DAY 25

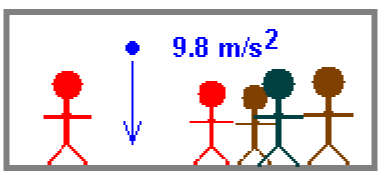
Summary of Topics Covered in Today's Lecture

General Relativity

Special Relativity deals with inertial (non-accelerating) reference frames. To deal with reference frames that are accelerating requires Einstein's theory of General Relativity.

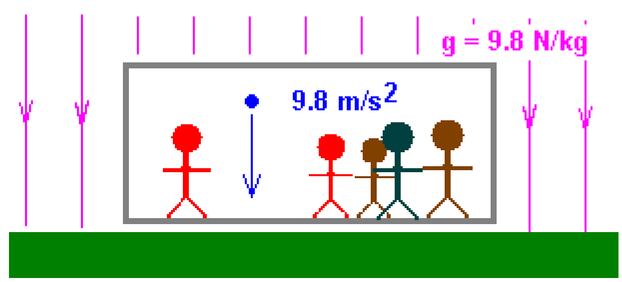
General Relativity is based on the same two ideas as Special Relativity (laws of physics same for all; speed of light same for all), plus one more:

There is no physical difference between a reference frame that lies in a uniform gravitational field and a reference frame that is undergoing constant acceleration.



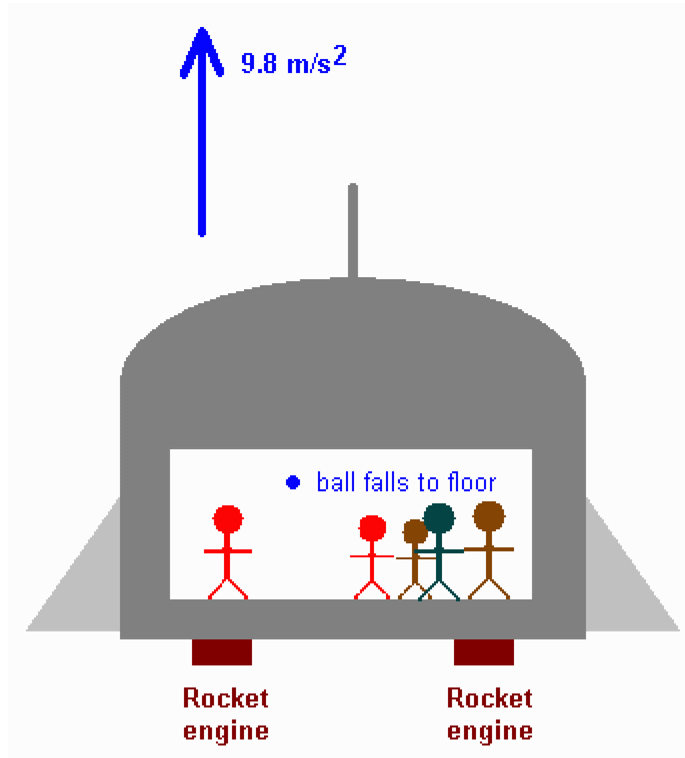
Consider a classroom, on Earth, in which a professor is given a lecture on falling objects. A golf ball dropped in the classroom accelerates downward at a rate of 9.8 m/s^2.

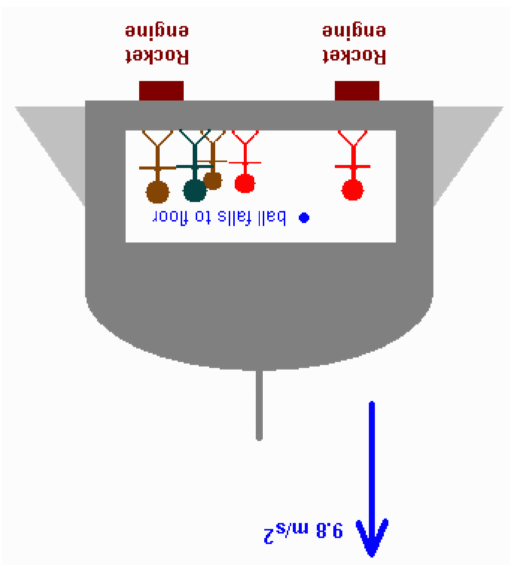
The class sees the ball accelerate because the classroom is in a uniform gravitational field of 9.8 N/kg directed downward. The force of gravity causes the acceleration, and the acceleration of 9.8 matches the gravitational field strength of 9.8 because inertia mass and gravitational mass are identical (see Day 1 – yes, Day 1).



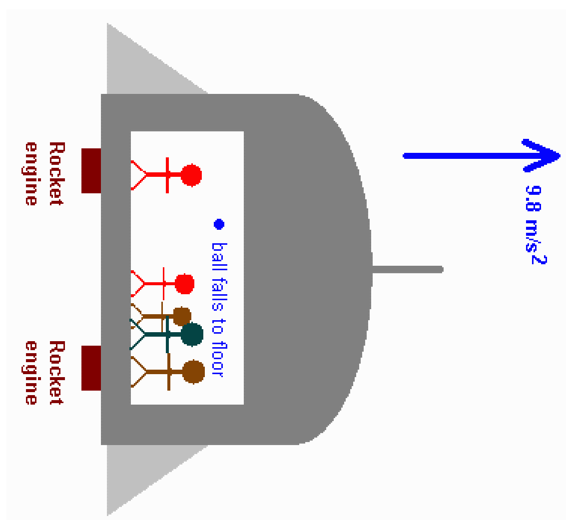
However, if the class were in a rocket ship that was accelerating upward at a constant 9.8 m/s^2, the class could not tell the difference between that and the uniform gravitational field. While the professor was holding the ball, it would accelerate with the ship. When the professor released the ball, it would stop accelerating and would move with constant velocity. However, the ship would continue to accelerate, and so the distance between the ball and the floor would decrease -- the ball would "fall" to the floor as though a force was acting on it. This is just like in a car -- when you step on the gas and accelerate forward, loose objects on the dashboard "fall" backward as though there were forces on them, and when you step on the brake loose objects "fall" forward.

To make things perfectly clear here, this has nothing to do with which way the ship is heading. Whether it is moving up, or moving down . . .





. . . or moving sideways . . .

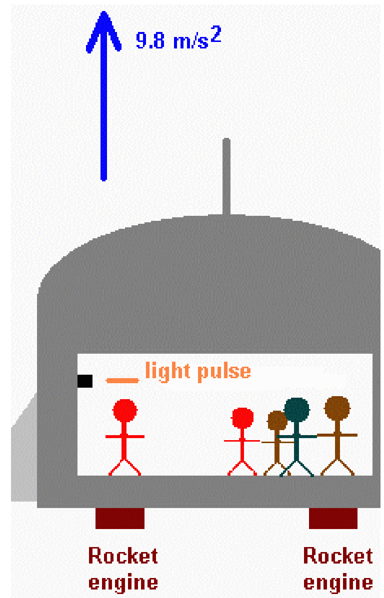


the ball always "falls" to the floor of the classroom at a rate of 9.8 m/s^2.

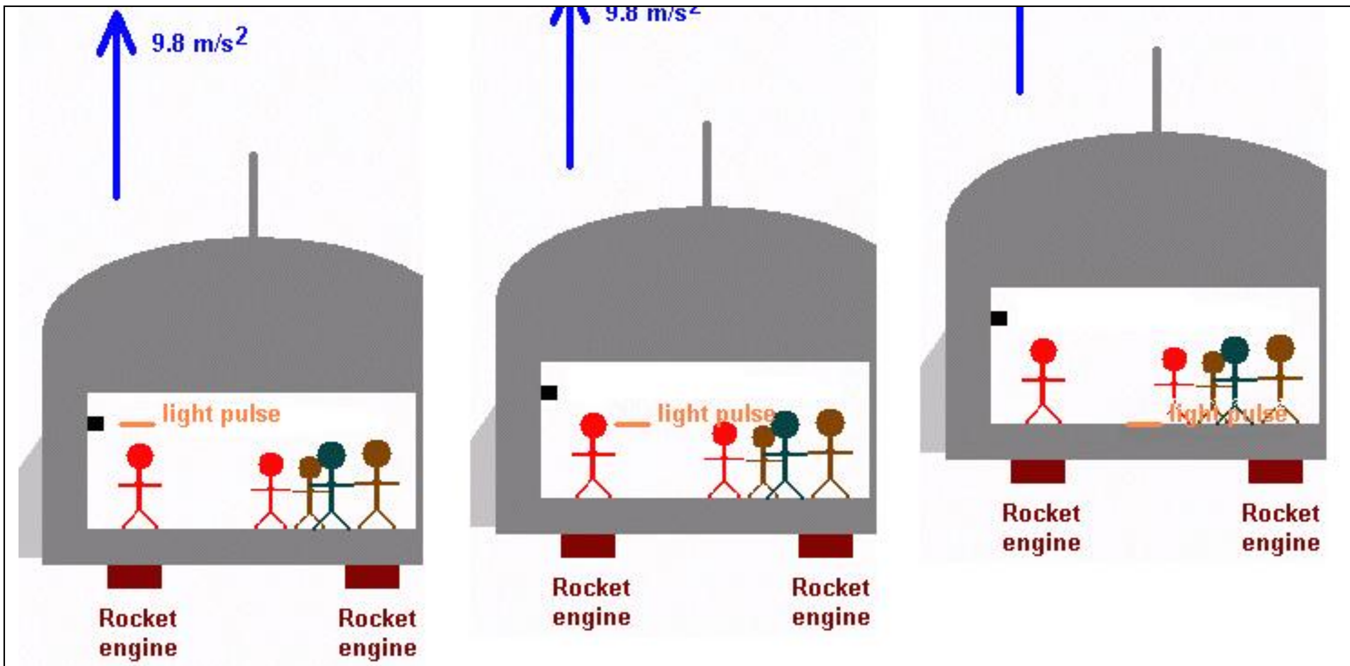
Einstein's theory explains why inertial mass (as in $\Sigma\mathbf{F} = m\mathbf{a}$) and gravitational mass (as in $\mathbf{F} = m\mathbf{g}$) are identical; it is because acceleration and gravity are identical. We discussed inertial vs. gravitational mass in some depth at the start of this semester – see Day 1.

There are a number of interesting consequences that spring from this concept that acceleration and gravity are equivalent. These include gravitational lensing, gravitational red shift & time dilation, gravitational waves, and a precession of orbits.

Gravitational Lensing



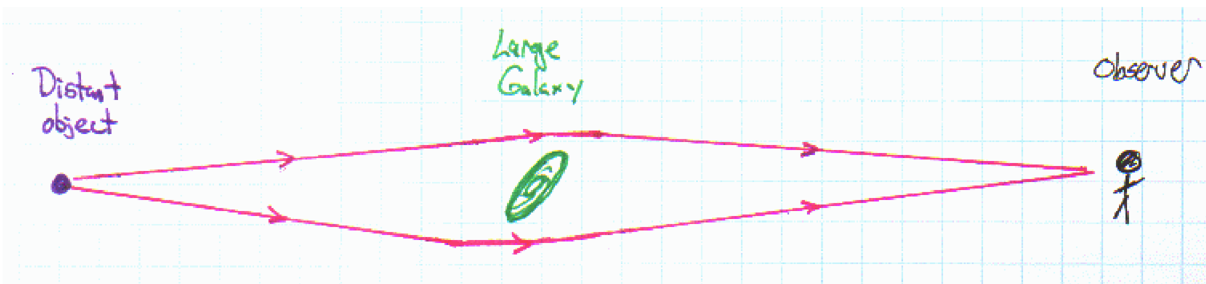
If gravity is physically identical to acceleration, then a gravitational field must bend a light beam. Suppose the "classroom in the rocket ship" has a lamp on the wall that emits a light pulse, as shown at right. Because the ship is accelerating upward, the light pulse will "fall" towards the floor of the classroom at a rate of 9.8 m/s^2. The light moves so rapidly across the room that it will "fall" very, very little as it crosses the room, but nonetheless, it will "fall" and if the acceleration of the ship was great enough, the "fall" could be significant, as in the figure below.



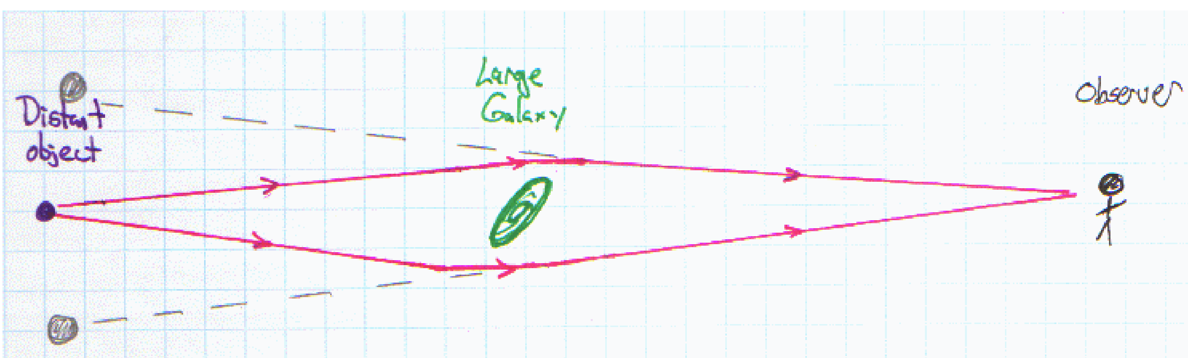
Here we see that, as the rocket accelerates forward, the light pulse is left behind. The result is that the pulse "falls to the floor" as seen by the occupants of the ship, who cannot tell if they are accelerating or if they are in a strong gravitational field. Note that the pulse moves horizontally with constant speed as seen from an inertial reference frame.

If a gravitational field is identical to an accelerating reference frame, then a light pulse emitted from a lamp on the wall of a classroom in a uniform gravitational field should also fall towards the ground. In fact, it does, but the effect is quite small. However, the effect of light being deflected by a gravitational field is well-documented.

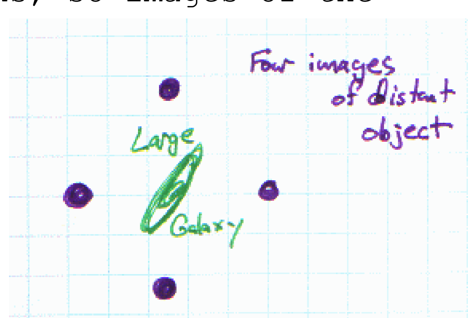
The most dramatic examples of gravity bending light are the examples of "gravitational lensing" found in astronomy. If a massive object such as a galaxy lies between an observer and a distant object, the galaxy's gravitational field can bend light from the distant object toward the observer as shown here.



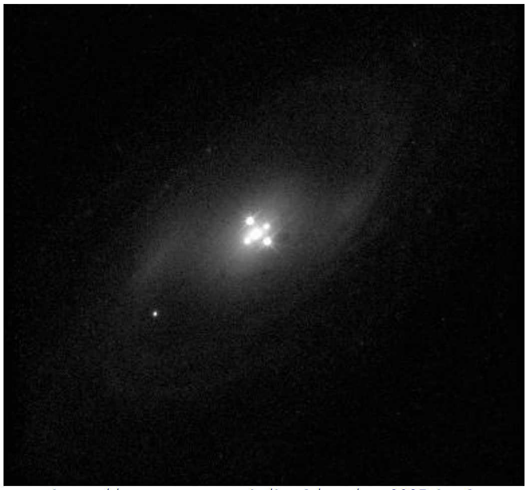
To the observer, light from the distant object seems to be coming from above and below the galaxy.



The result is that the observer sees an image of the distant object appearing above and below the galaxy. Of course this happens in three dimensions, so images of the distant object can appear to the left and right of the galaxy as well. This is what is known as an "Einstein Cross".



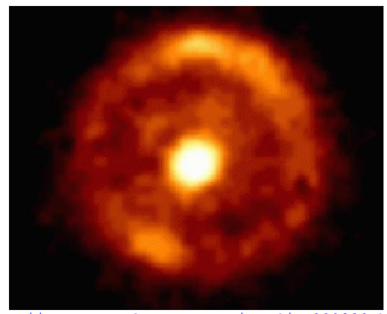
Einstein Cross



http://www.astr.ua.edu/keel/agn/qso2237.html

At left is a picture of a real Einstein Cross.

If the alignment is just right, it is possible for light from the distance object to converge from all directions around the galaxy toward the observer. The result of this sort of unique geometry is an "Einstein Ring" as pictured below.



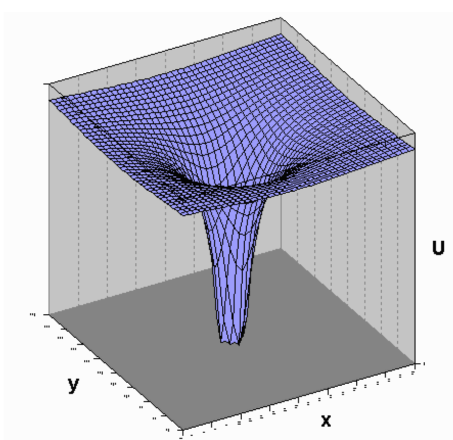
http://antwrp.gsfc.nasa.gov/apod/ap980330.html

However, gravitational lensing usually doesn't produce such neat, clean shapes. The usual result of gravitational lensing is multiple images, often distorted into arc shapes, as seen in the image below.

http://antwrp.gsfc.nasa.gov/apod/ap000423.html

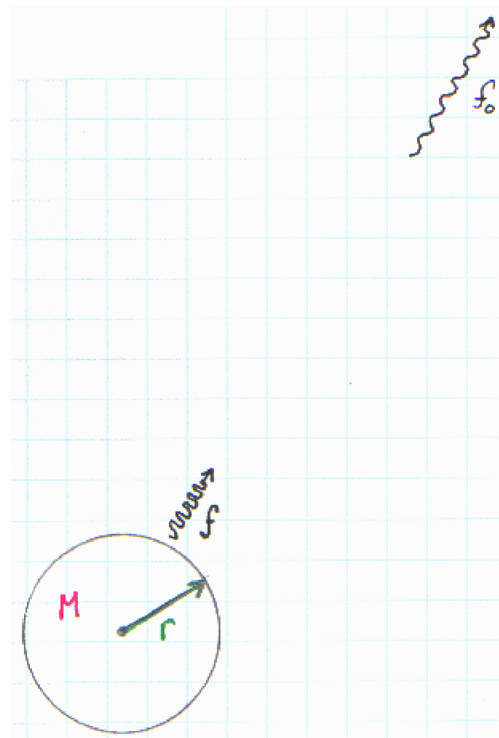
Here all the blue-colored images are all of the same object! The cluster of galaxies at the center of the picture has lensed the light from that distant object.

Since gravity does not exert a force on light (light has no mass so a "force" is meaningless), we have to think of gravity in other ways than as a force. Light follows a straight-line path when not in a gravitational field, so we think of gravity as a "warping" space. Light wants to travel in a straight line, but space itself is curved by the presence of a massive object, so the light travels in a curved path.



Gravitational Red Shift

Just as a ball that is launched upward from a planet's surface loses Kinetic Energy as it rises upward in the planet's gravitational field, so does light emitted from a planet's surface lose energy as it rises upward in the planet's gravitational field. However, light does not slow down. Rather, the energy of light is related to its frequency. High-frequency (short wavelength) light carries more energy; low-frequency (long wavelength) light carries less. Thus blue light is more energetic than red.



Consider a beam of light with frequency f emitted from a planet of mass M and radius r. When the light gets far from the planet (distance is large compared to the planet's radius), the frequency of the light will have decreased because the light will have lost energy, and it will have frequency f_0. The relationship between M, r, f_0, and f is

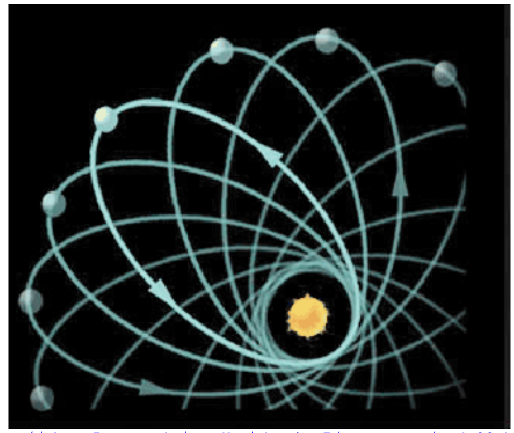
$$\frac{f_0}{f} = \sqrt{1 - \frac{2GM}{c^2 r}}$$

This change in frequency is called gravitational red shift (the name "red shift" is because the effect tends to make blue light less energetic and therefore redder in color).

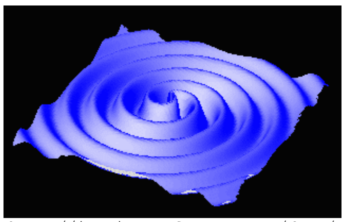
Changing the frequency of light also changes its wavelength, since speed does not change. As seen in the example problem below, the gravitational red shift means that there is also a gravitational time dilation. As we learned in Special Relativity, time dilation leads to effects on length, simultaneity, and a host of other phenomena.

Thus gravity is not just a warping of space; time is affected as well. Gravity is a warping of space and time (or space-time) by massive objects.

Other General Relativity Effects


http://phyun5.ucr.edu/~wudka/Physics7/Notes_www/node98.html

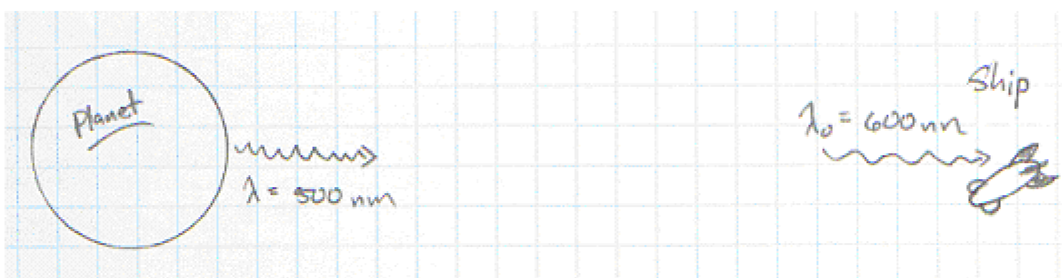
There are other effects of General Relativity. General Relativity has been used to explain a curious precession in the orbit of Mercury (shown greatly exaggerated here). General Relativity also predicts the existence of gravitational waves, created by oscillating masses the same way that oscillating charges create electromagnetic waves. Research into the existence of gravitational waves is on the cutting-edge of science today.


http://imagine.gsfc.nasa.gov/docs/features/topics/gwaves/gwaves.html

Example Problem #1

An alien on a planet's surface signals to its mother ship using a laser of wavelength 500 nm. The mother ship, which is at a great distance from the planet, receives the signals but by that point the laser beam which is carrying them has a wavelength of 600 nm. If the alien on the surface talks for 30 seconds, how long will the message be for the aliens on the mother ship?

Solution:



On the surface, the alien talks for 30 secnds, so the light pulse carrying the signal is $d = c\,t = 3 \times 10^8$ m/s (30 s) = 9×10^9 m long.

How many wavelengths is that?

$$9 \times 10^9 \text{ m} \; \frac{1\,\lambda}{500 \times 10^{-9} \text{ m}} = 1.8 \times 10^{16} \text{ wavelengths}$$

At the ship, all 1.8×10^{16} wavelengths arrive. The length of the pulse is the number of wavelengths times the wavelength (now 600 nm):

$d = 1.8 \times 10^{16} \times 600 \times 10^{-9}$ m $= 1.08 \times 10^{10}$ m

So the pulse will take

$$t = \frac{d}{c} = \frac{1.08 \times 10^{10} \text{ m}}{3 \times 10^{8} \text{ m/s}} = 36 \text{ seconds to be received.}$$

So the message is 30 seconds long when sent, 36 seconds long when received. The aliens on the ship will see their explorer on the surface as talking, moving, breathing, etc. a little slowly! Time is running more slowly on the surface!

Example Problem #2

In the above example, if the planet has the same diameter as Earth, what mass does it have? Give your answer in Earth masses.

Solution:

I look up Earth's mass and radius:

Earth mass = 5.98×10^{24} kg
Earth radius = 6.37×10^{7} m

$$\frac{f_o}{f} = \sqrt{1 - \frac{2GM}{c^2 r}}$$

but $f\lambda = v$ (and $v = c$ for light)
so $f = c/\lambda$

$$\frac{c/\lambda_o}{c/\lambda} = \sqrt{1 - \frac{2GM}{c^2 r}}$$

the c's cancel out...

$$\frac{\lambda}{\lambda_o} = \sqrt{1 - \frac{2GM}{c^2 r}} = \frac{500nm}{600nm} = \frac{5}{6}$$

square both sides...

$$1 - \frac{2GM}{c^2 r} = \left(\frac{5}{6}\right)^2$$

Solve for M...

$$M = -\left[\left(\frac{5}{6}\right)^2 - 1\right]\frac{c^2 r}{2G}$$

Plug in numbers...

$$M = -\left[\left(\frac{5}{6}\right)^2 - 1\right]\frac{\left(3\times10^8 \frac{m}{s}\right)^2 6.37\times10^7 m}{2(6.67\times10^{-11})\frac{Nm^2}{kg^2}}$$

$$M = 0.305556 \frac{5.733 \times 10^{24} \left(\frac{m}{s}\right)^2 m}{1.3334 \times 10^{-10} \frac{kgm/s^2\, m^2}{kg^2}} = 1.3132 \times 10^{34}\, kg$$

$$1.3132 \times 10^{34}\, kg \left(\frac{1 Earth}{5.98 \times 10^{24}\, kg}\right) = 2{,}196{,}000{,}000\, Earths$$