DAY 7 -- PHY 231 ONLY

Summary of Primary Topics Covered

Calculus with Energy and Power

In calss we often mentioned the calculus versions of various algebra equations. For example

Algebra	Calculus
$a=\Delta v/t$	$a=dv/dt$
$P=E/t$	$P=dE/dt$

One powerful use of calculus comes from using a computer to "fit" an equation to data so that we can then analyze the equation via calculus.

Momentum and Newton's 2nd Law of Motion the way Newton wrote it

$\Sigma F = \Delta p/t$ is actually closer to the way Newton actually wrote the 2nd law. Newton wrote the second law in derivative form in terms of momentum:

$$\Sigma F = dp/dt$$

since $p = m\,v$, you can use the Chain Rule on this

$$\Sigma F = dp/dt$$
$$\Sigma F = d(mv)/dt$$
$$\Sigma F = m(dv/dt) + (dm/dt)v$$

If mass is constant then $dm/dt = 0$.

$a = dv/dt$ so we plug that in and get

$$\Sigma F = m(a) + 0(v)$$

and that gives us the Newton's 2nd Law we know.

$$\Sigma F = m\,a$$

BUT, if mass is not constant then dm/dt is not zero.

$$a = dv/dt$$

$\Sigma F = m(a) + (dm/dt)v$

so we get a different form of Newton's 2nd Law.

$\Sigma F = m\,a + (dm/dt)v$

This is actually the full form of Newton's 2nd Law. We use it when mass is changing – like in a rocket that is burning up fuel and therefore losing mass.

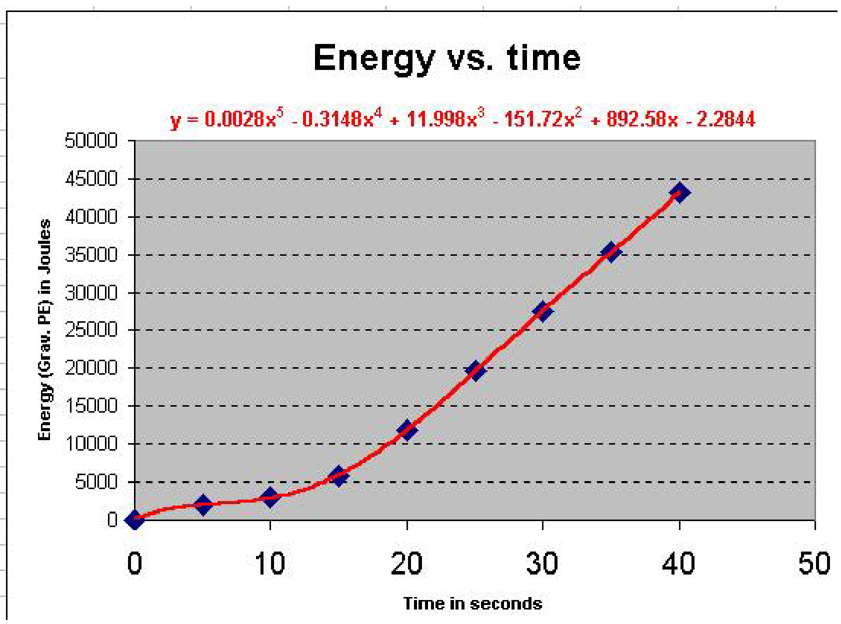
Example Problem #1 (PHY 231 ONLY):

A motor is used to lift a 400 kg mass. Here is data on the height of the mass vs. time, where t=0 when the lift is turned on. (a) Obtain an equation for the Energy of the mass as a function of time, and use that equation to get the power output of the motor as a function of time. (b) Graph the power output as a function of time. (c) If the motor consumes a steady 2000 W of electric power, when it the most heat being generated?

Time (s)	Height (m)
0	0
5	0.5
10	0.75
15	1.5
20	3
25	5
30	7
35	9
40	11

First I use EXCEL to calculate the PE of the mass at each time. Then I make a graph of PE vs. time. Then I fit a trend line to the graph. A 5th order polynomial fit the points well.

Time (s)	Height (m)	PE = mgy (Joules)
0	0	0
5	0.5	1960
10	0.75	2940
15	1.5	5880
20	3	11760
25	5	19600
30	7	27440
35	9	35280
40	11	43120



The computer gave me the equation

$y = 0.0028x^5 - 0.3148x^4 + 11.998x^3 - 151.72x^2 + 892.58x - 2.2844$

which in terms of what is plotted (Energy is y, time is x) is really

$E = 0.0028t^5 - 0.3148t^4 + 11.998t^3 - 151.72t^2 + 892.58t - 2.2844$

The power that went into giving the mass PE is given by

$P = dE/dt$

$P = d/dt\,(0.0028t^5 - 0.3148t^4 + 11.998t^3 - 151.72t^2 + 892.58t - 2.2844)$

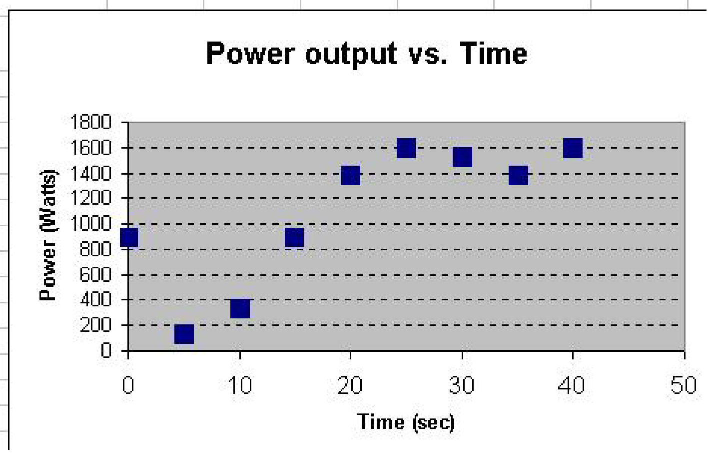
$\mathbf{P = 0.014t^4 - 1.2592t^3 + 35.994t^2 - 303.44t + 892.58}$

Graphing this using EXCEL gives me the plot below. I see there is an initial surge of power, then a drop, then the power rises and at around t=20 seconds becomes more or less constant at roughly 1500 W.

The most heat is being generated at the point where the least power is going into lifting the object. That would seem to be at t = 5 seconds, where less than 100 W of power was going into lifting the mass. Presumably the rest was going into heat. Maybe a belt was slipping somewhere.

B4 = 0.014*A4^4 - 1.2592*A4^3 + 35.994*A4^2 - 303.44*A4 + 892.58

Time (s)	Power (Watts)
0	892.58
5	126.58
10	338.38
15	898.58
20	1387.78
25	1596.58
30	1525.58
35	1385.38
40	1596.58



Example Problem #2 (PHY 231 ONLY):

A rocket in space (no forces acting on it other than its engine) has the following characteristics:

Mass = 1000 kg
Rate of fuel consumption = 1 kg/s
Velocity = 500 m/s
Engine Thrust = 2000 N

What is the rocket's acceleration? Does the fuel consumption significantly affect the acceleration?

ΣF	$= m\,a + (dm/dt)v$	*Rocket doesn't have constant mass because it is consuming fuel at a significant rate.*
2000 N	= 1000 kg (a) + (-1 kg/s) 500 m/s	*Rate of fuel consumption is the rate of mass change. It is mass loss so dm/dt = -1 kg/s*
2000 kgm/s^2	= 1000 kg (a) - 500 kgm/s^2	
2500 m/s^2	= 1000 a	
2.500 m/s^2	= a	

ANSWER: 2.5 m/s^2

If we ignore the mass loss and assume constant mass, then we get

ΣF	= m a
2000 kgm/s^2	= 1000 kg (a)
2000 m/s^2	= 1000 a
2.000 m/s^2	= a

2.5 vs. 2.0 is a 25% difference. The fuel consumption affects the acceleration significantly.