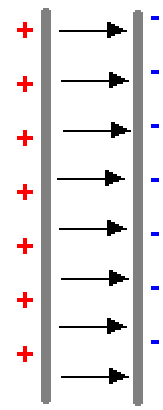
DAY 10

Summary of Topics Covered in Today's Lecture

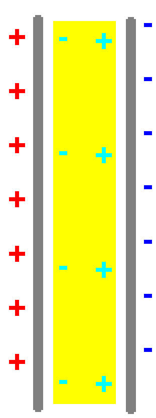
Capacitors



A capacitor is a device that stores electric charge. A capacitor consists of two conductors separated by an insulator. The ability of a capacitor to store charge depends on the size of the conductors, the separation of the conductors, and the insulator.

The most basic capacitor consists of two identical flat parallel conducting plates separated by a vacuum. This is called a *parallel-plate capacitor*. The area of each plate is A. The separation of the plates is d. According to Gauss's Law, the E-field between the two is constant (see Homework).

The capacitance of this kind of capacitor is $C = \varepsilon_0 A/d$. Larger plates (larger A) means "more room" for the charge, so it is easier to store charge on larger plates. Closer plates (smaller d) means that the opposite charges are closer to each other – the attraction between + and – means it is easier to add charge to the plates.



If a suitable insulator other than a vacuum is placed between the two plates, the insulator (known as a *dielectric*) will become "polarized" as its molecules are distorted by the E-field between the plates. This makes it even easier to add more charge to the two plates, because there is negative charge close to the positive plate that will attract additional charge on that plate, and there is positive charge close to the negative plate that will attract additional charge on that plate. However, as we discussed Day 2, if the E-field between the capacitor's plates gets strong enough, the molecules in the dielectric will not just be polarized by the field. If the field is strong enough, the field will rip the molecules apart, the insulator will become a conductor, and a spark will jump between the plates. The maximum field the dielectric can withstand without breaking down is sometimes referred to as the *dielectric strength*.

The capacitance of a parallel-plate capacitor with a dielectric is given by

$$C = \kappa \varepsilon_0 \frac{A}{d}$$

A is the area of the plates, d is the separation of the plates, ε_0 is the Permittivity of Free Space (ε_0 = 8.85 x 10^{-12} C^2/Nm^2 = $1/4\pi k$), and κ is called the dielectric constant (no units). The dielectric constant is a property of materials and usually has to be looked up in a table. κ = 1 for a vacuum. The dielectric constant for air is approximately 1, too.

The units of capacitance are the

$$\frac{C^2}{Nm^2}\frac{m^2}{m} = \frac{C^2}{Nm} = \frac{C^2}{J} = \frac{C}{V} = Farad$$

A Farad is abbreviated F.

Capacitors & Energy

Capacitance (C) gives the charge a capacitor can hold (Q) for a given potential difference (U):

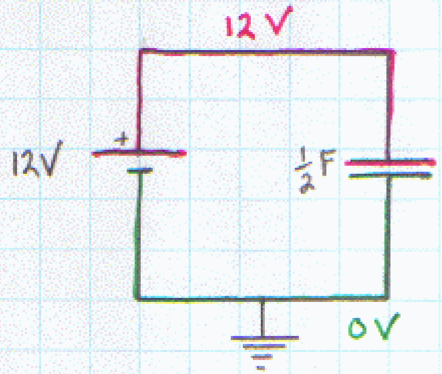
$$Farad = \frac{Coulomb}{Volt}$$

$$C = \frac{Q}{U}$$

By storing charge a capacitor also stores electrical potential energy. Recall that the potential energy of a charge q at a potential of U is

$$PE = qU$$

Now consider a circuit consisting of a ½ F capacitor and a 12 V voltaic cell, as shown below.



When electrostatic equilibrium is reached, the upper half of the circuit will be at $U = 12\,V$. The lower half will be at $U = 0\,V$. That's because in electrostatic equilibrium, all points on a conductor are at the same potential.

The charge stored is

$$Q = CU = \tfrac{1}{2}F(12\,V)$$

$$= \tfrac{1}{2}\left(\tfrac{C}{V}\right)(12\,V) = 6\,C$$

However, the energy stored is <u>not</u>

$$PE = QU = 6C(12\,V) = 6C\left(12\,\tfrac{J}{C}\right) = 72\,J$$

WRONG!!

Why is this wrong? Because the capacitor's voltage varies with charge. A discharged capacitor has $U = 0$. So while the capacitor may have $U = 12V$ fully charged, when empty it had $U = 0$. It cannot supply charge at a constant 12V. Rather it has an average voltage of $\tfrac{1}{2}(0 + 12V) = 6V$. Thus the energy stored is

$$PE = QU_{avg} = \tfrac{1}{2}QU = \tfrac{1}{2}(6C)(12V)$$

$$= 36\,J$$

So, the electrical energy stored in a capacitor holding charge Q at potential difference U is

$$E_{capacitor} = \frac{1}{2}QU$$

Electric Current and Ohm's Law

If a battery is connected to a device such as a light bulb, electrostatic equilibrium is not reached. Rather, the battery generates charge continuously. The charge flows through the bulb, heating the bulb's filament and producing light. The rate at which charge is generated and flows through this circuit is called electric current (I).

For a steady flow of charge current is

$$I = \frac{Q}{t}$$

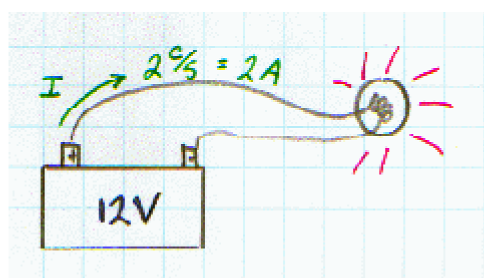
For a varying flow of charge current is

$$I = \frac{dQ}{dt}$$

(PHY 232 Only)

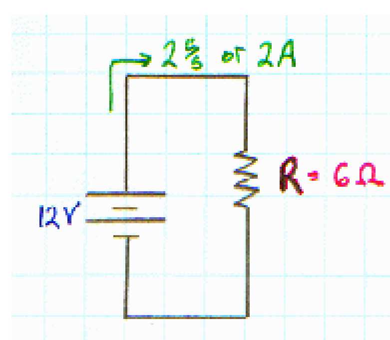
Current is measured in units of C/s or Amperes (1 A = 1 C/s).

Suppose that a 12 V battery is attached to a light bulb, with the result that a current of 2 C/s (2 A) flows through the circuit. The ratio between the potential difference of 12 V and the current of 2 A is called the *Resistance* (R) of the circuit. The resistance between two points in a circuit is defined as



$$R = \frac{U}{I}$$

where U is the potential difference between the two points and I is the current flowing between those two points. In the figure above, the two points are the positive and negative terminals of the battery. This simple definition has the rather grand name of *Ohm's*



Law. The units of resistance are V/A = Ω (Ohms). We can draw a diagram of the above circuit showing the voltage, current, and resistance in a simple form as shown at left.

Note – in physics we will always treat electric current as though it is a flow of <u>positive</u> charge, regardless of what type of charge is truly flowing in the circuit. As we noted a couple of days ago, it is not relevant whether the charges that flow are positive or negative – all that matters is that charge flows.

Electric Current and Power

We know that electrical potential energy is

$$PE = qU$$

In a typical current, flowing charge moves from higher potential to lower potential, and the charge loses PE. The rate at which the charge loses PE is power:

$$P = \frac{PE}{t}$$

If the potential is constant then we can write

$$P = \frac{PE}{t} = \frac{qU}{t} = \frac{q}{t}U = IU$$

Thus power is the product of the current and the change in potential. This is completely analogous to the flow of water in the case of gravity: A larger current of water, dropping through a larger potential, represents a greater release of power from gravitational potential energy.

Example Problem #1

What would be the diameter of a 1 F capacitor if its plates were separated by 1 cm of air?

Solution:

$$C = \kappa \varepsilon_0 A/d$$

$d = 1\ cm = .01\ m$
$\kappa = 1.0$ for air

$$1\ F = \frac{1\ (8.85 \times 10^{-12}\ C^2/Nm^2)\ A}{.01\ m}$$

$$1\ C^2/Nm = \frac{1\ (8.85 \times 10^{-12}\ C^2/Nm^2)\ A}{.01\ m}$$

$$1 = \frac{1\ (8.85 \times 10^{-12}\ 1/m)\ A}{.01\ m}$$

Canceling out units.

$$A = \frac{.01\ m}{1\ (8.85 \times 10^{-12}\ 1/m)}$$

$$A = 1.1299 \times 10^{9}\ m^2$$

$$1.1299 \times 10^{9}\ m^2 = \pi d^2/4$$

Area of a circle in terms of diameter.

$$d = 37{,}930\ m \text{ or } 37.93\ km$$

The capacitor would have diameter of almost 38 km. A Farad is a ***lot*** of capacitor!

Example Problem #2

Suppose you wanted to power an electric car by using capacitors to provide a surge of energy to accelerate a car, and then batteries to make the car cruise at a steady speed.

How much capacitance would be required to accelerate a 1500 kg car from 0 to 60 mph if the capacitors were charged via 48 V batteries?

Solution:

At 0 mph the car has KE = 0. At 60 mph the car has KE = ½ mv^2. So I need that much energy stored in the capacitor.

$E = \frac{1}{2} Q\,\mathcal{V} = \frac{1}{2} mv^2$

so $Q\,\mathcal{V} = mv^2$

but $C = Q/\mathcal{V}$ so $Q = C\,\mathcal{V}$

$(C\,\mathcal{V})\,\mathcal{V} = mv^2$

$C = mv^2/\mathcal{V}^2$

$C = m(v/\mathcal{V})^2$

$m = 1500$ kg, $\mathcal{V} = 48$ V, $v = 26.82$ m/s

$C = 1500\text{ kg }(26.82\text{ m/s } / \text{ } 48\text{ V})^2$
$= 468\text{ kg(m/s)}^2(1/\text{V})^2$ — a V is 1 J/C (here C means Coulomb not capacitance)
$= 468\text{ kg(m/s)}^2(\text{C/J})^2$
$= 468\text{ (kgm/s}^2)(\text{m})(\text{C/J})^2$
$= 468\text{ N(m)(C/J)}^2$ — a kgm/s^2 is a N
$= 468\text{ J(C/J)}^2$ — a Nm is a J
$= 468\text{ C}^2/\text{J}$
$= 468\text{ C(C/J)}$
$= 468\text{ C(1/V)}$
$= 468\text{ C/V}$
$= 468\text{ F}$

468 Farads of capacitance would be required.

Example Problem #3

When a light bulb is connected to a 6 V battery 2 A of current flow. Calculate the resistance of the bulb and the power output of the bulb.

Solution:

$U = 6\,V$
$I = 2\,A$

$R = U/I = 6V/2A = 3\Omega$

$P = IU = 2A\,(6V) = 12\,W$

Bulb resistance is 3 ohms, bulb power is 12 Watts.