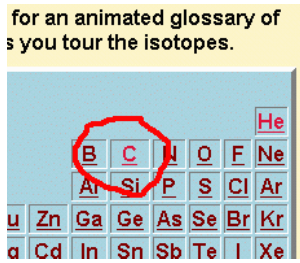


This concept holds true for anything that radiates energy -- a light bulb, a loudspeaker, an ultrasound transducer, a cell phone, a transmitter for a wireless web setup. The further you move away from the source the more area the power put out by the source is spread over and the weaker the intensity.

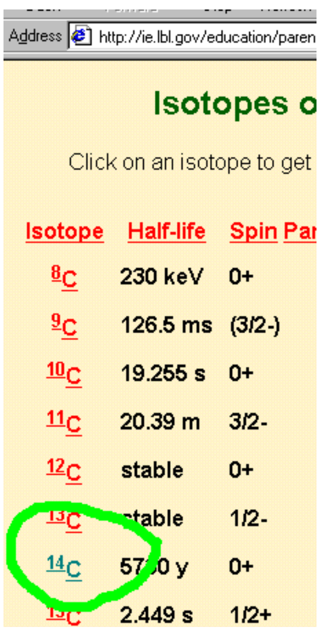
Example Problem #1:

a) Look up the decay mode of Carbon-14 and write down the nuclear reaction equation for its decay.
b) Look up the half-life of C-14 and its mass.
c) Determine the activity of 1 g of pure Carbon-14 in Bq and Ci.
d) Determine the flux rate a distance of 10 cm from the 1 g sample of C-14, for all particles emitted by the C-14.

I go to my Periodic Table with Isotope information and click on Carbon:



Then I scroll down to C-14.

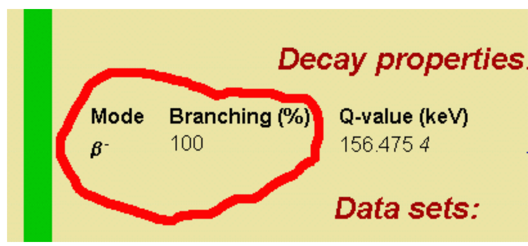

Address http://ie.lbl.gov/education/paren

Isotopes o

Click on an isotope to get

Isotope	Half-life	Spin Par
^{8}C	230 keV	0+
^{9}C	126.5 ms	(3/2-)
^{10}C	19.255 s	0+
^{11}C	20.39 m	3/2-
^{12}C	stable	0+
^{13}C	stable	1/2-
^{14}C	57[illegible]0 y	0+
^{15}C	2.449 s	1/2+

This tells me that C-14 undergoes β^- decay 100% of the time.

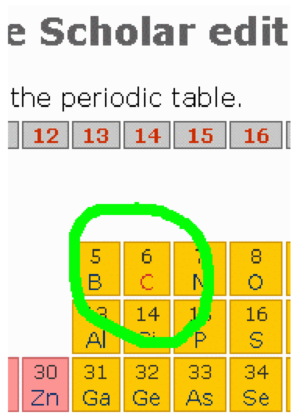


So I write the reaction equation for a β^- decay.

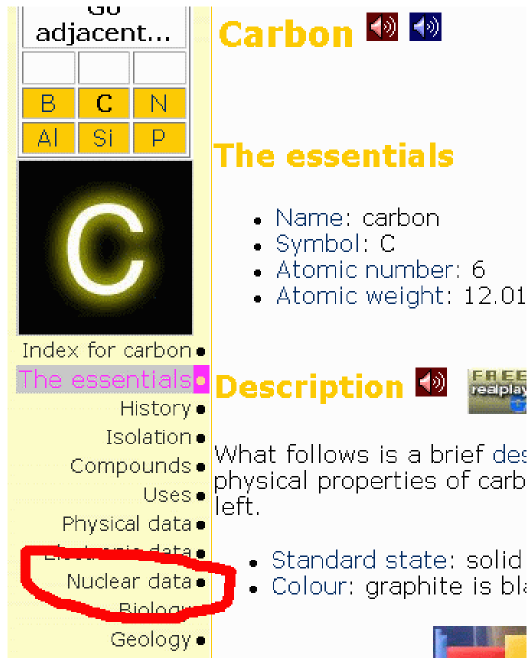
$$^{14}_{6}C \longrightarrow {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu} + {}^{14}_{7}N$$

The decay of C-14 produces β^- particles (electrons) and neutrinos.

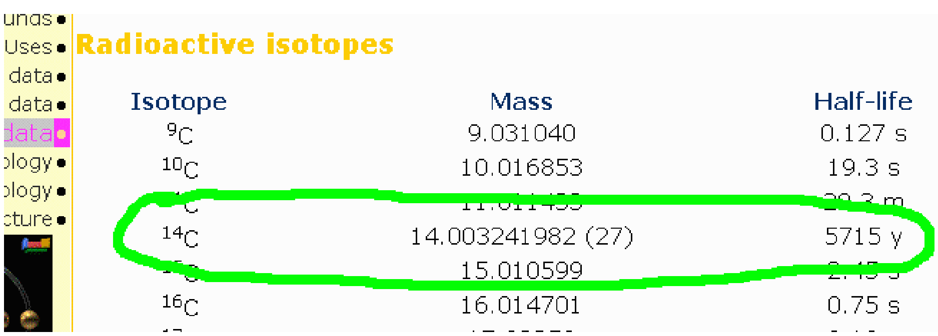
There's no mass information in the Periodic Table with Isotope Information, so I'll try the INTERACTIVE Periodic Table.
I click on Carbon.



I want data on the nucleus, so I click on Nuclear Data . . .



. . . and then I find C-14. Here's the mass and half-life. The half-life given here is slightly different than the half-life given on the other periodic table.



The mass of C-14 is $m_{C\text{-}14} = 14.0032\ u$ and its half-life is $T_{1/2} = 5715$ years.

To determine the activity I'll need to know how many nuclei are in a gram of C-14.

First I'll get the mass of C-14 in kg instead of u.

$$m_{C-14} = 14.0032\,u\left(\frac{1.66\times10^{-27}\,kg}{1\,u}\right) = 23.245\times10^{-27}\,kg$$

so mass of C-14 is 23.245×10^{-27} kg per nucleus

Now, I have .001 kg of C-14, and I know that 1 nucleus has mass of 23.245×10^{-27} kg. I'll use a ratio to figure out how many nuclei that is.

$$.001\,kg\left(\frac{1\text{ nucleus}}{23.245\times10^{-27}\,kg}\right) = 4.302\times10^{22}\text{ nuclei}$$

So $N = 4.302 \times 10^{22}$ nuclei. Now I can find the activity:

$$R = \frac{.693\,N}{T_{\frac{1}{2}}}$$

$$T_{\frac{1}{2}} = 5715\text{ yrs}\left(\frac{3.156\times10^{7}\,s}{1\text{ yr}}\right) = 1.804\times10^{11}\text{ sec}$$

I converted years to seconds because I want my activity in decays per second.

$$R = \frac{.693\,(4.302\times10^{22}\text{ nuclei})}{1.804\times10^{11}\text{ sec}}$$

$$= 1.653\times10^{11}\ \frac{\text{nuclei}}{\text{sec}}$$

So the activity is 1.653×10^{11} nuclei decaying per second or

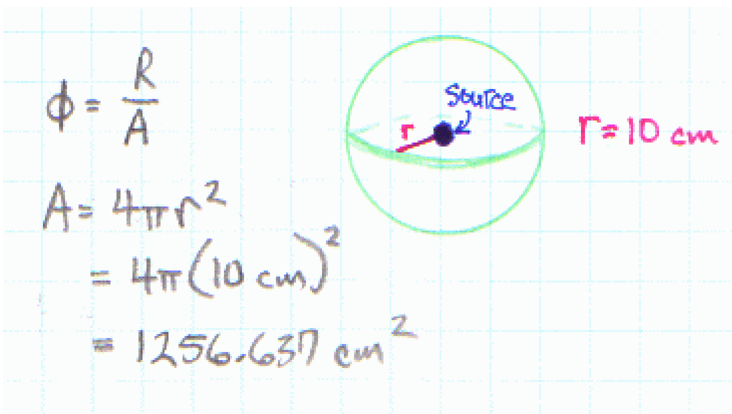
$R = 1.653 \times 10^{11}$ Bq.

To get this in Curies:

$$1.653\times10^{11}\,Bq\,\frac{1Ci}{37\times10^{9}\,Bq} = 4.47Ci$$

R = 4.47 Ci

Now to find the flux rates:



Each decay produces one β^-, so the electron flux rate is

$$\phi_{\beta^-} = \frac{1.653\times10^{11}\,\beta^-/\sec}{1256.637cm^2} = 131541560.5\beta^-/\sec/cm^2$$

The electron flux rate at a distance of 10 cm from the 1 gram sample of C-14 is 131.5 billion particles per second per square centimeter!

Each decay produces one ν, so the neutrino flux rate is

$$\phi_{\bar{\nu}} = \frac{1.653\times10^{11}\,\bar{\nu}/\sec}{1256.637cm^2} = 131541560.5\,\bar{\nu}/\sec/cm^2$$

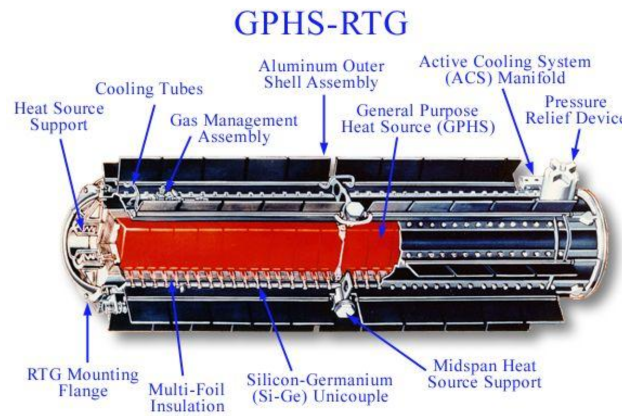
The neutrino flux rate at a distance of 10 cm from the 1 gram sample of C-14 is also 131.5 billion particles per second per square centimeter!

Betas and neutrinos are all the particles that are emitted.

These numbers are huge. They give you an idea of just how many nuclei are in a gram of something! There has to be an immense number of nuclei (4.302×10^{22} nuclei, actually) to be able to sustain that output for thousands of years.

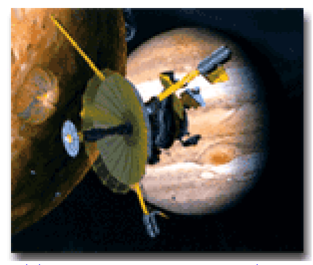
Example Problem #2:

Some NASA space probes carry Plutonium-238 on board them to generate power through the Plutonium-238's decay. These devices are known as Radioisotope Thermoelectric Generators or RTG's. This has caused some controversy since Plutonium is both toxic and radioactive, and since occasionally NASA missions blow up in the atmosphere and rain debris over the ground below.



http://spacepwr.jpl.nasa.gov/rtgs.htm

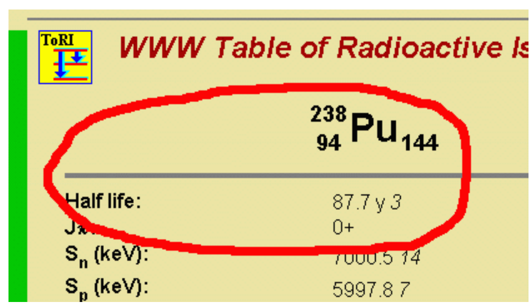
Write out the decay equation of Plutonium-238. Calculate the energy released per decay. Then figure out how many kg of Pu-238 would be required to generate the 570 Watts of power produced by the Galileo space probe's RTG. The Galileo probe recently wound down operation at Jupiter. Discuss how long this power source would last.



http://www.jpl.nasa.gov/galileo/

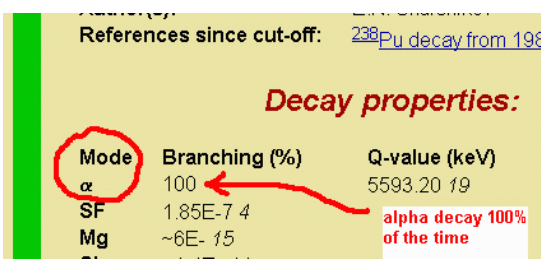
Solution:

The first thing to do is look up information on Pu-238 on a Periodic Table like in the example from the other day.



$^{238}_{94}Pu_{144}$

Half life:	87.7 y 3
Jπ:	0+
S_n (keV):	7000.5 14
S_p (keV):	5997.8 7



References since cut-off: 238Pu decay from 198

Decay properties:

Mode	Branching (%)	Q-value (keV)
α	100	5593.20 19
SF	1.85E-7 4	
Mg	~6E- 15	

alpha decay 100% of the time

So I now have some basic info on Pu-238:

Pu-238

$Z = 94$

$T_{1/2} = 87.7$ years

α decays 100% of the time

Now I'll work out the decay equation since I know that Pu-238 does an alpha decay:

Decay equation

$$^{238}_{94}Pu \longrightarrow {}^{4}_{2}\alpha + {}^{234}_{92}U$$

To figure out the mass defect I need to figure the total mass before and after the reaction. I look up the masses of Pu-238 and U-234 from the INTERACTIVE periodic table like in the example from Day 23.

Isotope	Mass	Half-life
^{236}Pu	236.04605	2.87 y
^{237}Pu	237.04840	45.7 d
^{238}Pu	238.04955	87.74 y
^{239}Pu	239.05216	24110 y

^{230}U	230.03393
^{231}U	231.03626
^{232}U	232.03715
^{233}U	233.039628
^{234}U	234.0409468 (24)
^{235}U	235.0439242 (24)
^{236}U	236.045561

These are the masses of the atoms – I have to subtract off the electrons to get the masses of the nuclei. The mass of the alpha particle is in the table in today's topics summary.

Mass of Pu-238 238.04955 u minus mass of 94 electrons

$$m_{Pu} = 238.04955 - 94(.0005486)$$
$$= 237.9979816\ u$$

Mass of U-234 234.0409468 minus 92 electrons

$$m_U = 234.0409468 - 92(.0005486)$$
$$= 233.9904756\ u$$

Mass of α $m_\alpha = 4.0015060\ u$

MASS BEFORE REACTION $\Rightarrow$ 237.9979816 u

MASS AFTER REACTION $\Rightarrow$ 233.9904756 u + 4.0015060 u = 237.9919816 u

Now I subtract to find the mass defect.

MASS DEFECT:

$$\begin{array}{r} 237.9979816\ u \\ -\ 237.9919816\ u \\ \hline 0.006000\ u \end{array}$$

mass defect is $m = 0.006u$

in kg this is

$$0.006\,u \; \frac{1.6605 \times 10^{-27}\ kg}{1\ u} = 9.963 \times 10^{-30}\ kg$$

With the mass defect nailed down, I can see how much energy was released because the mass defect turned into energy.

Now find out the energy released in this decay:

$$E = mc^2$$

$$E = 9.963 \times 10^{-30}\ kg \left(3 \times 10^{8}\ m/s\right)^2$$

$$\boxed{= 8.9667 \times 10^{-13}\ J}$$ in one decay

The RTG on the Galileo Probe has power output of $P = 570\text{ W} = 570\text{ J/s}$

$$570\,\frac{J}{s}\left(\frac{1\text{ decay}}{8.9667\times10^{-13}\,J}\right) = 6.35685\times10^{14}\text{ Nuclei Decay/sec}$$

That's the Activity Rate R

I can use the activity rate equation to find the number of Pu-238 nuclei in the RTG, and then use the number of nuclei to find the total mass of Pu-238 in the RTG.

$$R = \frac{.693\,N}{T_{1/2}}$$

$$T_{\frac{1}{2}} = 87.7\text{ yrs}\left(\frac{3.156\times10^{7}\text{ sec}}{1\text{ yr}}\right) = 2.76781\times10^{9}\text{ sec}$$

So let's find N

$$6.35685\times10^{14}\,\frac{\text{Nuclei}}{\cancel{\text{sec}}} = \frac{.693\,N}{2.76781\times10^{9}\,\cancel{\text{sec}}}$$

$$N = 6.35685\times10^{14}\text{ Nuc}\times 2.76781\times10^{9}/.693$$

$$N = 2.5389\times10^{24}\text{ Nuclei}$$

Each atom of Pu-238 has mass of 238.04955u so the total mass of Plutonium in the RTG is

$$m_{TOT} = 2.5389\times10^{24}(238.04955u) = 6.04384\times10^{26}u$$

in kg this is

$$6.04384\times10^{26}u\left(\frac{1.6605\times10^{-27}\text{ kg}}{1u}\right) = 1.00358\text{ kg}$$

So the probe is powered by 1.0 kg of Pu-238 (assuming the Pu-238 is pure).

How long will this power source last? Well, the Plutonium has a half-life of 87.7 years, so after 80 years it will still be putting out more than 50% of its original power! It really just keeps going and going and going . . .

http://www.energizer.com/bunny/default.asp