DAY 23 – Homework

1. At what speed must a meterstick move past you if it is to have length 50 cm? 10 cm? 1 cm? Give answers in terms of c.

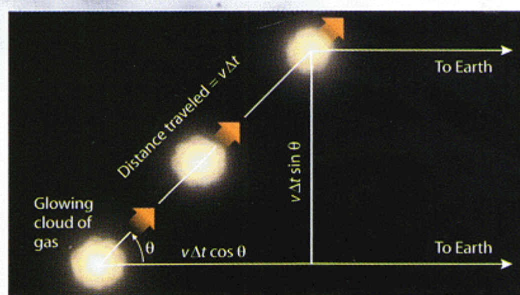
2. Superluminal motion is a phenomenon in astrophysics where material in space (usually a jet of gas ejected from a galaxy) appears to be moving faster than the speed of light. Here is a discussion of superluminal motion from *Sky & Telescope* magazine:

AN ILLUSORY BREAK WITH RELATIVITY

Superluminal motions, in which matter apparently exceeds the speed of light, are a beautiful illusion created by the laws of special relativity. In general, a jet's axis of motion will be inclined at some angle with respect to our line of sight, with one pole-aligned beam heading toward us and the other away from us. If the radiating clouds of gas within the jet are moving at near-light speeds, the approaching cloud will almost catch up with the radiation it has just emitted. Consequently, we on Earth receive a continuous succession of radiation from the moving cloud in a time-compressed fashion, like a fast-forwarded video. If an observer naively calculates the cloud's motion across the plane of the sky, he or she will erroneously conclude that the cloud exceeded the speed of light — a result forbidden by Albert Einstein's relativity theory.

Of course, once special relativity is folded into the calculation one finds that the speed of the jet is very close to — but certainly less than — that of light. The receding side of the jet suffers from the opposite effect, and we see it moving in the sky more slowly than it truly is.

The diagram shown here illustrates the concept. When any object emitting electromagnetic radiation moves toward Earth at a near-light speed of v, it "chases" its own light (or radio



waves). Say a cloud of plasma is ejected by a microquasar, and it emits radio waves at two times Δt seconds apart. Images taken from Earth at two separate times will suggest that is has moved a distance of $v\Delta t\sin\theta$, where θ is the angle between the cloud's direction of motion and our sightline to the microquasar, in just $\Delta t\,[1-(v/c)\cos\theta]$ seconds. (Here c is the speed of light, roughly 300,000 kilometers per second.) The cloud's apparent across-the-sky velocity is then $v_{app} = v\sin\theta/[1-(v/c)\cos\theta]$. As an example, if $v = 0.8c$ and $\theta = 30°$, then $v_{app} = 1.3c$ — an apparent (but ultimately explicable!) violation of Einstein's speed limit.

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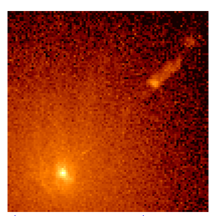
Discuss and derive their formula for apparent velocity (v_{app}).

A superluminal jet in the galaxy M-87.



http://science.msfc.nasa.gov/newhome/headlines/ast11jun97_1.htm

3. A proton is shot down a 1.5 km long tube at a speed of .95 c. How long will it take the proton to traverse the tube, as measured by the proton? How long is the tube, as measured by the proton?

4. How much of a time dilation effect is there if an object is traveling at 1/10 the speed of light?

5. In Star Trek motion through normal space is called "Impulse" speed.
"Ahead ¼ Impulse" means go forward at a leisurely rate.
"Ahead ½ Impulse" means go forward smartly.
"Ahead Full Impulse" means "floor it".
If various trekkies have explained this to me properly,
Full Impulse is actually defined as traveling significantly

less than the speed of light, so as to keep time dilation effects at bay.

Make an accurate plot of time dilation (as a percentage – a dilation of 50% being that time dilation makes 1 minute of proper time take 1.5 minutes as seen by an outside observer) vs. speed for speeds ranging from 0 to c. Discuss at what speed time dilation becomes significant.