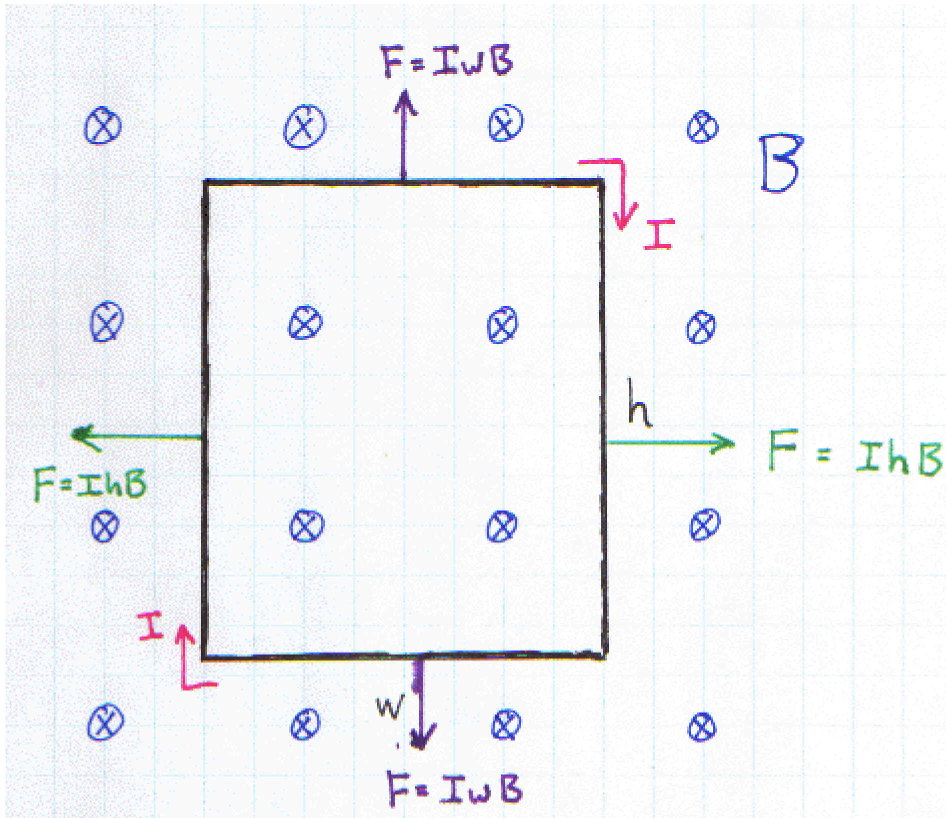
DAY 12

Summary of Topics Covered in Today's Lecture

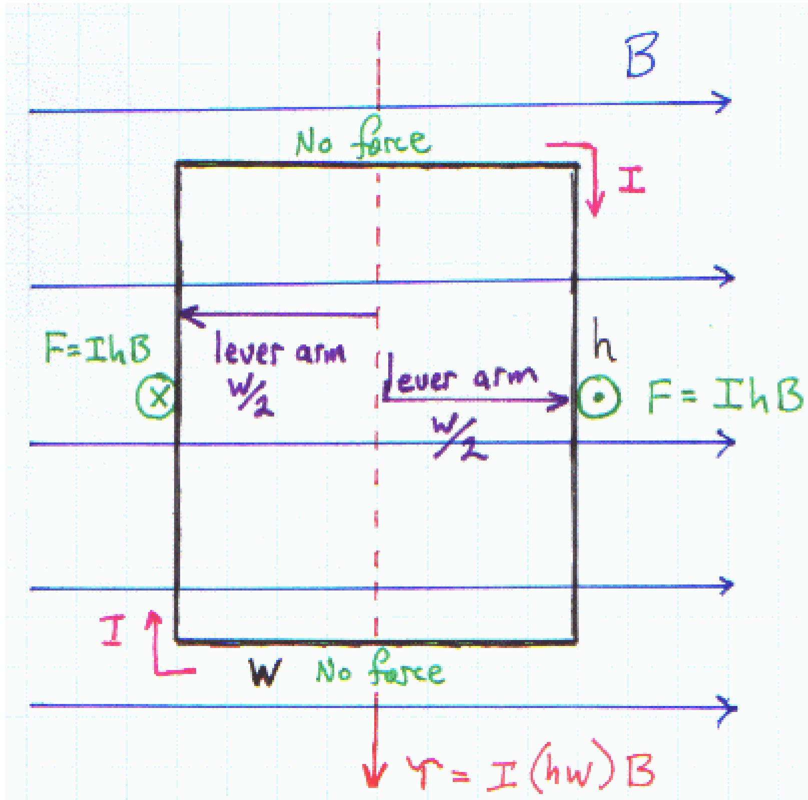
Magnetic Fields Exert Torques on a Loop of Current

Imagine a wire bent into the shape of a rectangle with height h and width w. The wire carries a current I. This rectangular loop of wire is placed in a magnetic field so that the plane of the loop is perpendicular to the field as shown below.



The force on each side of the loop is equal to the current times the length of that side times the magnetic field strength. The direction of each force is as shown, according to the RHR. All the forces cancel out, so $\Sigma F = 0$. All the forces lie in the same plane, so there are no torques on the loop, either, meaning $\Sigma\tau = 0$. The loop does not move.

However, if the rectangular loop of wire is placed in a magnetic field so that the plane of the loop is parallel to the field as shown at left, the situation is somewhat different.



First, there is no force on the top and bottom of the loop. That is because in those sections of the loop the current runs parallel to the field, and the cross product of two parallel quantities is zero.

Second, the forces on the sides are once again equal to the current times the length of that side times the magnetic field strength. Once again they point in opposite directions, so they cancel out and ΣF = 0 for the loop. However, they are not in the same plane. Each force is acting at the end of a level arm whose length is ½ w. The torque created by each of these forces is

$$\tau = \vec{r} \times \vec{F} = rF = \left(\frac{1}{2}w\right)(IhB)$$

Since there are two torques the net torque on the loop is

$$\sum \tau = w(IhB) = I(wh)B$$

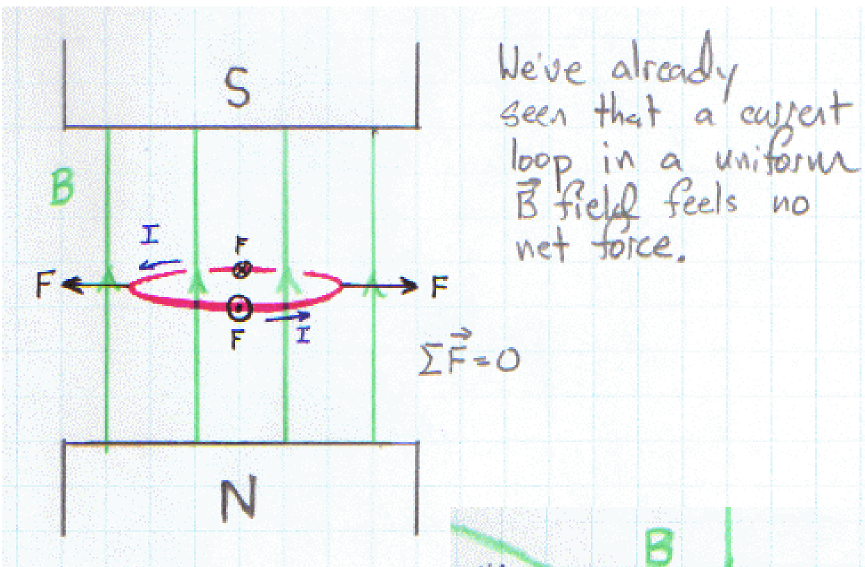
However, the width (w) times the height (h) is just the area of the loop, so the net torque on the loop is just

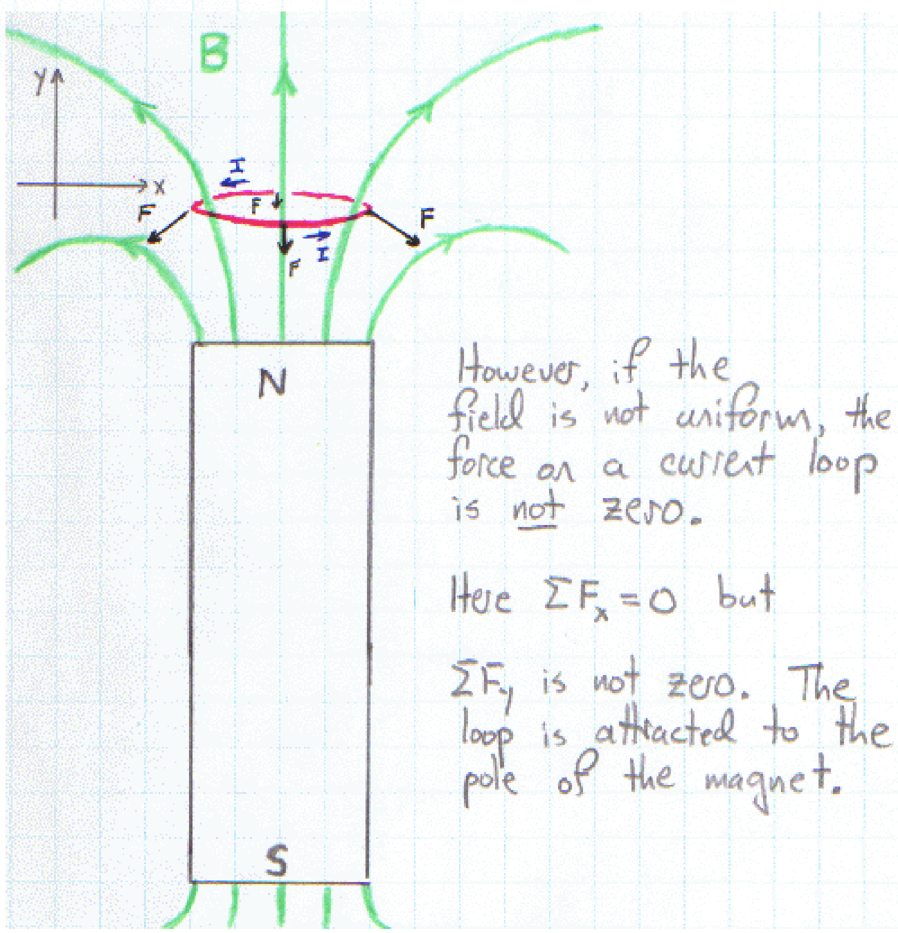
$$\sum \tau = IAB$$

This is the maximum torque that a current loop in a uniform magnetic field can experience - and it occurs when the plane of the loop is parallel to the field. The minimum torque is of course zero - and it occurs when the plane of the loop is perpendicular to the field.

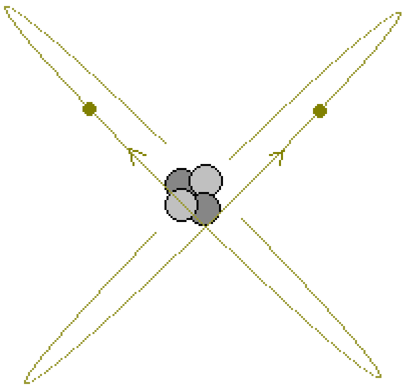
Torque on a current loop is the basic principle behind the electric motor.

A Non-Uniform Magnetic Field

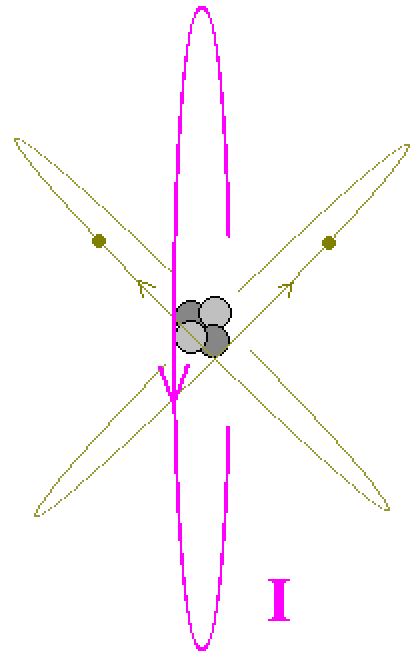




Magnets Attracting Materials



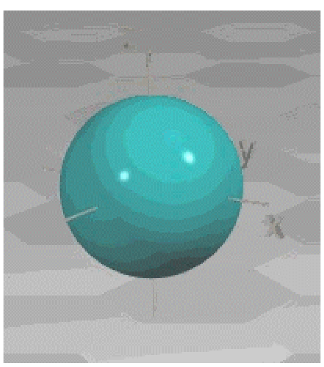
Atoms consist of a positive nucleus with negative electrons orbiting about it. A primitive picture of an atom would be something like the figure at left.



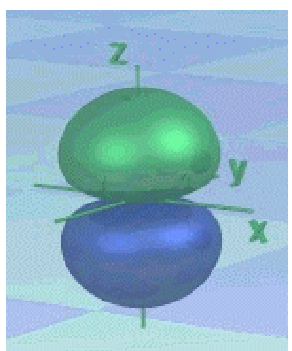
The motion of the electrons is a current, and the overall motion of electrons produces the equivalent of a current loop as in the figure at right.

One could see that the number of electrons and the arrangements of their orbits might influence the "current loop effect".

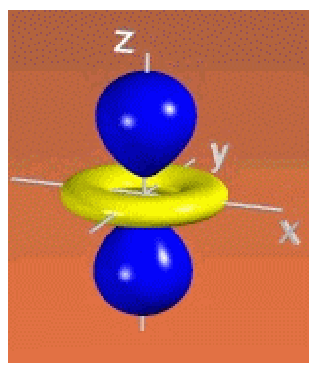
In reality the shapes of the "orbitals" of electrons around atoms are very complex. They do not look like the simple paths shown in the figures above, but instead look something like the figures below.



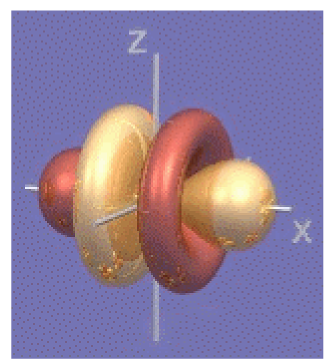
1s



2p



3d



4f

http://www.shef.ac.uk/chemistry/orbitron/AOs/4f/index.html

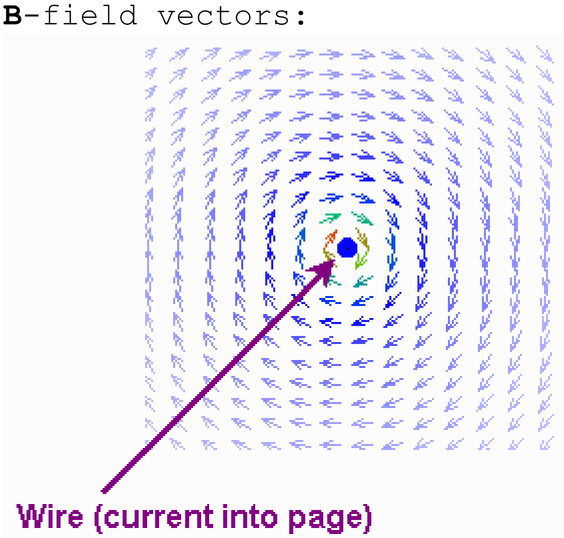
However, the basic idea of electron motion creating what is essentially a current loop within the atom still stands. The structure of the electron orbitals determines how significant the "current loop effect" is and how it behaves. This "current loop" can then be attracted to a magnetic pole.

In most materials the atomic current loops are more or less randomly oriented. When the material is placed in a magnetic field the loops tend to orient themselves with that field. Materials are classified by whether their atomic current loops orient themselves so as to be attracted to a magnetic pole or repelled from a magnetic pole. If the material tends to be attracted to a magnetic pole it is said to be *paramagnetic* (paramagnetic materials include Aluminum and Oxygen). If it tends to be repelled it is said to be *diamagnetic* (diamagnetic materials include Copper, Gold, and Mercury). If the material is strongly attracted to a magnetic pole it is said to be *ferromagnetic* (ferromagnetic materials include Iron, Nickel, and Cobalt).

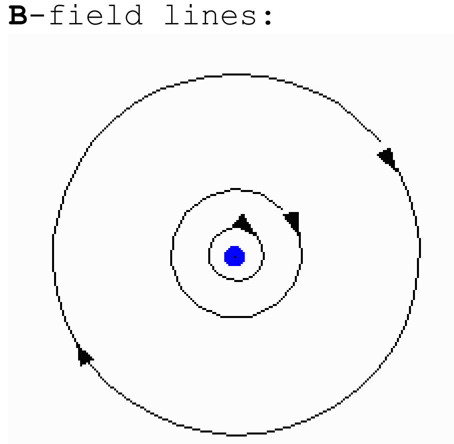
The Source of Magnetic Fields

Magnetic fields may exert forces on moving charges, but they are also generated by moving charges. Experiments show that a wire carrying current generates a magnetic field around it:

B-field vectors:



B-field lines:



Symmetry requires that the magnetic field be radially symmetric in this case. The direction of the field is given by another version of the Right-Hand-Rule – if you point the thumb of your right hand in the direction of the current your fingers will curl in the direction of the B-field.

Note that the wire does not produce recognizable magnetic poles.

The magnetic field produced by a current can be determined by Ampere's Law. Ampere's law says that *if you follow a closed path, the product of the distance along that path and the B-field parallel to that distance is equal to the current flowing through that closed path, times a constant known as the permeability of free space* (the constant is written as $\mu_0 = 4\pi \times 10^{-7}$ N/A^2).

Ampere's Law is a calculus law, but the fact is most of the time you solve it by simple geometric means (see the example problems).

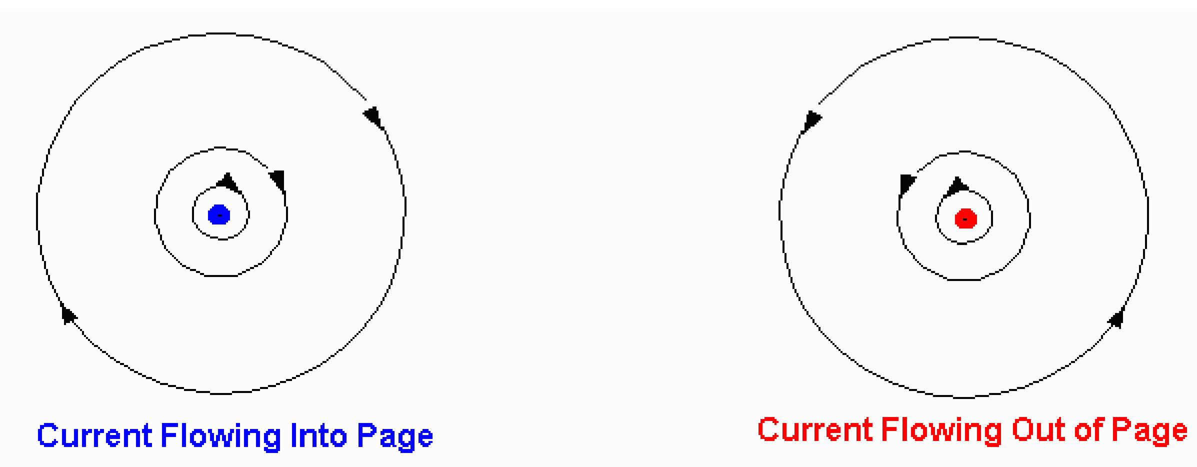
PHY 232 Only

In mathematical form, Ampere's Law is written as the integral of field and distance around a closed path

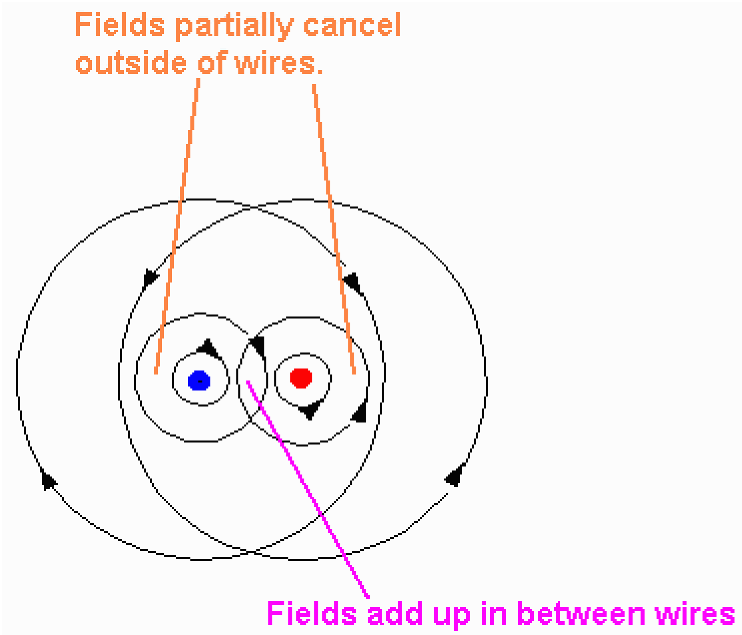
$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_0 I_{enclosed}$$

The Magnetic Field of a Loop – and of Many Loops

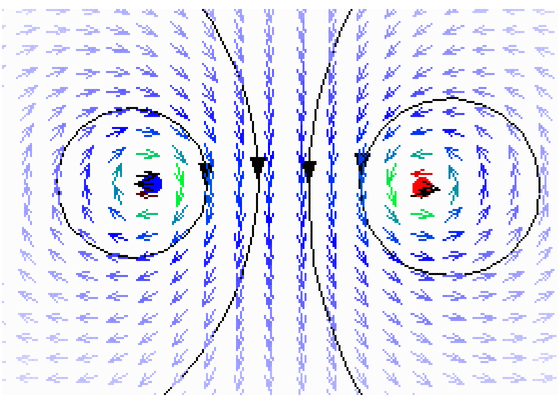
If two wires with current are brought close to one another you get a result something like this:

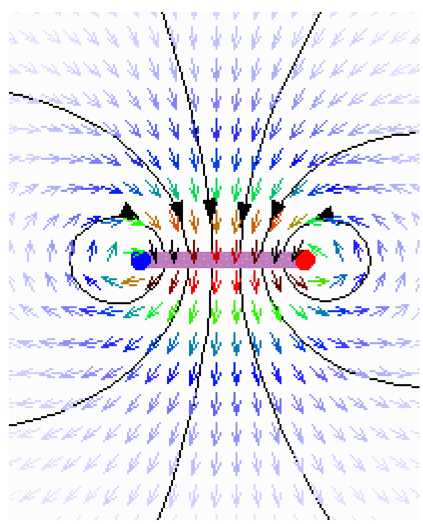


Bringing them a little closer together we see that their magnetic fields point in the same direction between the wires, but in opposite directions outside the wires.

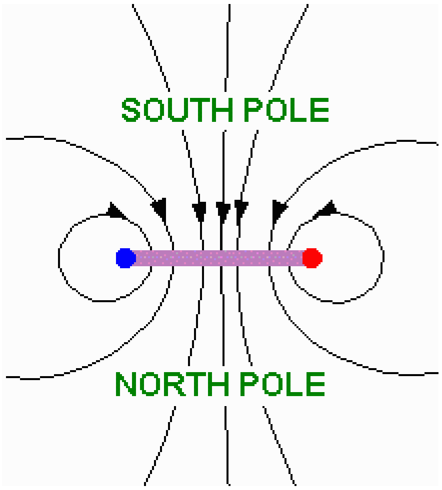


The result is this (showing both field lines and field vectors in this picture):





If a wire is bent into a loop, then on one side current is flowing one way, and on the other the current is flowing in the opposite direction. This is similar to the two straight wires problem given above. As seen at right, the field for a coil looks similar to the field for the two wires, at least when viewed edge-on. Of course if your rotated the figure at right by 90° to the left it would look no different. If you rotated the "two wires" figure above by 90° to the left then you'd be looking at two long straight wires sitting horizontally.



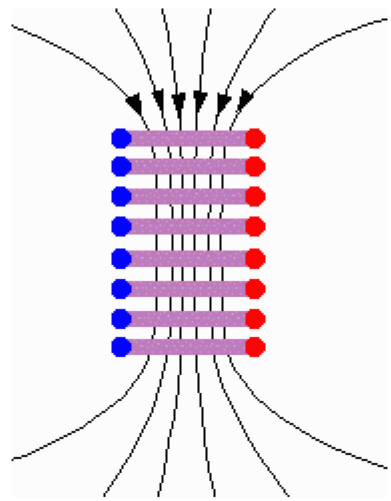
Note that the field produced by a loop of wire also has a recognizable North Pole and South Pole. What makes a North Pole? The divergence of field lines in the case of North Poles. What make a South Pole? The convergence of field lines.

This kind of magnetic field -- one with two poles that face away from each other -- is known as a *magnetic dipole*. Current loops are always magnetic dipoles.

If we put many current loops together we get a coil, or *solenoid*. The field inside a solenoid is relatively uniform and is given by

$$B = \mu_0 \frac{N}{L} I$$

Here N is the number of loops in the solenoid, L is the length of the solenoid, and I is the current flowing through the solenoid.



This is how a typical *electromagnet* is constructed.

Example Problem #1

A motor is constructed of an "armature" of 1000 loops of wire round into a circle of radius 10 cm. The armature is placed in a magnetic field of strength 5 T and a current of 2 A passes through it. Find the maximum torque the motor can generate. If the motor spins at 750 RPM, what is the maximum power it can generate?

Solution:

Torque on one loop is

$\tau = IAB$

$A = \pi r^2$

$\tau = \pi r^2 IB = 3.1416(.1\ m)^2(2\ A)(5\ T)$
$= .31416\ m^2(A)(N/Am)$ *Cancel out the A and m*
$= .31416\ Nm$

Maximum torque is .31416 Nm or $\pi/10$ Nm.
The torque for all 1000 loops is then 314.16 Nm.

To figure the power I have to go back to Physics 1:

$P = \tau\ \omega$

$\omega = 750\ RPM = 12.5\ rev/sec = 25\ \pi\ 1/sec$

$P = (314.16\ Nm)(25\ \pi\ 1/sec) = 7854\ \pi\ J/sec$ *1 Nm = 1 J*
$= 24674\ W$
$= 33.1\ Hp$ *746 W = 1 Hp*

The maximum power output of the motor is 33.1 Hp.
Of course the maximum torque cannot be maintained, so this figure is somewhat high.

Example Problem #2 (PHY 232 ONLY)

21. A strong magnet is placed under a horizontal conducting ring of radius r that carries current I, as shown in Figure P29.21. If the magnetic field **B** makes an angle θ with the vertical at the ring's location, what are the magnitude and direction of the resultant force on the ring?

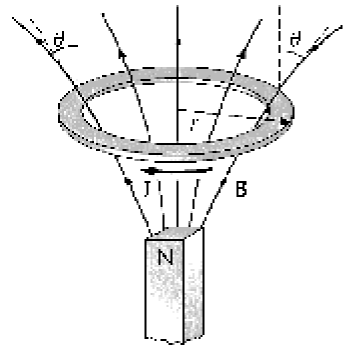


Figure P29.21

Solution

As is shown in the figure below, the magnetic force on each bit of ring is radial inward and upward, at an angle θ above the radial line, according to:

$$|d\mathbf{F}| = I|d\mathbf{s} \times \mathbf{B}| = I\,ds\,B$$

The radially inward components tend to squeeze the ring, but cancel out as forces. The upward components $I\,ds\,B\sin\theta$ all add to

$$\mathbf{F} = I(2\pi r)B\sin\theta \text{ up}$$

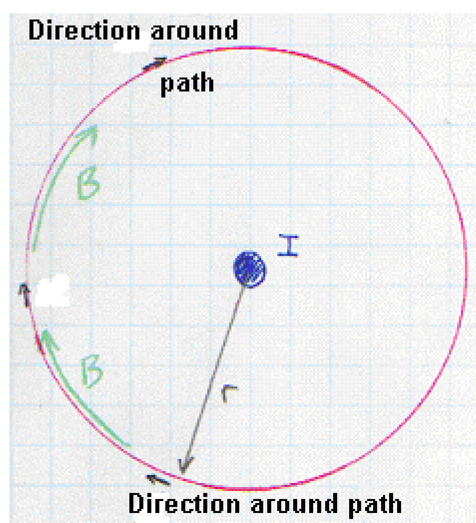
The magnetic moment of the ring is down. This problem is a model for the force on a magnetic dipole in a nonuniform magnetic field, or for the force that one magnet exerts on another magnet.

http://www.brookscole.com/physics_d/templates/student_resources/003026961X_serway/guide/ch29-21.html

Example Problem #3

Use Ampere's Law to determine the magnetic field produced by a long straight wire carrying current I. What is the magnetic field 1 cm from a wire carrying 10 A of current?

Algebra Solution:



The field around a wire is circular. So, I will use an Ampere's law path that is a circle, centered on the wire. The current enclosed by the path is just I.

The distance around a circular path is just the circumference: $2\pi r$.

Since the problem is radially symmetric B can't change as I go around the path. Ampere's Law says the product of B and the distance around must equal $\mu_o I_{enclosed}$. So this means

$B(2\pi r) = \mu_o I_{enclosed}$

So $B = \mu_o I_{enclosed}/(2\pi r)$

Now to calculate the field:

$I = 10$ A
$r = 1$ cm $= .01$ m
$\mu_o = 4p \times 10^{-7}$ N/A^2

$B = (4\pi \times 10^{-7}$ N/A$^2)(10$ A$)/(2\pi\ .01$ m$) = .0002$ N/Am $= 0.2$ mT

The field is 0.2 milliTeslas.

Calculus (PHY 232 ONLY) Solution:

The field around a wire is circular. So, I will use an Ampere's law path that is a circle, centered on the wire. The current enclosed by the path is just I.

(The algebra solution can be used for some problems. But to be able to solve all problems requires knowledge of calculus.)

Now to calculate the field:

$I = 10$ A
$r = 1$ cm $= .01$ m
$\mu_o = 4\pi \times 10^{-7}$ N/A^2

$B = (4\pi \times 10^{-7}$ N/A$^2)(10$ A$)/(2\pi\ .01$ m$) = .0002$ N/Am $= 0.2$ mT

The field is 0.2 milliTeslas.