DAY 18 – Homework

1. When you tune a cello or other stringed instrument, you increase or decrease the tension in the strings. You do not change the length of vibrating portion of the string. Therefore, you do not change the wavelength of the fundamental or any of the harmonics. Nonetheless, the frequency of the string changes. What property of the waves on the string must you be changing when you change the tension?

2. In the figure at right is shown all the notes of a piano, along with the frequencies of those notes (given in Hz).

   As stated in example #1 for today, sound travels at roughly 340 m/s and a tube that is open on both ends has a fundamental with a wavelength twice the length of the tube.

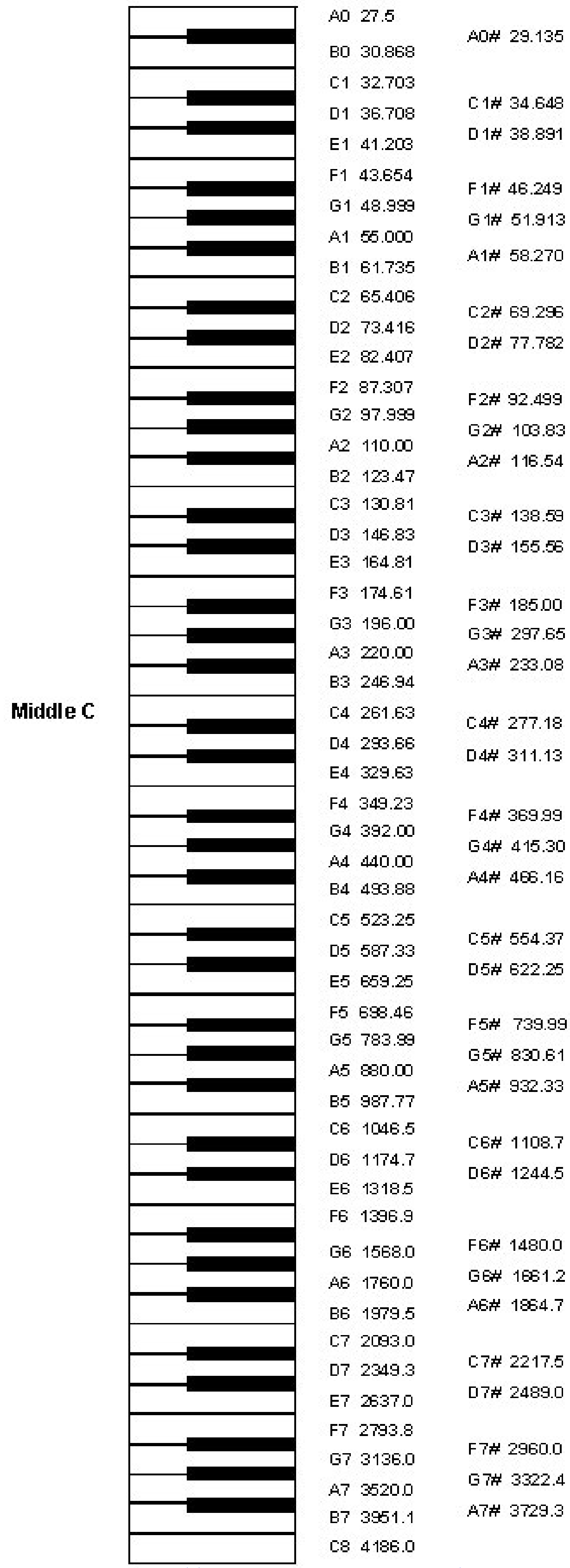
   You decide to create a musical instrument from PVC pipe that uses fundamental vibrations of the air in the pipes to produce sound. Determine the lengths of pipe required to produce all the notes between C3 and C5 on the piano. Make a to-scale drawing of this set of pipes. This instrument would be a primitive pipe organ.

3. The speed of sound in air is a function of temperature. One approximate equation for this is

   $$v = (331\,m/s)\sqrt{1 + \frac{T_{inCelsius}}{273^{o}\,C}}$$

   What problems will this cause for your organ in problem 2?

4. A stiff wire is clamped on one end, and free on the other. The



http://www.vibrationdata.com/piano.htm

boundary conditions in this system are that there is a node at the clamped end and an antinode at the free end. Sketch some of the standing waves possible in this system.

5. Determine an equation for the wavelengths of standing waves in the above system:

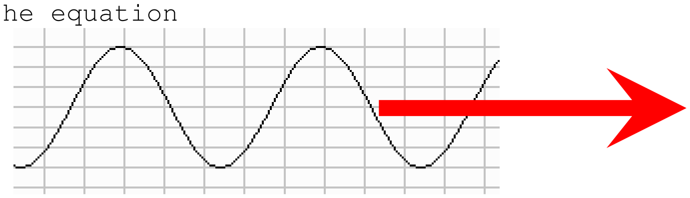
   λ_n = … ?

6. A string clamped at both ends resonates in its 3^{rd} harmonic when driven at 200 Hz. What other frequencies will produce resonance? Do not go above the 5^{th} harmonic.

7. What do you notice about the frequencies of the notes on the piano? Discuss whether the following is true – every A note on a piano can in theory be obtained from a string with fundamental frequency f_{FUND} = 27.5 Hz.

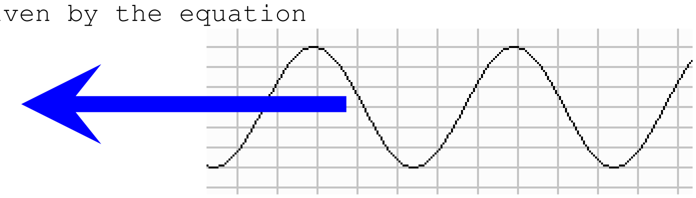
8. A wave of wavelength λ, period T, and phase angle ϕ = 0 that is moving to the right is given by the equation

$$f(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)$$



   Another wave with the same wavelength and period and phase angle that moves to the left is given by the equation

$$g(x,t) = A\sin\left(\frac{2\pi}{\lambda}x + \frac{2\pi}{T}t\right)$$



   Now, determine y(x,t) = f(x,t) + g(x,t) to get the equation for a standing wave. Discuss your results.

   HINT -- Sum formulas for sine and cosine

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$