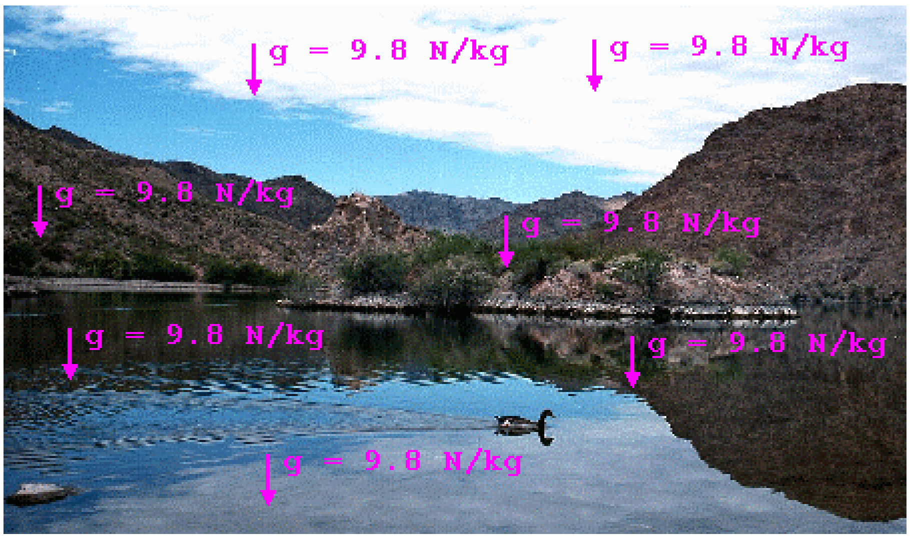
DAY 2

Summary of Topics Covered in Today's Lecture

Uniform Gravitational and Electric Fields

The gravitational field at the Earth's surface is uniform (constant). This means that wherever you go, the gravitational field strength is always the same (more or less -- there are minor fluctuations), and the direction is always downward:

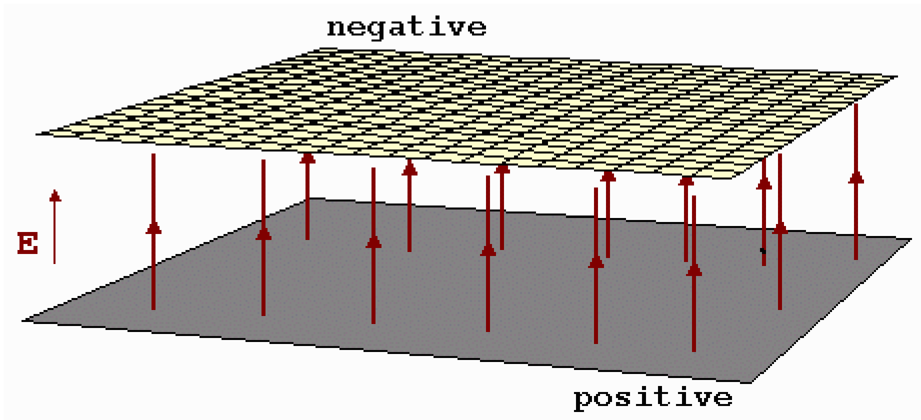


We know how massive objects move in a uniform gravitational field. If thrown directly upward, or dropped, they undergo free-fall. Their trajectory is a straight line, and they move with constant acceleration. If thrown at an angle they undergo projectile motion and have parabolic trajectories.

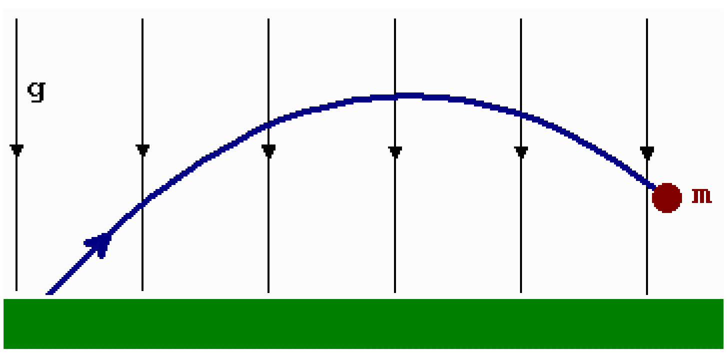
We will often use *field lines* to represent fields. Instead of drawing many arrows to indicate a uniform gravitational field (as shown in the figure above), we can represent the field with lines that point in the direction of the field, as shown in the figure at right.

The spacing of the field lines indicates field strength. More closely-spaced lines indicate a stronger field; more widely-spaced lines indicated a weaker field; even spacing everywhere indicates a uniform field.

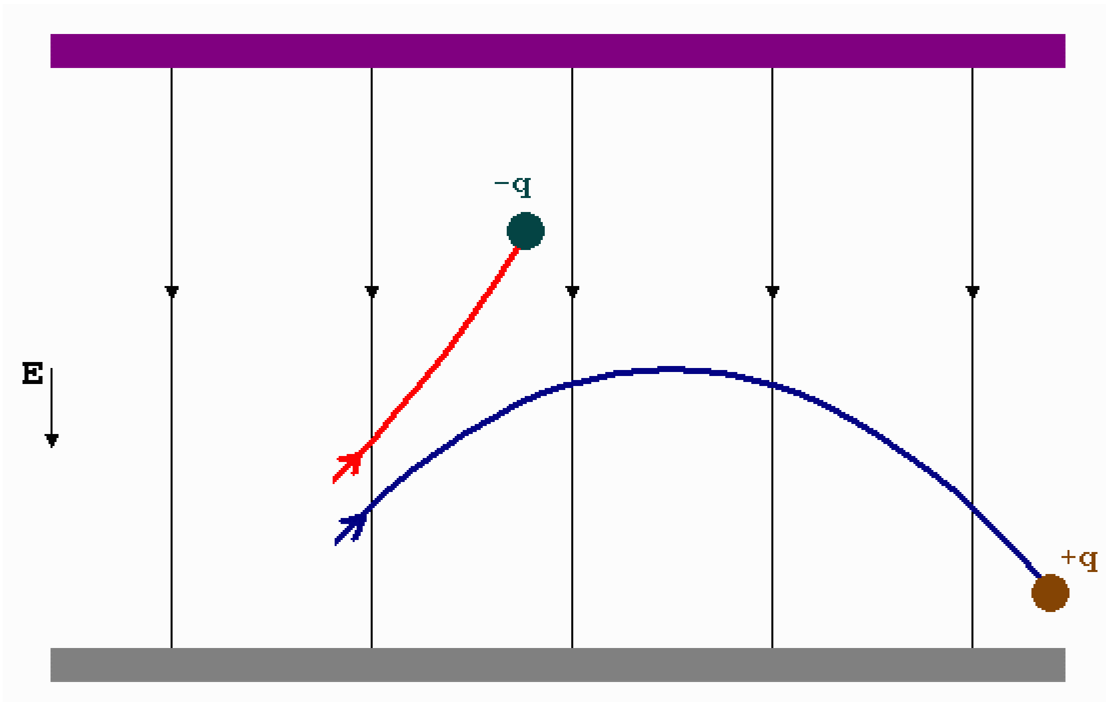
A uniform electric field can be produced by a large, flat sheet of charges, or by two sheets of opposite charge that are parallel to each other:



The motion of a charged particle in a uniform electric field is then exactly analogous to the motion of a massive particle in a uniform gravitational field: constant acceleration with either linear or parabolic trajectories. One difference between the motions of massive particles in a gravitational field and charged particles in an electric field is that of course gravity is always attractive, so the trajectory of massive particles always curves so the concave side of the parabola faces the same direction that the field lines point;



whereas the parabolas of charged particle trajectories can face in different directions in an electric field, depending on the charge that is moving. Positive charges have trajectories whose curves are such that the concave side of the parabola faces the same direction as the electric field lines; negative charges have trajectories whose curves are such that the concave side of the parabola faces the opposite direction as the electric field lines.

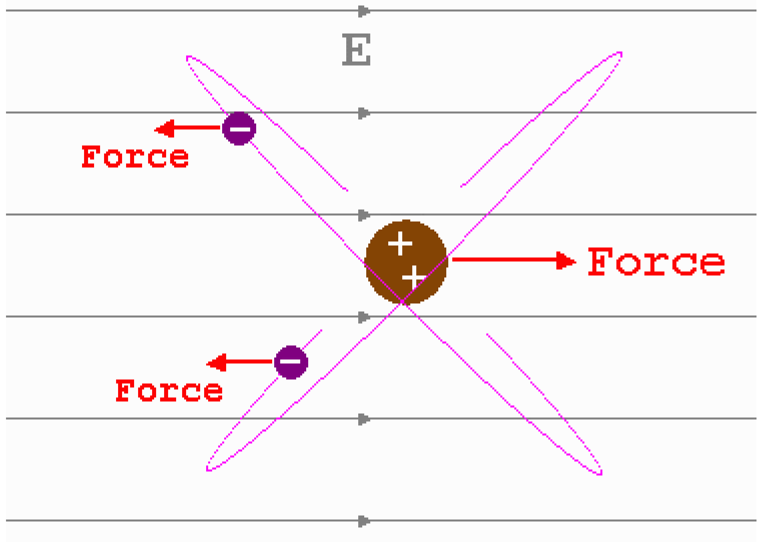


Electric Fields, Conductors, and Insulators

Electrical conductors are materials that allow electric charge to move through them freely. These include most notably metals, as well as certain solutions of things dissolved in water (for instance, common tap water -- tap water has many things dissolved in it). In metals, for instance, the bonds between atoms are such that electrons can easily move from atom to atom. In solutions, both positive and negative charges can move.

Electrical insulators are materials that will not allow charge to move through them. These include many substances, including most plastics, rubbers, glasses, ceramics, woods, gasses (including air), pure water (pure liquid H_20) and more. In insulators all the electrons within the material are tightly bound to particular atoms and are not free to move.

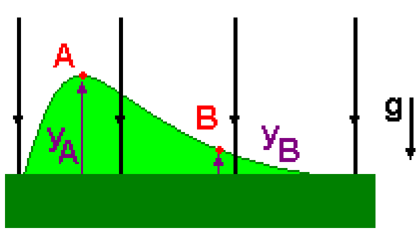
However, in the presence of an electric field the nuclei of the atoms that make up the insulator and the electrons bound to those nuclei, are pulled in opposite directions. These forces tend to pull the atom apart -- nucleus one way and electrons the other. The electrons are, of course, attracted (and bound) to the nucleus. But if the electric field is strong enough, at least some electrons can be stripped from the atom. Now charges can move in the material and it loses its insulating properties.



In air this breakdown of insulating properties occurs for an electric field of E = 30,000 V/cm (a rough value -- the actual value in any given situation depends on various factors such as humidity). When the breakdown occurs, the subsequent motion of charges generates heat, light, and sound, and we see a spark.

More on Potentials

So far we've been ignoring the question of negative signs when dealing with potentials. We've just figured out whether an object in a system gains or loses potential energy (PE) by looking at a picture of the system. This will not work as our problems get more complex, plus it is just a sloppy way to do math.



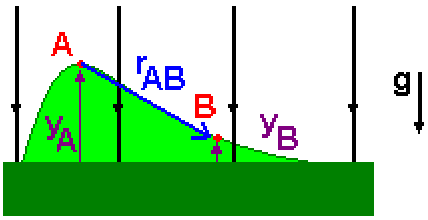
Refer to the figure. Suppose I move from a high point (A) on a hill down to a low point (B). The bottom of the hill is at zero gravitational potential.

The difference in gravitational potential between A and B (ΔU_{g-AB}) is

$$\Delta U_{g-AB} = U_{gB} - U_{gA} = gy_B - gy_A = g(y_B - y_A)$$

ΔU_{g-AB} doesn't depend on horizontal changes in position – only vertical changes. Put another way, only changes in position that are parallel to the direction of the field are important. The fact that the potential depends on parallel quantities means that we can write change in potential with a vector DOT product:

$\Delta U_{g-AB} = -\mathbf{g} \cdot \mathbf{r}_{AB}$



or generally speaking

$\Delta U_g = -\mathbf{g} \cdot \mathbf{r}$ (for electrical fields this is $\Delta U_E = -\mathbf{E} \cdot \mathbf{r}$)

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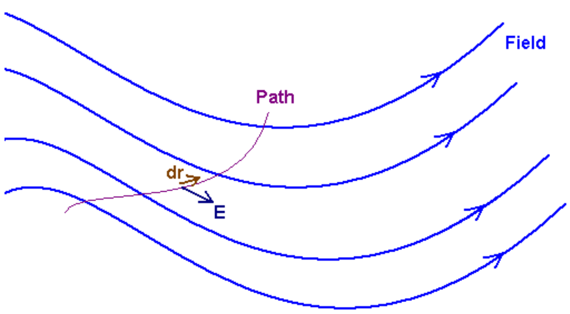
In a varying field this concept still holds, but only at the level of infinitesimals. Instead of being an equation in algebraic variables, our equation for ΔU_g becomes an equation in differential (infinitesimal) variables:

$dU_g = -\mathbf{g} \cdot d\mathbf{r}$

There are equivalent equations for electric fields:

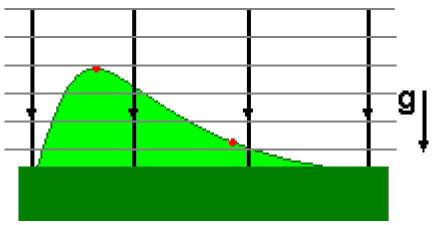
$\Delta U_E = -\mathbf{E} \cdot \mathbf{r}$ $dU_E = -\mathbf{E} \cdot d\mathbf{r}$



$-\mathbf{E} \cdot d\mathbf{r}$ gives the change in potential dU_E along a very small piece of a path (the length of $d\mathbf{r}$) that passes through some field.

Equipotential Surfaces

Change in potential is zero if the field and **r** vectors are perpendicular. This means that, no matter how radical the field may be, potential does not change (it is constant) so long as you move perpendicularly to that field. Any path that is taken through a field that is always perpendicular to that field is a path of constant potential. Such paths are known as equipotential surfaces. In a uniform field these paths are straight lines.



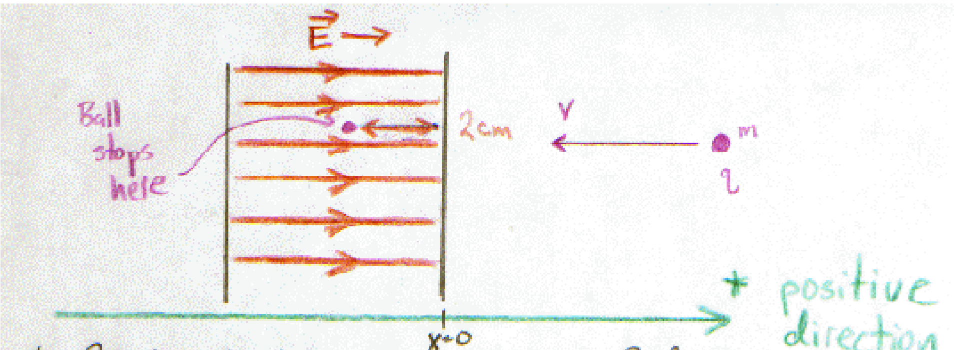
Note -- In the examples that follow I have indicated concepts from Physics I that are used in the solutions, for those who may need to do review.

Example Problem #1

A small ball of mass 5 grams carrying a positive charge of 100 e moving at 50 m/s to the left enters a region where there is a uniform electric field. The ball comes to a halt in 2 cm. What is the electric field (both strength and direction)?

Solution:

This solution uses a kinematic equation of motion and Newton's Laws from Physics I



The fact that the ball halts means the E-field is directed to the right, so as to exert a force to decelerate the ball.

$m = 5g = .005\ kg$

$q = 100e = 100(1.6\times10^{-19}C) = 1.6\times10^{-17}C$

$v_0 = -50\ m/s$

$v = 0$

$x_0 = 0$

$x = -2cm = -.02\ m$

I'm defining positive as being to the right.

Find acceleration of ball:

$v^2 = v_0^2 + 2a(x - x_0)$

$0^2 = (-50\ m/s)^2 + 2a(-.02\ m - 0)$

$0 = 2500\ m^2/s^2 - (.04\ m)a$

$a = \frac{2500\ m^2/s^2}{.04\ m} = 62,500\ m/s^2$

Use Newton's 2nd Law to find Force on ball:

$\sum F = ma$

$= .005\ kg(62,500\ m/s^2) = 312.5\ N$

Now find E

$F = qE \qquad E = \frac{F}{q} = \frac{312.5\ N}{1.6\times10^{-17}C} = 1.95\times10^{19}\ N/C$

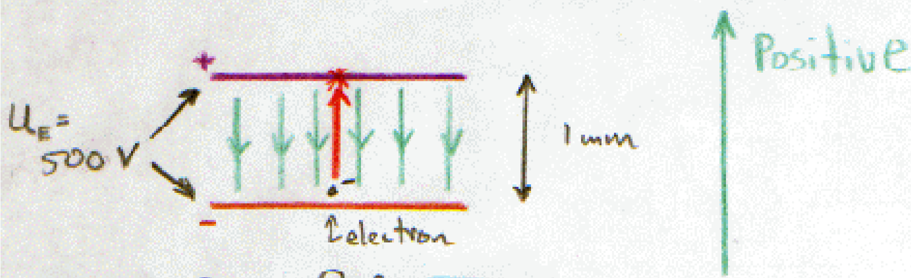
$\vec{E} = 1.953\times10^{19}\ N/C$ directed to right

Example Problem #2

In an experiment, an electron is pulled off a negatively charged flat surface when a positively charged flat surface that has a potential of 500 V relative to the electron's surface is brought to within 1 mm of the electron's surface. The electron is repelled from the negative surface and attracted to the positive surface. (a) Using fields and forces, calculate the speed of the electron when it hits the positive surface. (b) Using energies and potentials, calculate the speed of the electron when it hits the positive surface.

Solution:

This solution uses a kinematic equation of motion and Newton's Laws from Physics I, and concepts of Kinetic and Potential Energy from Physics I. You must also look up the mass and charge of an electron.



Using forces + fields:

$U_E = 500\,V$ so $U_E = Ey$ will give me E ($y = 1$ mm)

$$500\,V = E(.001\,m)$$

$$\frac{500\,J/C}{.001\,m} = E = 500{,}000\,N/C$$ (directed down) (because electron travels up).

Force on Electron is $F = qE = ma$

$$a = \frac{q}{m}E \qquad E = -500{,}000\,\frac{N}{C}$$ Negative

Now I can find speed at impact using $v^2 = v_0^2 + 2a(y - y_0)$

$v_0 = 0$

$y_0 = 0$

$y = .001\,m$

$$v^2 = 0^2 + 2\left(\frac{q}{m}E\right)(.001\,m - 0)$$

$$v^2 = 2\left(\frac{-1.6\times10^{-19}\,C}{9.11\times10^{-31}\,kg}\right)\left(-500{,}000\,\frac{N}{C}\right)(.001\,m)$$

$$v^2 = 1.75631\times10^{14}\,\frac{C\,(kg\,m/s^2)\,m}{kg\,C}$$

$$v = 1.3252591\times10^{7}\,m/s$$

The electron is moving at 1.325×10^{7} m/s when it hits.

Using Energy:

$U_E = 500V = 500\ J/C$

PE of electron is initially $PE = qU_E = 1.6\times10^{-17}C(500\ J/C) =$

$= 8\times10^{-17}J$ (For time being I'm just ignoring negatives in energy and am just figuring out gains + losses by looking)

Remember:

I see this electron is losing PE and gaining KE.

$PE_{lost} = KE_{gained}$

$8\times10^{17}J = \frac{1}{2}mv^2$

$8\times10^{17}J = \frac{1}{2}(9.11\times10^{-31}kg)v^2$

$1.7563117\times10^{14}\frac{J}{kg} = v^2$

$\frac{J}{kg} \rightarrow \frac{Nm}{kg} \rightarrow \frac{\frac{kg\,m}{s^2}m}{kg} = m^2/s^2$ Working out the units

$V^2 = 1.7563117\times10^{14}\ m^2/s^2$

$V = 1.325\times10^{7}\ m/s$ Same answer!

Example Problem #3

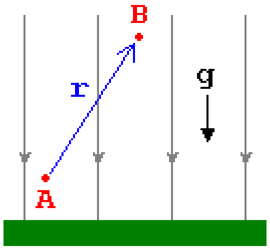
Point A is 2 ft off the ground. Point B is 10 ft off the ground. Use the dot product method to find the difference in potential between A & B in J/kg.

Solution:

First convert to SI metric units:

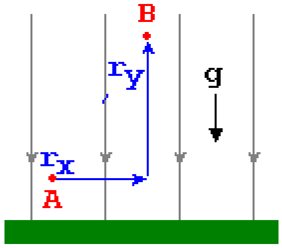
A is 0.6096 m off the ground.
B is 3.0479 m off the ground.

$\Delta U_g = -\mathbf{g} \cdot \mathbf{r}$



I've drawn in r in the figure at left. Now look at the vectors ---I will split **r** into a x & y components as shown at right.

$$\Delta U_g = -\mathbf{g} \cdot (\mathbf{r}_x + \mathbf{r}_y)$$
$$= -\mathbf{g} \cdot \mathbf{r}_x - \mathbf{g} \cdot \mathbf{r}_y$$



However, g and r_x are perpendicular, so their dot product is zero. So I only have to worry about r_y.

$$\Delta U_g = - \mathbf{g} \cdot \mathbf{r}_y$$
$$= - g r_y \cos(\theta)$$

r_y is not hard to calculate:

$r_y = 3.0479 \text{ m} - 0.6096 \text{ m} = 2.4383 \text{ m}$

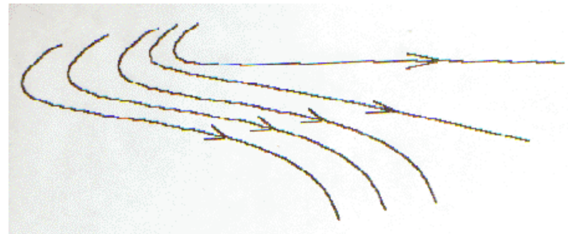
Since **g** points down and $\mathbf{r}_y$ points up, θ is 180°. Now I plug this into my equation

$$\Delta U_g = - \mathbf{g} \cdot \mathbf{r}_y$$
$$= - g r_y \cos(\theta)$$
$$= - (9.8 \text{ N/kg})(2.4383 \text{ m}) \cos(180) = 23.8953 \text{ Nm/kg} = 23.8953 \text{ J/kg}$$

So the change in potential is 23.9 J/kg.

Example Problem #4

Draw the equipotential surfaces in the field shown. Be accurate -- all equipotential surfaces <u>must</u> cross <u>every</u> field line perpendicularly.



Solution: