DAY 8

Summary of Primary Topics Covered

Density

Density is the relationship between an object's mass and the amount of volume it takes up. A kilogram of water takes up a volume of 1 liter, so the density of water is 1 kg/l. A kilogram of feathers would take up a lot more space than 1 liter.

The symbol usually used for density is the Greek letter "rho" (ρ):

$$\rho = \frac{m}{V} = \frac{mass}{volume}$$

Density is a characteristic of a material. For instance, lead is very dense – roughly eleven times the density of water. Styrofoam is not very dense – roughly 1/20 the density of water. You usually have to look up the density of a material in some sort of table. Tables of densities have been added to the class web page.

Weight

We've already encountered weight in our discussions. Weight is the force that gravity exerts on an object. Weight is given by

$$W = mg$$

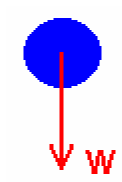
Where m is the mass of an object and g is the strength of the gravitational field. Near the surface of Earth,

$$g = 9.8N/kg = 2.203lb/kg$$

So a 1 kg object weighs 9.8 N or 2.203 lb.

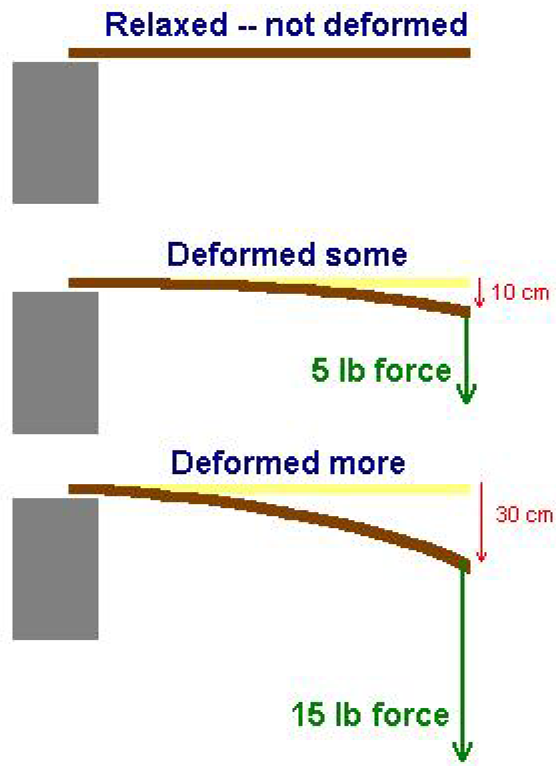
An object is said to be in "free-fall" if the only significant force acting on it is gravity. This is most true for large, dense objects that are not moving too fast. A brick dropped off the edge of a table would be a free-falling object.

A free-body diagram for an object in free-fall looks like this:



$\Sigma F = W$	The only force acting on it is weight.
$m\,a = m\,g$	$\Sigma F = ma$ and $W = mg$
$a = g$	m cancels out
$a = 9.8$ N/kg	Grav. strength is 9.8 N/kg
$a = 9.8\ (kgm/s^2)/kg$	1 N = 1 kgm/s^2
$a = 9.8\ m/s^2$	kg's cancel out
	The object accelerates at 9.8 m/s^2
$9.8\ m/s^2 \ \dfrac{2.237\ \text{mph}}{1\ \text{m/s}} = 21.9$ mph/s	or 21.9 mph/s
	9.8 m/s^2 or 21.9 mph/s is called a "g".

Elastic Forces – Hooke's Law



We've already encountered elastic forces in our discussions, too. A force required to deform an object (like a spring or a flexible stick) that is proportional to the amount the object is deformed is called a Hooke's Law Force. Hooke's Law is

$$F = -kx$$

Where F is the amount of force required to deform the object, k is the stiffness or Spring Constant of the object, and x is the amount the object is deformed. The minus sign indicates that the object will tend to spring back in the direction opposite of which it was deformed. In the picture at right, a 5 lb force bends the rod 10 cm, and a 15 lb force deforms the rod 30 cm. So the spring constant will be

k = 0.5 lb/cm.

Most objects only follow Hooke's Law if they are slightly deformed (so that they will <u>return to their original shape</u> when the deforming force is released). If they are deformed so radically that they are permanently altered in shape, then you can't count on Hooke's Law. And obviously Hooke's law won't work if an object breaks either.

Example Problem #1:

A lead sphere 10 cm in diameter is attached to a spring with a spring constant of 500 N/m. How much does the ball weigh and how much will the spring stretch? Give answers in English units.

$d = 10 \text{ cm}$

$r = 5 \text{ cm}$

Volume $V = \frac{4}{3}\pi r^3$ for sphere

$V = \frac{4}{3}\pi (5 \text{ cm})^3 = 523.599 \text{ cm}^3$

density of lead (from tables) $\rho = 11.34 \text{ g/cm}^3$

mass $m = \rho V = \left(11.34 \frac{\text{g}}{\text{cm}^3}\right)\left(523.599 \text{ cm}^3\right)$

$m = 5937.610 \text{ g} = 5.93761 \text{ kg}$

Weight $W = mg = (5.93761 \text{ kg})\left(9.8 \frac{N}{kg}\right) =$

$W = 58.1886 \text{ N} = 13.082 \text{ lbs}$

SPHERE WEIGHS 13.1 lbs

$F = kx$ $\quad F = W = 58.1886 \text{ N}$

$k = 500 \text{ N/m}$

$58.1886 \cancel{N} = 500 \frac{\cancel{N}}{m} x$ N's cancel

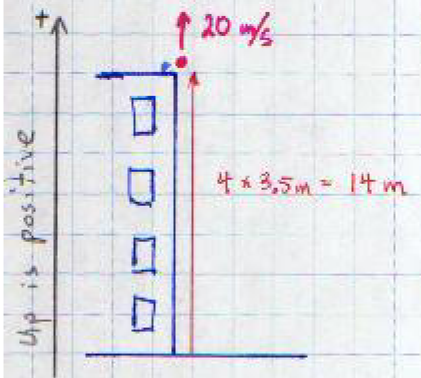
$.116377 \text{ m} = x$

$.116377 \text{ m} \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = .382 \text{ ft}$ or 4.58 inches

SPRING STRETCHES 4.6 inches

Example Problem #2:

A rock is tossed upwards at a speed of 20 m/s from the top on a 4-story building (each story is 3.5 m). How long before it hits the ground?



$V_0 = 20$ m/s
$X_0 = 14$ m
$a = -9.8$ m/s²

Starting values

$a = -9.8$ m/s² because the object is free-falling

$X = 0$ m (ending value)

I want time.

$$X = X_0 + V_0 t + \frac{1}{2} a t^2$$

$$0 = 14\text{m} + 20\,\text{m/s}\; t + \frac{1}{2}(-9.8\,\text{m/s}^2)\, t^2$$

$$0 = 14\text{m} + 20\,\text{m/s}\; t - 4.9\,\text{m/s}^2\; t^2$$

cancel meters – that just leaves seconds.

$$0 = -4.9t^2 + 20t + 14$$

Use quadratic formula

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-4.9)14}}{2(-4.9)}$$

$$t = \frac{-20 + 25.9692}{-9.8} \quad \text{or} \quad t = \frac{-20 - 25.9692}{-9.8}$$

~~$t = -3.9308$ sec~~ or $t = 4.6907$

Can't have negative time

The rock hits the ground in 4.7 sec