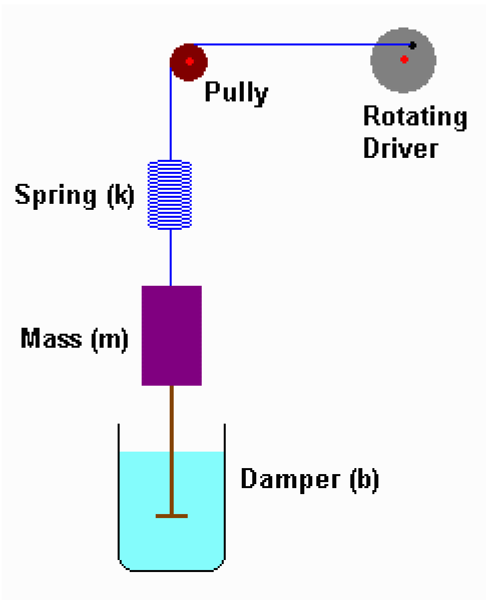
DAY 21

Summary of Topics Covered in Today's Lecture

The Electrical Oscillator

We can now finish out our analogy between mechanical and electrical phenomena.



In studying the mechanical oscillator, we add up the forces in the system and arrived at the equation

$$ma + bv + kx = 0$$

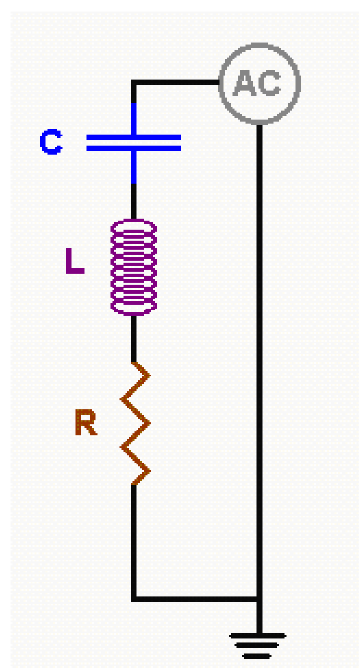
$$m\frac{\Delta v}{t} + bv + kx = 0$$

Calculus:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

We found a solution for this, and learned about damped oscillations. We saw that, if a driving force was added to the system we got resonant behavior.

Now with an electrical circuit containing a resistance, a capacitance, and an inductance, we add up the voltages in the circuit using Kirchoff's Loop Law. If there is no driving voltage and I start from the grounded point, all the voltages in the circuit must add up to zero:



$$U_L + U_R + U_C = 0$$

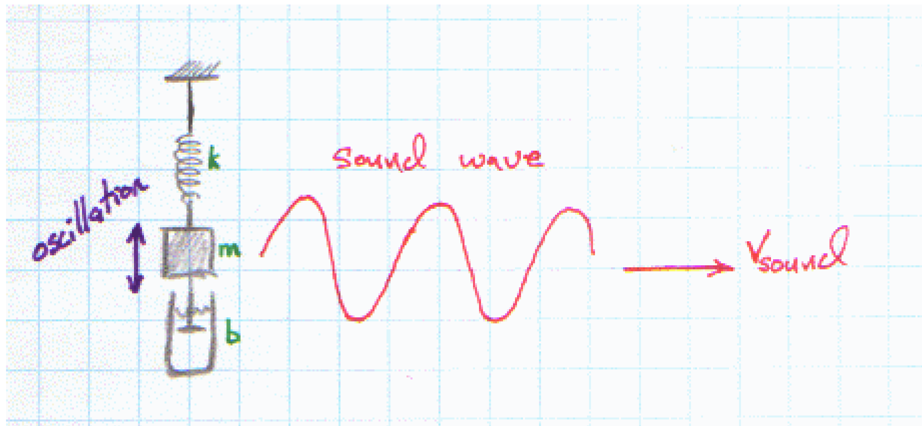
$$L\frac{\Delta I}{t} + RI + \left(\frac{1}{C}\right)Q = 0$$

$$L\frac{dI}{dt} + RI + \left(\frac{1}{C}\right)Q = 0$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \left(\frac{1}{C}\right)Q = 0$$

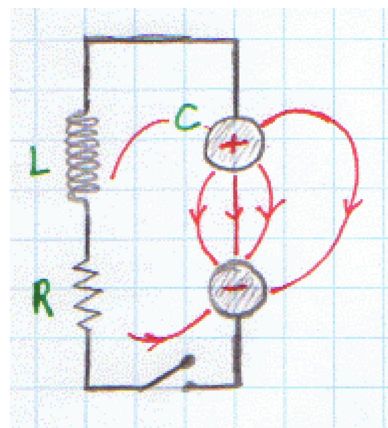
The equation is exactly the same, mathematically speaking, as the equation for the mechanical oscillator. This means all the behavior found in a mechanical oscillator can be found in this system as well. Sinusoidal motions, damped oscillations, resonant behavior – it's all here.

Oscillators and Waves – Mechanical and Electrical



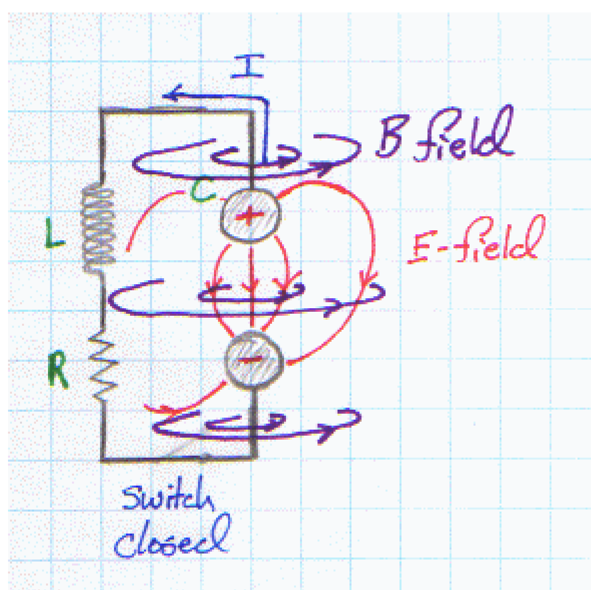
As we mentioned a couple of days ago, a mechanical oscillator can create sound waves in air.

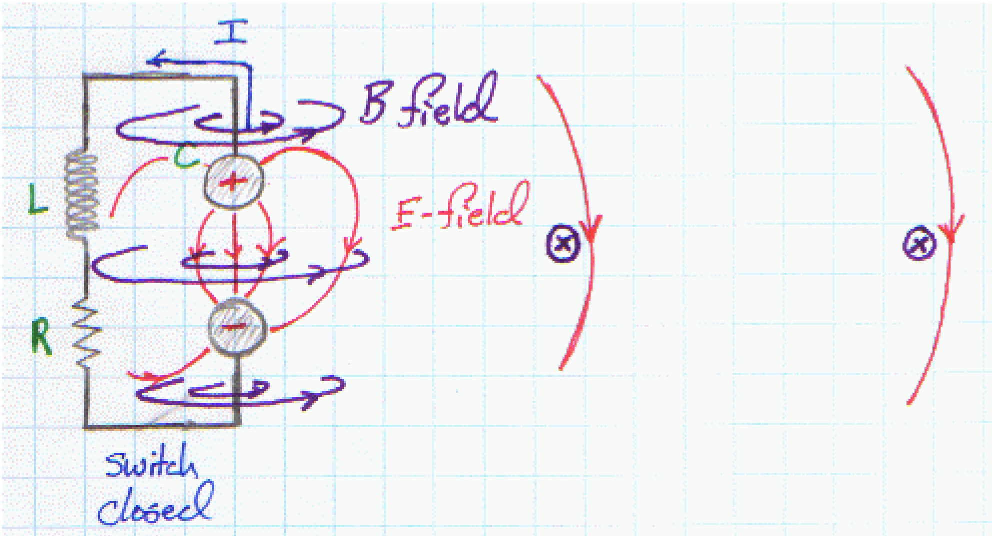
Following the analogy we have been using (the analogy between mechanical oscillators and electrical oscillators) we might expect that an electrical oscillator could serve as a source of waves.



Consider the oscillator shown at right. The capacitor in this oscillator consists of two conducting spheres. The capacitor is fully charged and the switch in the circuit is open, so it cannot discharge. An electric field exists due to the capacitor.

Now we close the switch. The capacitor begins to discharge. Current flows in the circuit in counter-clockwise direction, creating a magnetic field, whose direction is given by the right-hand-rule.

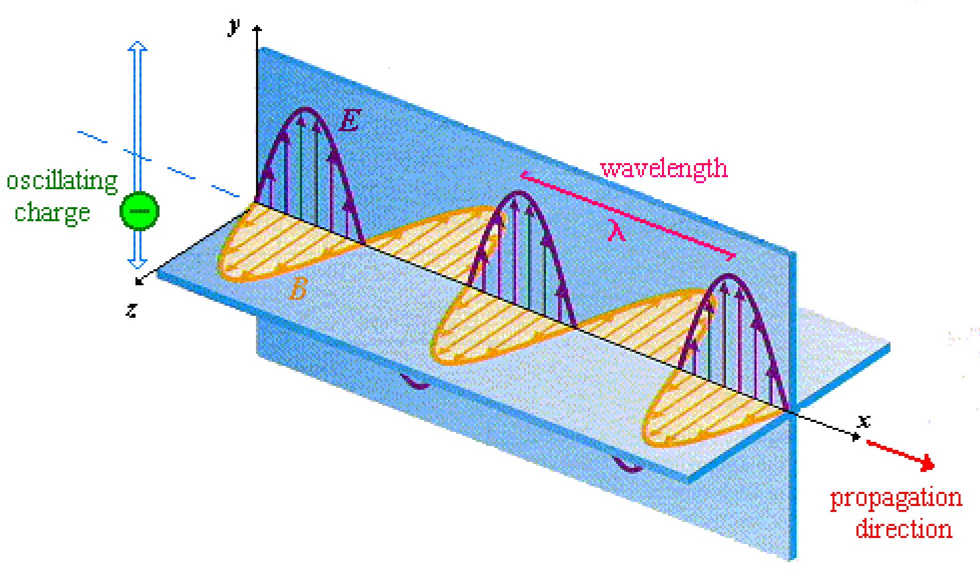




These electric and magnetic fields extend infinitely far out into the space around this oscillator. Of course the further one goes from the oscillator the weaker the fields get, but they never completely disappear.

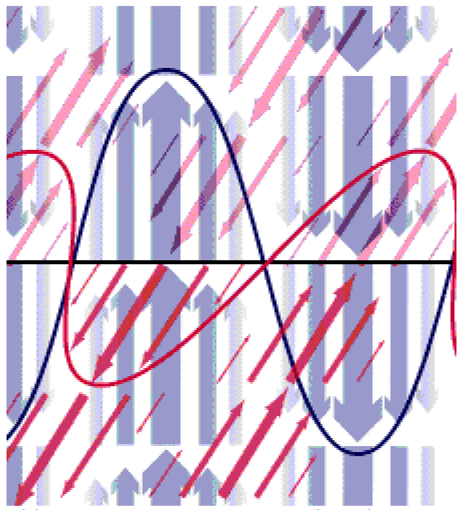
This is an oscillator circuit. Charge will oscillate back and forth between the two spheres.

As the charge oscillates the **E** & **B** fields will oscillate, too. These oscillating fields propagate outward from the circuit at a finite speed. The sequence of alternating fields, moving away from the circuit, is a wave. Since the wave consists of both electric and magnetic fields, this is called an *electromagnetic wave*.



http://www.cbu.edu/~jvarrian/252/emwave.html

One can derive *differential equations* for an electromagnetic (EM) wave from Ampere's and Faraday's Laws. Those equations indicate that an EM wave travels at a speed of $v = 3.0 \times 10^8$ m/s.

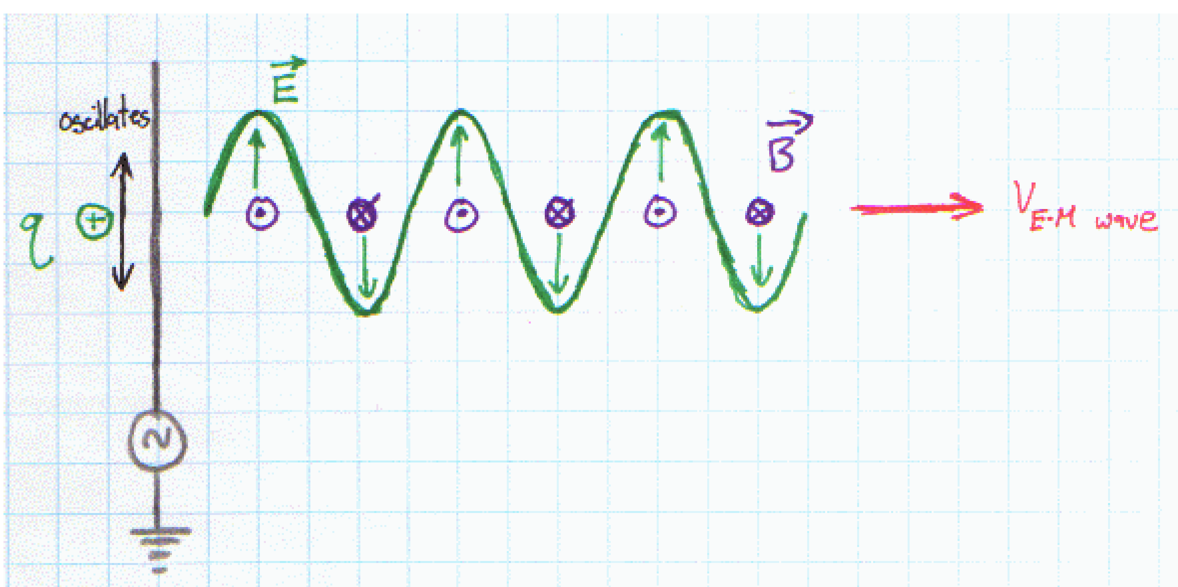


http://imagers.gsfc.nasa.gov/ems/waves2.html

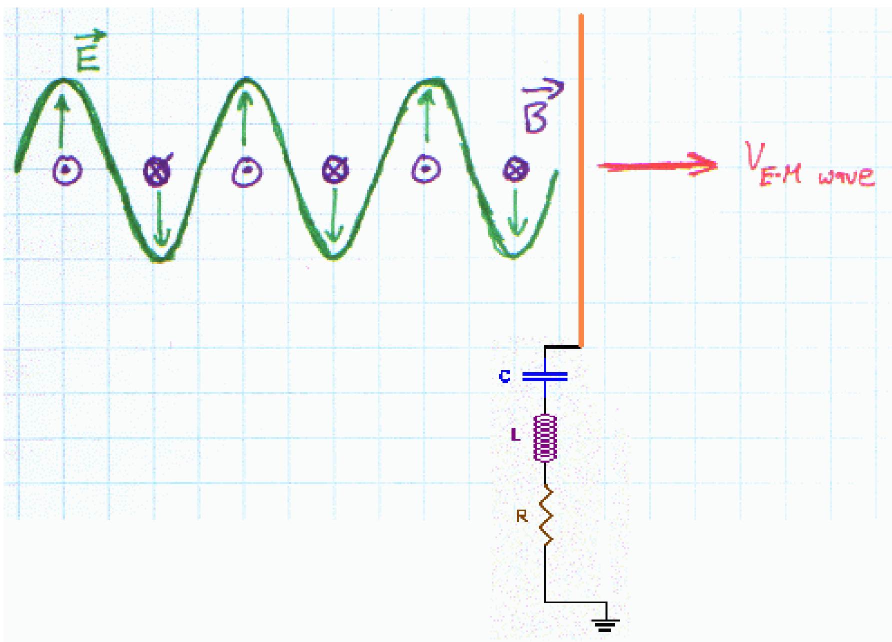
Note how Oscillating mass serves as a source for sound waves while oscillating charge serves as a source for EM waves.

EM Waves and Antennae

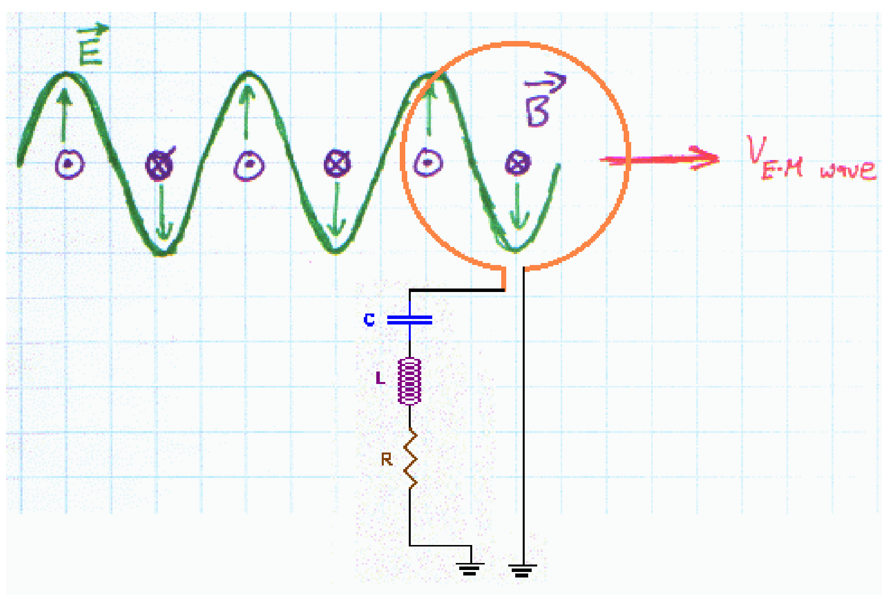
It is easy enough to create a source for EM waves – one only needs to hook a wire up to an oscillating (AC) voltage source to create oscillating charge in an antenna. This is a broadcast antenna – it produces EM waves.



A receiving antenna can be made from a conducting rod that is placed parallel to the direction of the **E**-field. The oscillations of the **E**-field exert forces on charges in the rod, causing them to oscillate and produce an oscillating voltage. This voltage can then be used as a driver to drive an oscillator circuit. If the circuit is tuned (by means of a variable capacitor or inductor) so that it resonates with the voltage produced by the EM wave, significant current can be produced in the oscillator circuit, and the signal detected and amplified. This is how a radio tuner works. This type of antenna, which detects the E-field component of an EM wave, is called a rod antenna.



Another type of receiving antenna can be made from a conducting loop that is placed so as to create maximum flux due to the B-field (so the plane of the loop is perpendicular to the direction of the **B**-field). The oscillations of the **B**-field induce an oscillating voltage in the loop. This voltage can then be used as a driver to drive an oscillator



circuit. If the circuit is tuned (by means of a variable capacitor or inductor) so that it resonates with the voltage produced by the EM wave, significant current can be produced in the oscillator circuit, and the signal detected and amplified. This type of antenna, which detects the B-field component of an EM wave, is called a loop antenna.

Types of EM Waves

Light is one form of EM wave – but there are others. We must keep in mind that all of these "forms" differ from one another only in terms of their frequency/wavelength. Just as various frequencies of sound have different names (bass, treble, etc.) but are still sound, so the various frequencies of EM wave have different names but are still EM waves.

	Sound Waves	**EM Waves**
Low Frequency	BASS	Radio
		Microwaves
		Infrared
Middle Frequency	MID-RANGE	Visible Light
		Ultraviolet
High Frequency	TREBLE	x-rays
Very High Frequency	ULTRASOUND	γ-rays

Therefore there is no more difference between an x-ray and a radio wave than there is between a high-pitched whistle from a piccolo and a low-pitched rumble from a pipe organ.

The Aether – the Medium for EM Waves

Sound waves travel through a medium – usually air when we are hearing them. In the late 1800's, as the physics of EM waves was being developed, it was presumed that since these were waves they must also travel through a medium. This hypothesized medium was called the "Aether". Since EM waves travel through space (the light from the stars is an EM wave and it travels through space to reach Earth), the aether must exist all throughout the universe yet must allow matter to pass through it easily.

In the 1880's two scientists at what is now Case-Western University in Cleveland, Ohio performed an experiment to try to detect the aether. This experiment is now considered to be quite famous and is known as the *Michelson-Morely experiment* (for more on the experiment, and a great "flashlet" animation, visit **http://galileoandeinstein.physics.virginia.edu/lectures/michelson.html**). The experiment failed to detect anything. In fact, no experiment has ever been successful at detecting an aether.

Example Problem #1

Determine an equation that will tell you when an electrical oscillator is critically damped, and then calculate the resistor needed to critically damp an oscillator that has an inductance of 1 Henry and a capacitance of 1 Farad. Show that the units work out properly.

Solution:

From our study of spring-mass (mechanical) oscillators, we know that a mechanical oscillator is critically damped when

$$b = 2\sqrt{mk}$$

By analogy, we should replace b with R, m with L, and k with 1/C. That would give us

$$R = 2\sqrt{L\left(\frac{1}{C}\right)} = 2\sqrt{\frac{L}{C}}$$

So if L = 1 H and C = 1 F, let's find R.

$$R = 2\sqrt{\frac{1H}{1F}} = 2\sqrt{\frac{H}{F}}$$

The math was simple, the units aren't. Somehow this must reduce to Ohms (which are Volts per Amp). I recall that a Farad measures capacitance, which is charge per volt, so I can replace F with Coulombs per Volts. In an example problem just the other day we broke Henry's down into Joules per square Ampere. From this we can work out the units.

$$R = 2\sqrt{\frac{H}{F}} = 2\sqrt{\frac{J/A^2}{C/V}}$$

A volt is a Joule per Coulomb

$$R = 2\sqrt{\frac{J/A^2}{C/V}} = 2\sqrt{\left(\frac{J}{A^2}\right)\frac{V}{C}} = 2\sqrt{\left(\frac{V}{A^2}\right)\frac{J}{C}} = 2\sqrt{\left(\frac{V}{A^2}\right)V} = 2\sqrt{\left(\frac{V^2}{A^2}\right)}$$

$$R = 2\frac{V}{A} = 2\Omega$$

So if $L = 1$ H and $C = 1$ F, the oscillator is critically damped if $R = 2\ \Omega$.

Example Problem #2

Does the frequency of an EM wave have any impact on how well it will be detected by an antenna?

Solution:

Yes, but only for loop antennae. The voltage in a rod antenna is due to **E**-fields. Recall from early in the semester that the potential difference created by an E-field is given by

$$\Delta U = -\mathbf{E}\cdot\mathbf{r}$$

So this would indicate that the voltage in a rod antenna depends on the amplitude of the **E**-field in the wave, the size of the antenna, and the orientation of the antenna with respect to the field. But the voltage in a loop antenna is due to induction. For induction

$$\Delta U = -\Delta\Phi_B/t$$

Where $\Phi = \mathbf{B}\cdot\mathbf{A}$. This indicates that the voltage in a loop antenna depends on the amplitude of the B-field in the wave, the size of the antenna, the orientation of the antenna, and the rate of change! A higher-frequency wave will have a greater rate of change.