DAY 13

Summary of Topics Covered in Today's Lecture

Solenoids and Materials

The field produced by a solenoid can be either strengthened or weakened if a material is placed inside the solenoid to serve as the *core* of the solenoid. With a core, the solenoid's field is given by

$$B_{solenoid} = K_m \mu_0 \frac{N}{L} I$$

Where K_m is called the *relative permeability* of the core. In turn,

$$K_m = 1 + \chi_m$$

Where χ_m is called the *magnetic susceptibility* of the core material.

For paramagnetic materials χ_m is positive but very small, and K_m is slightly greater than 1. A paramagnetic core will slightly increase the strength of the solenoid's B-field. This occurs because the magnetic dipoles of the atoms in a paramagnetic material will weakly line up with the magnetic field created by the solenoid current, adding slightly to the overall field.

For diamagnetic materials χ_m is negative but very small, and K_m is slightly less than 1. A diamagnetic core will slightly decrease the strength of the solenoid's B-field. This occurs because the magnetic dipoles of the atoms in a diamagnetic material will weakly line up to oppose the magnetic field created by the solenoid current, subtracting slightly from the overall field.

For ferromagnetic materials χ_m is positive and very large. K_m is much greater than 1. A ferromagnetic core will greatly increase the strength of the solenoid's B-field. This occurs because the magnetic dipoles of the atoms in a ferromagnetic material strongly line up to with the magnetic field created by the solenoid current, greatly enhancing the overall field.

Furthermore, ferromagnetic materials can become permanently magnetized, because once the magnetic dipoles of the atoms in the material are lined up with the field produced by the current, they tend to stay lined up. This is because they have a strong tendency to line up with each other's field's -- once lined by the current's field, they stay that way. Thus ferromagnetic materials "remember" the magnetic field produced by the coil, even after the current has been turned off. This "magnetic memory" is the principle behind magnetic storage devices such as cassette tapes, VHS video tapes, computer hard drives, computer floppy disks, and all other magnetic media.

Strength of Solenoids

Note that formulas for B only give the field strength, not the "pole strength" that says how strongly a magnetic solenoid attracts something. *An extremely strong but perfectly uniform field will exert no attractive force on a current loop* -- at best it can exert a torque. Pole strength and attraction depends on having *a strong field that diverges (N pole) or converges (S pole)*.

Lots of Examples

Today we also do example problems that used Ohm's Law and the equation for the magnetic field of a solenoid.

Example Problem #1

A solenoid is created by closely wrapping copper wire of diameter *d* around a nail that measures 30 cm long by 1 cm diameter. The relative permeability of the nail is 200. The solenoid is connected to a 6 V battery and generates a magnetic field B. Determine how the field created by the solenoid depends on *d*. Make a graph of B vs. d.

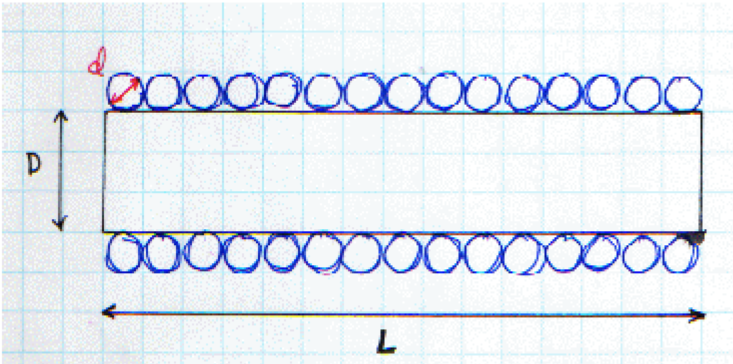
NOTE -- The resistance R of a length l of wire of cross-sectional area A is given by the equation

$R = \rho l/A$

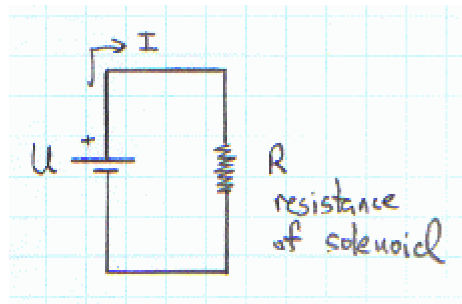
Where ρ is known as the *resistivity* of the material the wire is made of. ρ is a basic property of material, like its color or density or elastic modulus, and typically can be looked up in a table (a table is ρ values has been added to the class web page). ρ is low for good conductors like silver or copper, higher for less good conductors like iron, and very high for insulators.

Solution:

The solenoid looks something like this:



The solenoid has a resistance R. The solenoid and the battery make a simple circuit:

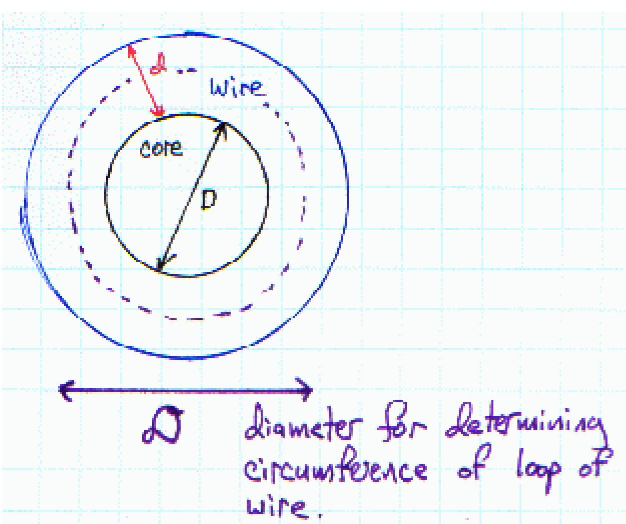


The magnetic field of the solenoid is given by

$$B = K \mu_0 \frac{N}{L} I$$

$I = \frac{U}{R}$ Current through solenoid

How much wire is needed to wrap solenoid?



$$\text{Circ} = \pi \mathcal{D} = \pi (D + d)$$

That's one loop. The solenoid has N loops so the length of wire needed is

$$\ell = N(\text{Circ}) = N\pi(D + d)$$

The resistance of the wire is

$$R = \rho \frac{\ell}{A} = \rho \frac{N\pi(D+d)}{\frac{\pi}{4} d^2} = \rho \frac{4N(D+d)}{d^2}$$

So current is

$$I = \frac{\mathcal{U}}{R} = \frac{\mathcal{U} d^2}{4N\rho(D+d)}$$

$$B = K\mu_0 \frac{N}{L} I$$

$$= K\mu_0 \frac{\cancel{N}}{L} \frac{\mathcal{U} d^2}{4\cancel{N}\rho(D+d)}$$

$$B = \frac{K\mu_0 \mathcal{U} d^2}{4L\rho(D+d)}$$

Check units

$$\frac{\frac{N}{A^2}[V]\cancel{m^2}}{m[\Omega \cancel{m}][\cancel{m}]} = \frac{\frac{N}{A^2} V}{m\,\Omega} = \frac{\frac{N}{\cancel{A^2}} \cancel{V}}{m \frac{\cancel{V}}{\cancel{A}}}$$

$$= \frac{N}{Am} = T$$

Teslas is correct for $\vec{B}$ field

Now let's plug some values into the equation:

$K = 200$ $\quad\quad$ $L = .3\,m$

$\mu_o = 4\pi \times 10^{-7}\ \frac{N}{A^2}$ $\quad\quad$ $\rho = 1.68 \times 10^{-8}\ \Omega m$ for copper (from table)

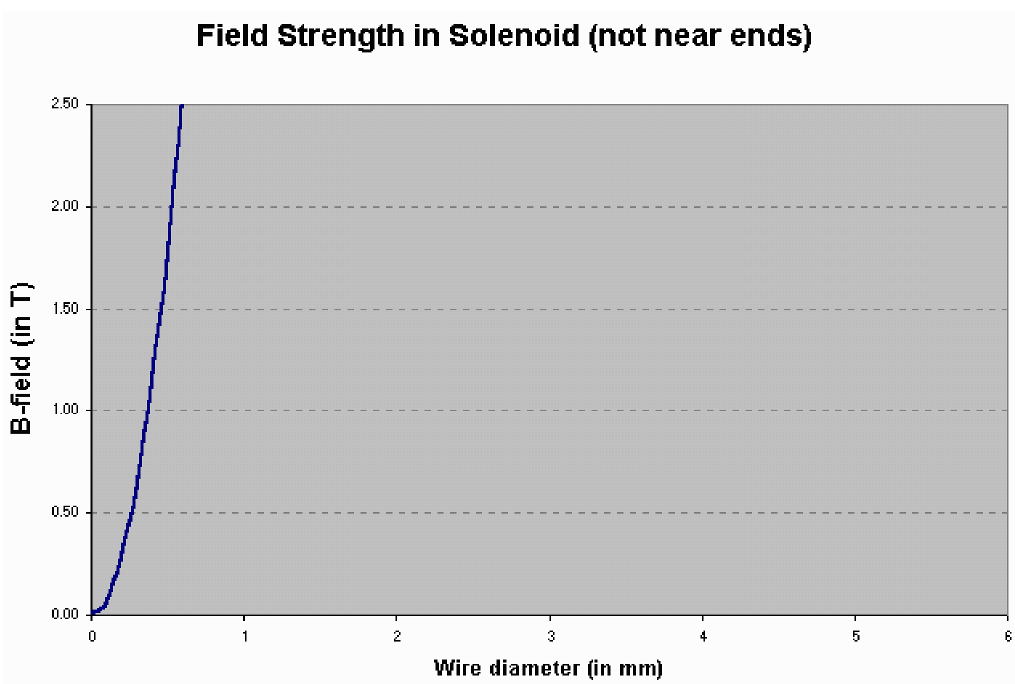
$U = 6V$

$D = .01\,m$

$$B = \frac{200\,(4\pi \times 10^{-7})(6)\,d^2}{4(.3)(1.68 \times 10^{-8})(.01 + d)}$$

$$= 74800\ \frac{d^2}{.01 + d}\ [T] \text{ if } d \text{ is in } m$$

Graph that equation and we see that the field gets stronger with larger diameter wire.

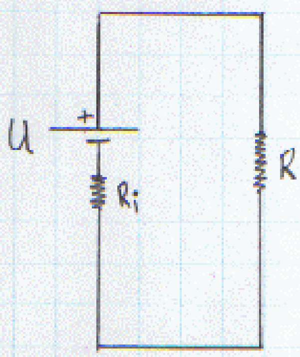


Example Problem #2

The previous example implicitly assumed that there was no limit to the current that a battery can produce. In reality every battery is limited by its own internal resistance (R_i). Rework the problem for a 6V battery with internal resistance of 0.1 Ω (this means the battery could produce a maximum current of I = U/R = 6V/0.1Ω = 60 A).

Solution:

This is just like the above problem, except for the circuit diagram and the current.



$$I = \frac{U}{R_i + R}$$

From the previous problem I know

$$R = \rho \frac{4N(D+d)}{d^2}$$

$$I = \frac{U}{R_i + \frac{4\rho N(D+d)}{d^2}}$$

$$B = K\mu_0 \frac{N}{L} I$$

$$= K\mu_0 \frac{N}{L} \frac{U}{R_i + \frac{4\rho N(D+d)}{d^2}}$$

N is the length of the core divided by the diameter of the wire

$$N = \frac{L}{d}$$

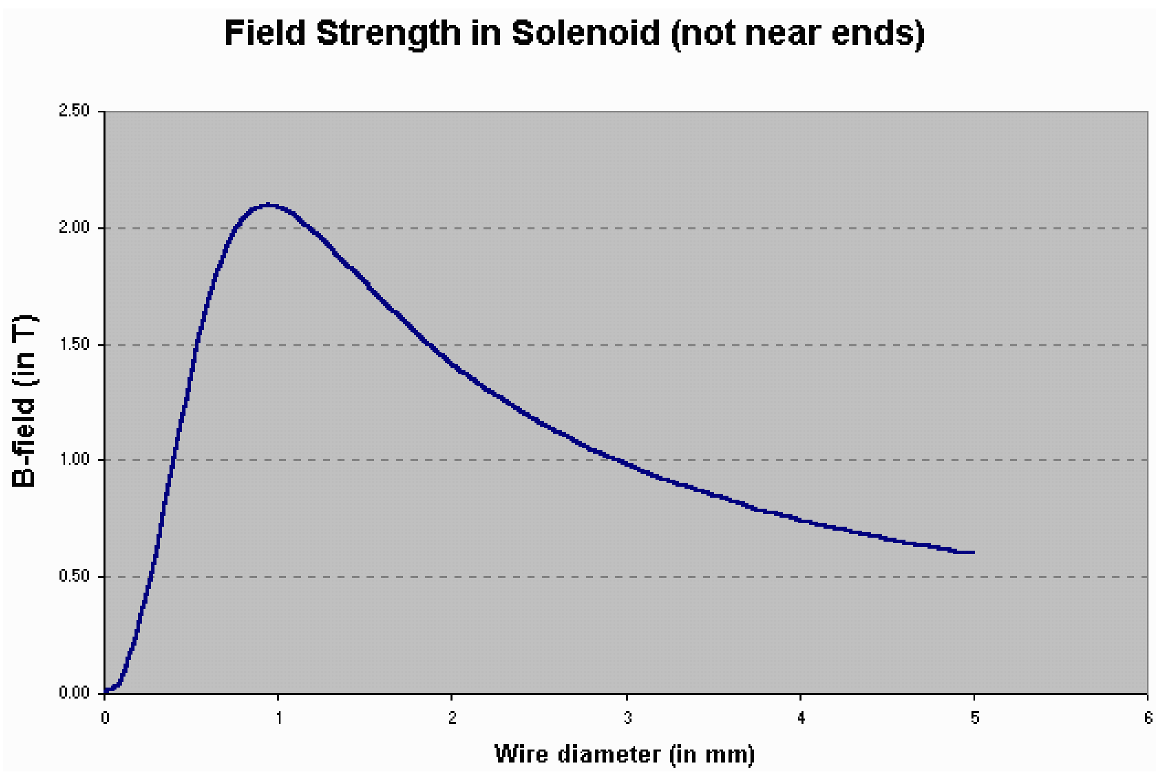
$$B = K\mu_0 \frac{\cancel{L}/d}{\cancel{L}} \frac{U}{\left(R_i + \frac{4\rho L/d\,(D+d)}{d^2}\right)}$$

$$B = \frac{K \mu_0 U}{d\left(R_i + \frac{4\rho L(D+d)}{d^3}\right)}$$

$$= \frac{200(4\pi \times 10^{-7})(6)}{d\left(.1 + \frac{4(1.68 \times 10^{-8})(.3)(.01+d)}{d^3}\right)}$$

$$= \frac{.001508}{d\left(.1 + \frac{2.016 \times 10^{-8}(.01+d)}{d^3}\right)}$$

Graphing this equation:



Note that for small wires the graph looks the same as in example #1, but as the wires get larger it is clear that there is a maximum field possible with this battery. This is the more realistic way to do this problem.