DAY 16 -- Homework

1. Since F = – b v then the damping coefficient must have units of N per m/s. Show that units of N per m/s work in the equation F = –b v. Show also that the units work out for the critical damping coefficient equation.

$$b_c = 2\sqrt{mk}$$

2. Explain how a kid's "pumping" while on a swing works -- from the standpoint of resonance.

3. The motion of the damped oscillator is given by

$$x = A\cos(\varpi t + \phi)$$

   where

$$A = A_0 e^{-\frac{b}{2m}t}$$

$$\varpi = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

   An oscillator has mass 10 kg and undamped frequency of 4 Hz. (a) Determine the spring constant of the oscillator. (b) Determine what damping coefficient is require for critical damping. (c) Make a plot of x vs. t for the case where the oscillator is undamped (b = 0) and starts with amplitude A_0 = –10 cm. Plot for 10 cycles. (d) What is ϕ in part c? (e) Make additional plots for the cases where the oscillator is lightly damped, underdamped, and critically damped. Use EXCEL and starting amplitudes of either A_0 = 10 cm or A_0 = –10 cm.

4. At what frequency would you drive a 1 meter long pendulum in order to get it to oscillate with high amplitude?

5. Your car has 14" diameter rims with tires are 5" tall. Thanks to an out-of-balance tire, your steering wheel shimmies violently at 63 mph, but is hardly noticeable at speeds much above or below that. What is the resonant frequency of your car's suspension (in Hz)? Explain why bad shocks will make the effect of an out-of-balance tire worse.

PROBLEMS 6-8
PHY 231 ONLY

6. Derive an equation for the frequency at which peak amplitude occurs in a driven harmonic oscillator. Show that your equation reduces to

$$\varpi_{peak} = \sqrt{\frac{k}{m}}$$

for lightly damped systems. Hint – use the old "take a derivative to find a maximum/minimum" technique.

7. By direct substitution, show that

$$x = A\cos(\varpi t + \phi)$$

where

$$A = A_0 e^{-\frac{b}{2m}t}$$

$$\varpi = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

is the solution to

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

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8. Think you're *tough*? Think you are a math *wizard*? A ***genius***?

OK, by direct substitution, show that

$$x = A\cos(\varpi_D t + \phi)$$

where

$$A = \frac{F_{ext}/m}{\sqrt{\left(\varpi_D^2 - \frac{k}{m}\right)^2 + \left(\frac{b\varpi_D}{m}\right)^2}}$$

is the solution to

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = F_{ext}\cos(\varpi_D t).$$