DAY 24

Summary of Topics Covered in Today's Lecture

Paradoxes in Special Relativity

It is possible to get quite flummoxed by apparent paradoxes in Special Relativity. In dealing with such problems, it is important to keep in mind all aspects of relativity – time dilation, length contraction, simultaneity – while also being alert for "symmetric" situations which are not so symmetric.

Relativistic Momentum & Kinetic Energy

Momentum and Kinetic Energy have different forms under Special Relativity. We are not going to derive these – we are simply going to introduce the equations:

Momentum:

$$\mathbf{p} = \gamma\, m\, \mathbf{v}$$

Kinetic Energy:

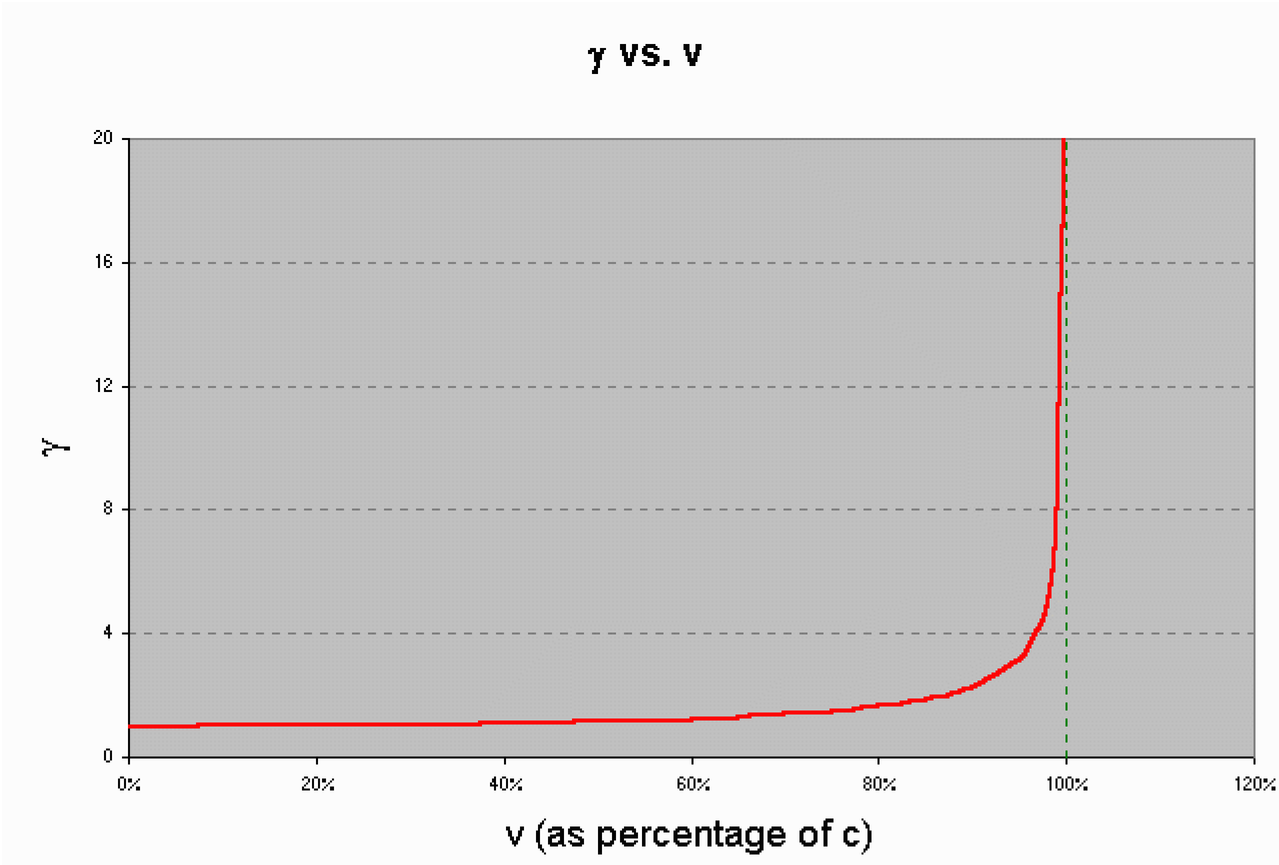
$$KE = (\gamma - 1)\, m\, c^2$$

The term $m\,c^2$ is called the "rest energy" of a mass. This is the famous

$$E = m\, c^2$$

concept – it says that mass and energy are equivalent. Matter is essentially a form of energy. Even a small amount of matter represents an enormous quantity of energy. Conversion of matter-to-energy is the mechanism thought to power stars.

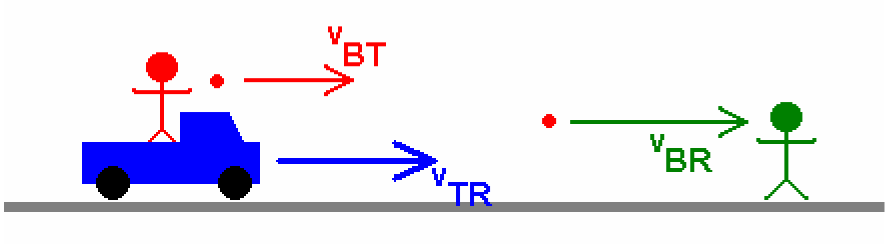
Speed of Light as the Upper Limit for Speed



As v → c, γ → infinity. Therefore a mass-having object moving at the speed of light represents infinite momentum and infinite energy. It would require infinite force and infinite energy to accelerate even an electron to the speed of light. Since infinite force and infinite energy do not exist in the universe, no mass-having particle can reach the speed of light. And if the particle cannot be accelerated <u>to</u> the speed of light, it cannot be accelerated to <u>faster</u> than the speed of light.

Velocity Addition

Let's go back to a concept we discussed a few days ago. Jack can throw a dense ball (air resistance is not an issue) at a speed of 50 mph.



A truck is moving down the road at 60 mph, and Jack is in the back of the truck. The 60 mph figure is the speed of the truck measured with respect to the road:

$v_{\text{TRUCK with respect to ROAD}} = v_{TR} = 60 \text{ mph}$

Jack throws a ball forward at 50 mph. This is the speed of the ball with respect to Jack (and, since Jack is in the truck, with respect to the truck):

$v_{\text{BALL with respect to TRUCK}} = v_{BT} = 50 \text{ mph}$

The speed of the ball with respect to the road is then

$v_{\text{BALL with respect to ROAD}} = v_{BR} = v_{TR} + v_{BT}$

$v_{BR} = v_{TR} + v_{BT} = 60 \text{ mph} + 50 \text{ mph} = 110 \text{ mph}$

Simple enough. This is called velocity addition. However, things can't work this way at high speeds. After all, if the truck is moving at 0.60 c and Jack throws the ball forward at 0.50 c, this basic velocity addition, which is part of Newtonian/Galilean Relativity, would have the ball moving at 1.10 c. We've learned that that is not possible.

In fact, if you include the effects of Special Relativity, the equation for adding velocities becomes

$$v_{BR} = \frac{v_{TR} + v_{BT}}{1 + \frac{v_{TR} v_{BT}}{c^2}}$$

For small v's, the term $v_{TR}v_{BT}/c^2$ is essentially zero, and the results are the same as the Newtonian/Galilean velocity addition. However, for large velocities the results are much different, as we shall see in the examples.

Example Problem #1

How much energy is stored in a 500 g lab mass? How long could this energy power a 200 Hp motor?

Solution:

m = 0.5 kg

$E = m c^2$
$E = 0.5 \text{ kg } (3 \times 10^8 \text{ m/s})^2$
$E = 4.5 \times 10^{16} \text{ kgm}^2/\text{s}^2$
$E = 4.5 \times 10^{16} \text{ J}$

P = E/t

P = 200 Hp = 149,200 W

$149{,}200 \text{ J/s} = 4.5 \times 10^{16} \text{ J/t}$

$t = 3.01609 \times 10^{11}$ sec = 9600 years

Example Problem #2

If the truck in the discussion above is moving at 0.60 c and Jack throws the ball forward at 0.50 c (both of which are at least theoretically possible), basic velocity addition from Newtonian/Galilean Relativity would have the ball moving at 1.10 c (which is not theoretically possible). What will be the actual speed of the ball with respect to the road?

Solution:

$$v_{BR} = \frac{v_{TR} + v_{BT}}{1 + \frac{v_{TR} v_{BT}}{c^2}}$$

v_{TR} = 0.60 c
v_{BT} = 0.50 c

$$v_{BR} = \frac{0.60c + 0.50c}{1 + \frac{0.60c(0.50c)}{c^2}} = \frac{1.10c}{1+.3} = \frac{1.10}{1.3}c = .846c$$

So the ball will be moving at 85% the speed of light, measured with respect to the road.

Example Problem #3

Graph the KE of a 1 kg mass vs. its speed for speeds of 0 to c using the classical physics definition of KE (½ m v^2). On the same plot graph the KE of the mass using the relativistic definition of KE. At what speed does the difference between them reach 5%?

Solution:

I'll use EXCEL or some other plotter to graph both of these, and I'll calculate the percent difference [(1^{st}-2^{nd})/1^{st}] between the two values. I see that there is essentially no difference between what the two KE formulas yield until I reach speeds of about 10 million meters per second (1/30 c). Then the difference begins to gradually increase, as seen in the plot. The relativistic KE is greater.

There's a 5% difference between the two methods of calculating KE at approximately 75 million meters per second, or 1/4 c.

Speed (m/s)	KE = 1/2 mv^2	KE = $(\gamma-1)mc^2$	% Difference
0	0.000E+00	0.000E+00	0.0%
1,000,000	5.000E+11	5.000E+11	0.0%
2,000,000	2.000E+12	2.000E+12	0.0%
3,000,000	4.500E+12	4.500E+12	0.0%
4,000,000	8.000E+12	8.001E+12	0.0%
5,000,000	1.250E+13	1.250E+13	0.0%
6,000,000	1.800E+13	1.801E+13	0.0%
7,000,000	2.450E+13	2.451E+13	0.0%
8,000,000	3.200E+13	3.202E+13	0.1%
9,000,000	4.050E+13	4.053E+13	0.1%
10,000,000	5.000E+13	5.004E+13	0.1%
11,000,000	6.050E+13	6.056E+13	0.1%
12,000,000	7.200E+13	7.209E+13	0.1%
13,000,000	8.450E+13	8.462E+13	0.1%
14,000,000	9.800E+13	9.816E+13	0.2%
15,000,000	1.125E+14	1.127E+14	0.2%
16,000,000	1.280E+14	1.283E+14	0.2%
17,000,000	1.445E+14	1.448E+14	0.2%
18,000,000	1.620E+14	1.624E+14	0.3%
19,000,000	1.805E+14	1.810E+14	0.3%
20,000,000	2.000E+14	2.007E+14	0.3%