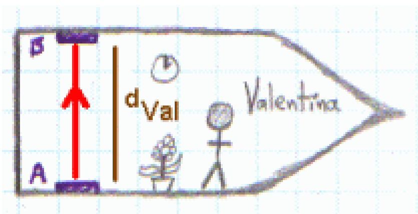
DAY 23

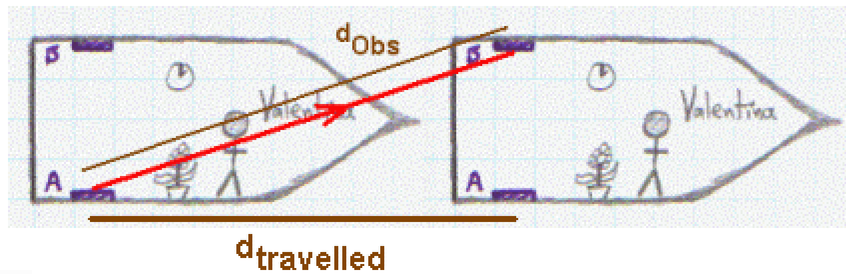
Summary of Topics Covered in Today's Lecture

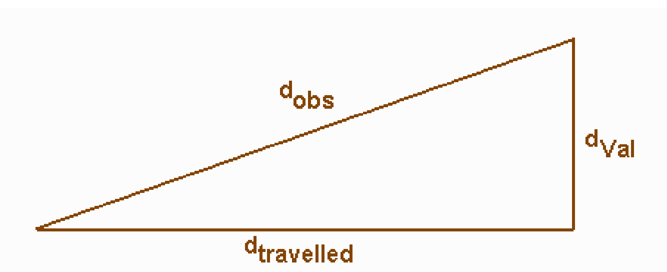
Time Dilation and Length Contraction

Continuing from the previous lecture...

Let's make a triangle out of these two distances, plus the distance traveled by the ship if it is moving at speed v:







Then using the Pythagorean Theorem:

$$d_{Val}^2 + d_{travelled}^2 = d_{obs}^2$$

where

$$d_{Val} = c\ t_{Val}$$
$$d_{travelled} = v\ t_{obs}$$
$$d_{obs} = c\ t_{obs}$$

$$(c\ t_{Val})^2 + (v\ t_{obs})^2 = (c\ t_{obs})^2$$
$$c^2\ t_{Val}^2 + v^2\ t_{obs}^2 = c^2\ t_{obs}^2$$
$$c^2\ t_{Val}^2 = c^2\ t_{obs}^2 - v^2\ t_{obs}^2$$
$$c^2\ t_{Val}^2 = (c^2 - v^2)t_{obs}^2$$
$$c^2\ t_{Val}^2\ /(c^2 - v^2) = t_{obs}^2$$
$$t_{Val}^2\ /(1 - v^2/c^2) = t_{obs}^2$$

$$t_{Val}\ /(1 - v^2/c^2)^{1/2} = t_{obs}$$

$$\boldsymbol{\gamma\ t_{Val} = t_{obs}}$$

where

$$\boldsymbol{\gamma = 1/(1 - v^2/c^2)^{1/2}}$$

This means that any event that occurs on board the ship and is measured to last for time t_{Val} will, if seen by an observer outside the ship who sees the ship as moving at some speed v, last longer than t_{Val} by a factor of γ. This is what is known as *time dilation*.

Because the time measured by two different people is dependent on their relative velocities, distance measurements are also affected, also by an amount of γ. This effect is known as *length contraction*.

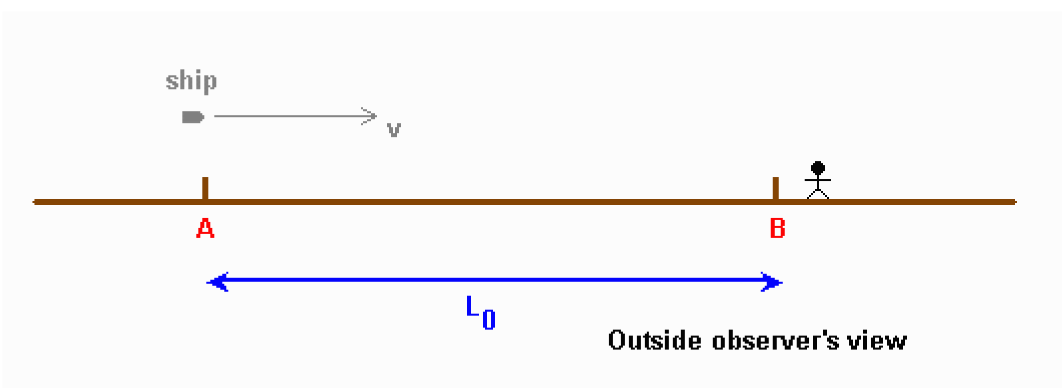
Proper Time and Length

The time that an event takes as measured in a frame of reference at rest with respect to that event is called the *proper time*. For example, suppose that the ship in our diagram at top is moving such that $\gamma = 2$. Now, Valentina needs to "answer the call of nature", and spends 5 minutes in the ship's bathroom as measured by a clock on board ship. The proper time for the bathroom stop is 5 minutes. As observed by the outside observer, that stop took 10 minutes:

$$\gamma \, t_{Val} = t_{obs}$$

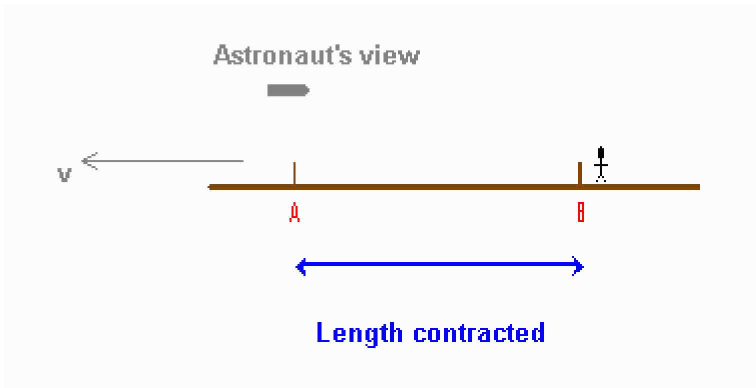
$$2 \ (5 \text{ min}) = 10 \text{ min}$$

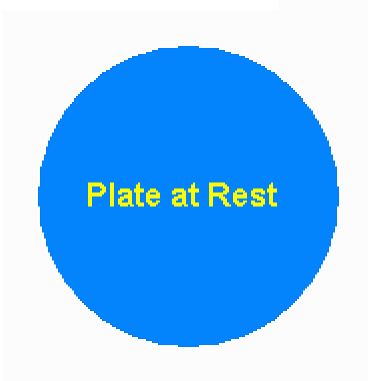
Suppose further that at the moment Valentina steps into the bathroom, the outside observer sees the ship pass a marker (A). At the moment Valentina steps out of the bathroom the outside observer sees the ship pass a second marker (B). The distance between A & B as measured by the outside observer, <u>who is at rest with respect to them</u>, is L_0.



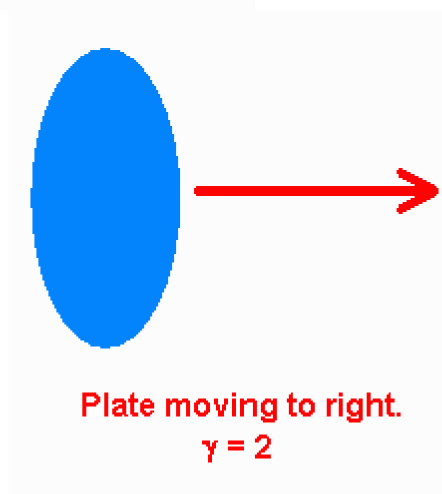
This L_0 is called the proper distance between A & B, or the *proper length*. If the outside observer knows the speed of the ship, the observer can use that speed, and the time it took the ship to go from A to B, to measure L_0: $L_0 = v \times 10$ minutes.

However, from Valentina's point of view, A, B, and the outside observer are all moving past at speed v. Furthermore, the time between A & B was 5 minutes. Therefore, Valentina measures the distance between A & B to be v x 5 minutes; that's half the distance measured by the outside observer. Valentina sees the distance from A to B as being shortened. This is the length contraction effect.





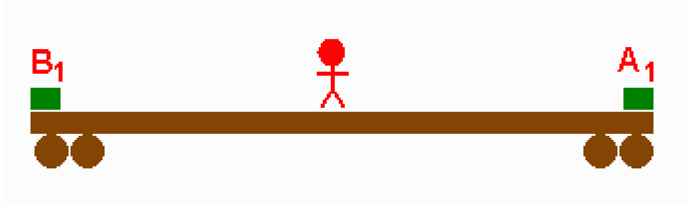
Length contraction only occurs in the direction of motion. So if a round plate passes by at high speed, it becomes elliptical.



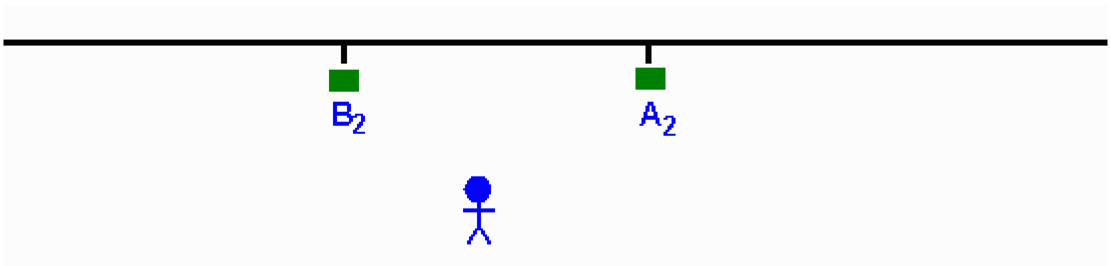
Simultaneous Events

Length contraction leads to the conclusion that two events that are simultaneous in one reference frame are not simultaneous in another reference frame if there is relative motion between the two.

Suppose two people do an experiment. One person – Jane – will ride on a high-speed rail car. Jane will take two clocks with her. She synchronizes her two clocks. Pictured below are Jane, her rail car, and her two clocks (one at A_1, the other at B_1):

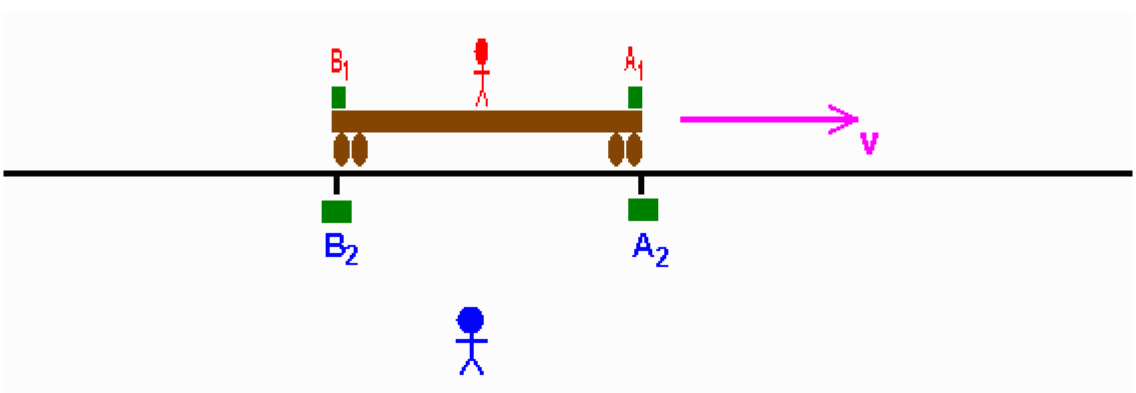


A second person – Sally – will set up an observation post on the side of the track. Sally also has two synchronized clocks at A_2 & B_2.



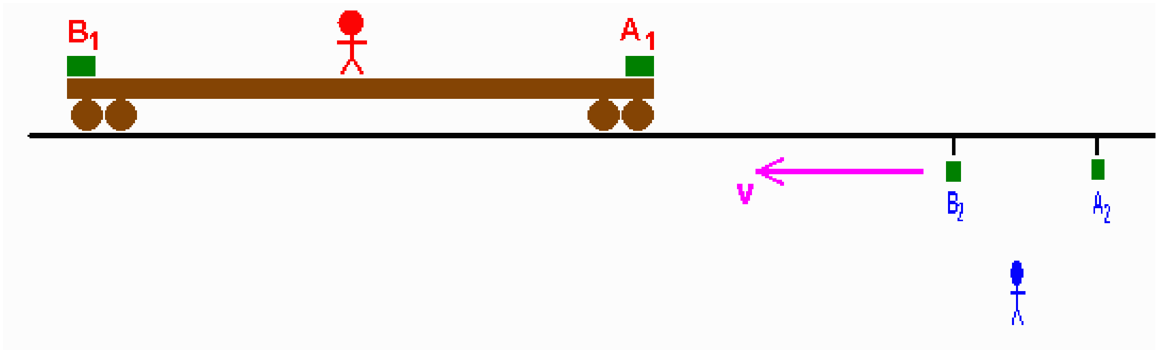
In the experiment, Jane will move past Sally at high speed. Each will take stop-action photos of the situation.

From Sally's point of view, here is what happens:
Jane moves by at high speed (v). Jane is length-contracted.
The lining up of A_1 over A_2 is simultaneous with the lining up of B_1 over B_2:

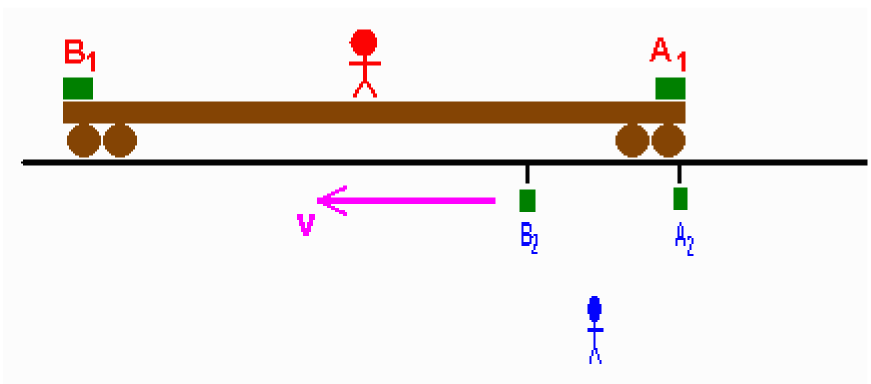


Sally takes a photo of this line-up.

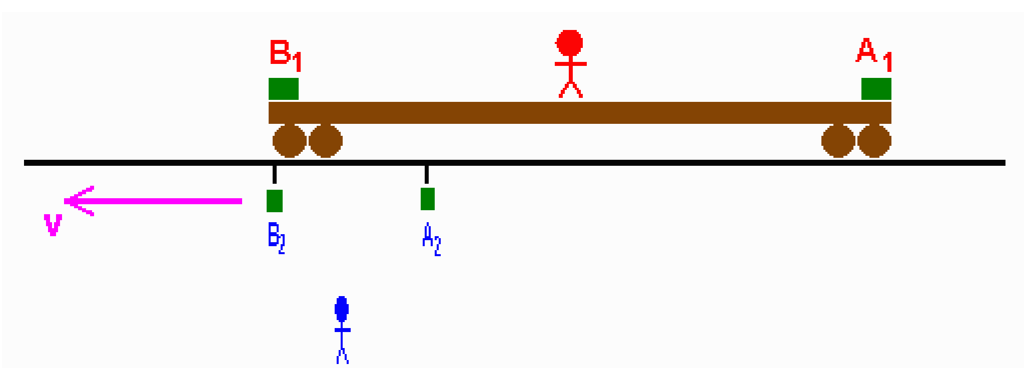
From Jane's point of view, Sally is length-contracted and moving toward her at high speed:



To Jane, the line-up of A_1 & A_2 is not simultaneous with the line-up of B_1 & B_2. First A_1 & A_2 line up:



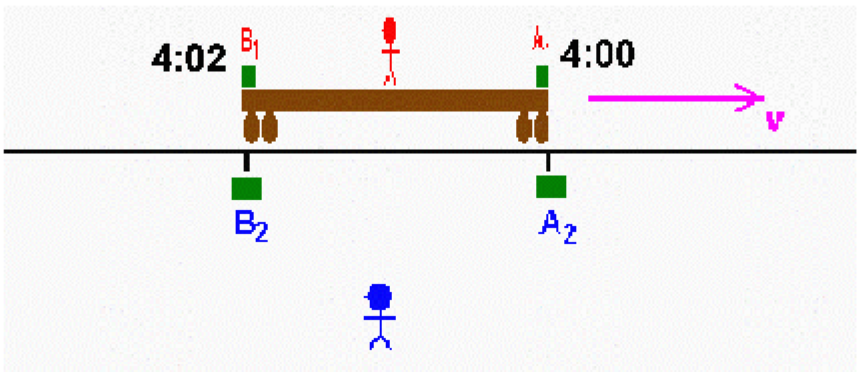
Then B_1 & B_2 line up:



Jane takes a photo of both of these line-ups.

Note that what was simultaneous to Sally was not simultaneous to Jane. In Jane's reference frame, the A's lined up before the B's did. Therefore, Jane's clock at A_1 reads one time when it lines up with A_2. A few moments later, Jane's clock at B_1 reads a later time when it lines up with B_2.

However, Sally sees these line-ups as simultaneous. So she will see Jane's A_1 clock showing an earlier reading than Jane's B_1 clock. She will claim Jane's clocks are not synchronized. For instance, suppose when the A's line up Jane's clock at A reads 4:00, and when the B's line up Jane's clock at B reads 4:02. Sally will see 4:00 at one end of Jane's car and 4:02 at the other end as occurring simultaneously – and all the times in between along the length of the car – past, present, and future all together!

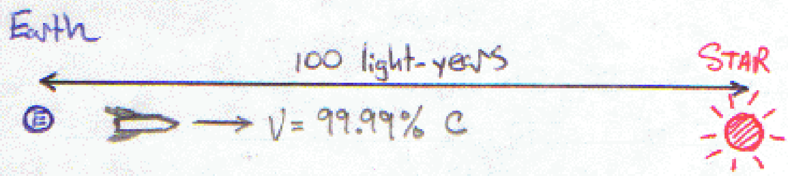


Jane will likewise claim Sally's clocks are not synchronized and see various times in seemingly odd ways.

Example Problem #1

A space ship travels at 99.99% the speed of light from Earth to a distance star. The star is at rest relative to Earth and its distance has been measured by triangulation with satellites to be 100 light-years from Earth (1 light-year = the distance light travels in 1 year; **1 light-year = c x 1 yr**). How long will it take the ship to travel from Earth to the Star as measured by someone on Earth and as measured by someone on the ship? What is the distance from Earth to the star as measured by someone on the ship?

Solution:



<u>As measured by person on Earth</u>

$$d_o = 100 \text{ lyr} = 100(c)(1\text{yr})$$

This is "proper distance" from Earth to Star, because it is distance measured by someone at rest in that Earth-Star reference frame.

time for ship to make journey is

$$t_E = \frac{d_o}{V} = \frac{100(c)(1\text{ yr})}{.9999c} = 100.01 \text{ yrs}$$

<u>However, due to time dialation, the person on the ship sees less time go by.</u>

$$t_{ship} = \frac{t_E}{\gamma}$$

$$\gamma = \left(1 - (v/c)^2\right)^{-\frac{1}{2}}$$

$$\gamma = \left(1 - (.9999)^2\right)^{-\frac{1}{2}}$$

$$\gamma = 70.71$$

$$t_{ship} = \frac{100.01 \text{ yr}}{70.71}$$

$$t_{ship} = 1.41 \text{ yr}$$

So person on the ship says
"I've been moving toward this star at $v = 99.99\% c$ (or it has been moving toward me). It took 1.41 years to make the trip. So the distance was ...

$$d_{ship} = v\, t_{ship}$$

$$= .9999\, c\, (1.41\ yr)$$

$$\approx 1.41\ (c)(1\ yr)$$

$$d_{ship} \approx 1.41\ \ c\, yr$$

... 1.41 light-years (approx)