DAY 6

Summary of Primary Topics Covered

Work

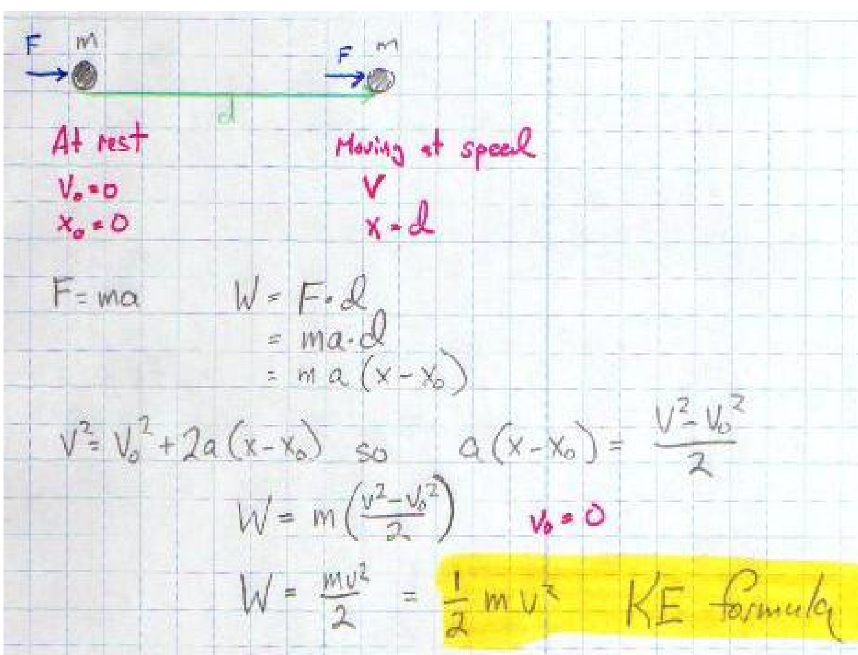
Work is defined as the product of a force and the distance moved by that force in a direction parallel to the force. The "parallel" requirement is a hallmark of a specific type of product, known as the DOT product. We will encounter the DOT product in detail later in the semester. A dot is used as the multiplication symbol in our work equation:

$$W = F \cdot d$$

Using this equation, we can see from where the formulas for Kinetic Energy and Potential Energy are derived.

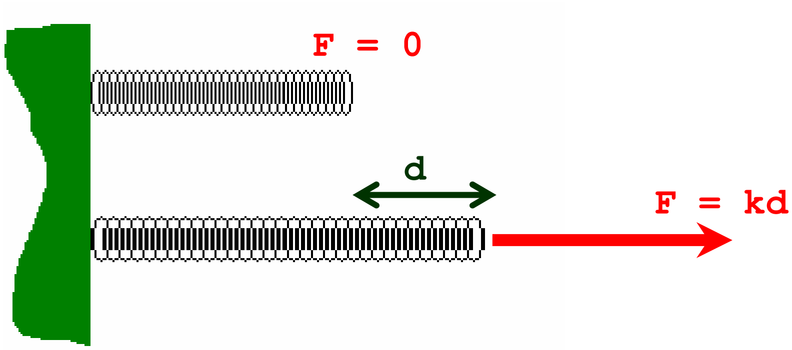
Deriving Kinetic Energy formula

A force F is applied to an object of mass m that is at rest. It accelerates up to speed v in a distance d. The object gains Kinetic Energy because it is moving.



Deriving Elastic Potential Energy formula

A spring with force constant k is stretched a distance d. The spring gains Potential Energy because it is stretched.



$W = F \cdot d$

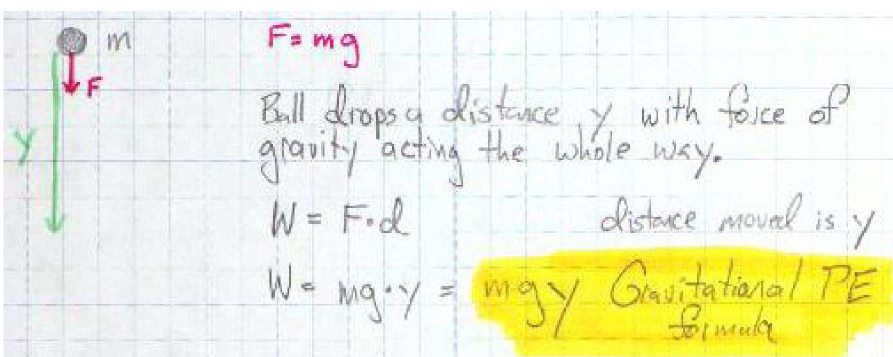
$F_{avg} = ½(0 + kd) = ½\ kd$

$W = ½\ kd \cdot d = ½kd^2$

If we use x for distance instead of d we have the elastic potential energy formula $W = ½\ kx^2$.

Deriving Gravitational Potential Energy formula

An object of mass m drops a distance y. The gravitational field strength is g.



Heat Energy

We don't have a heat energy formula, but we do have a variable to use in representing heat -- Q.

Momentum

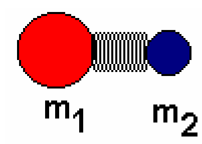
Momentum: $p = mv$

The concept of Momentum comes from Newton's 3rd Law of Motion:

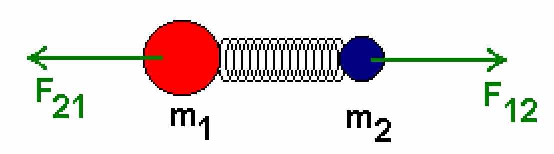
> *If one object exerts a force on a second object, the second object exerts an equal and opposite force back on the first object.*

$$F_{12} = -F_{21}$$

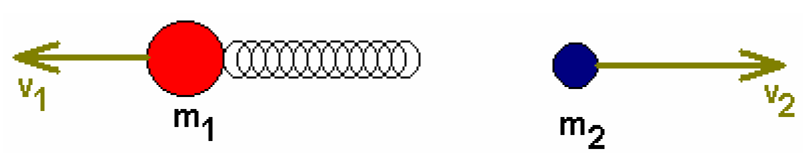
For example, consider two heavy masses connected by a light compressed spring. Everything is at rest.



The spring is released, creating a repulsive force between the two masses.



The two forces are equal and opposite. Furthermore, they both act for the same amount of time. Once the spring loses contact, there is no more force between the objects and they both fly off with whatever velocity they attained while the forces were acting on them.



$F_{21} = - F_{12}$		*Newton's 3rd Law*
$m_1a_1 = - m_2a_2$		*F = ma (Newton's 2nd Law)*
$m_1(\Delta v_1/t) = - m_2(\Delta v_2/t)$		*a = Δv/t*
$m_1(v_1/t) = - m_2(v_2/t)$		*Both start at rest ($v_0 = 0$) so $\Delta v = v - v_0 = v - 0 = v$*
$m_1v_1 = - m_2v_2$		*times cancel out*

Momentum: p = m v

Experiments have shown that the total momentum in a system is always constant. Momentum is <u>conserved</u>.

Momentum and Newton's 2nd Law

Combining Newton's 2nd Law with our momentum equation gives us a new way to write Newton's 2nd Law:

$$\Sigma F = m\ a$$

$$= m\ (\Delta v/t) \qquad a = \Delta v/t$$

$$= m\ (v - v_0)/t \qquad \Delta v = v - v_0$$

$$= (mv - mv_0)/t$$

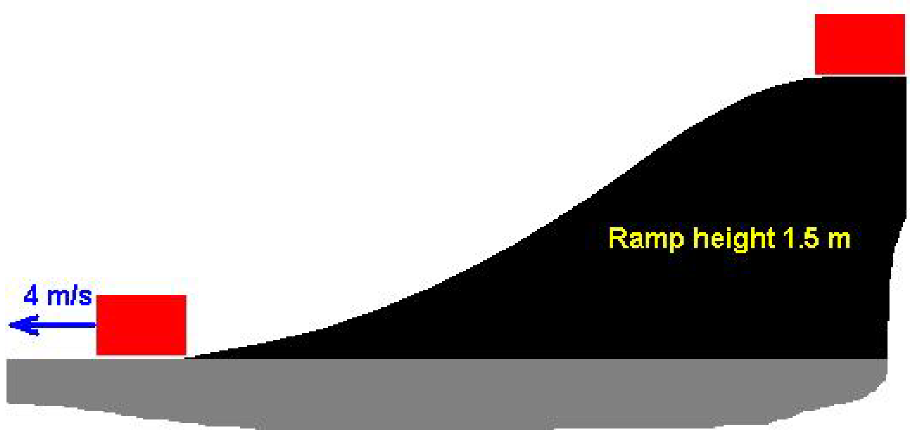
$$= (p - p_0)/t$$

$$\mathbf{\Sigma F = \Delta p/t}$$

Net force is change in momentum over elapsed time.

Example Problem #1:

A 50 kg crate slides down a ramp as shown. When it reaches the bottom it is moving at 4 m/s. How much heat energy was generated?



Energy at top = Energy at bottom

At the top the energy is all gravitational PE. At bottom the crate has KE and some energy has been converted to heat (Q) as it slid down.

$$PE = KE + Q$$

$$mgy = \tfrac{1}{2}\ m\ v^2 + Q$$

$$(50\ kg)(9.8\ N/kg)(1.5\ m) = (0.5)(50\ kg)(4\ m/s)^2 + Q$$

$$735\ Nm = 400\ kgm^2/s^2 + Q$$

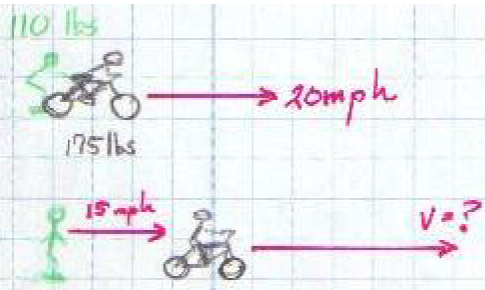
Nm and kgm^2/s^2 are both Joules

$$735\ J = 400\ J + Q$$

$$Q = 335\ J$$

Example Problem #2:

A bicyclist (175 lbs including the bike) is pulling a roller blader (110 lbs) along at a speed of 20 mph. The roller blader lets go, giving the cyclist a good shove. After the let-go, the roller blader is moving at 15 mph. How fast is the cyclist moving after the let-go (in mph)? Frictional effects are small.



First convert stuff to metric SI units

$20 \text{ mph} \left(\frac{1 \text{ m/s}}{2.237 \text{ mph}}\right) = 8.9405 \text{ m/s}$ $15\text{mph} = 6.7054$ mph

$110 \text{ lbs} \left(\frac{1 \text{ kg}}{2.203 \text{ lb}}\right) = 49.9319 \text{ kg}$

$175 \text{ lbs} \left(\frac{1 \text{ kg}}{2.203 \text{ kg}}\right) = 79.4371 \text{ kg}$

Conservation of Momentum

$P_{before} = P_{after}$

Before $P_{blader} + P_{biker} = P_{blader} + P_{biker}$ After

Before = After

$(49.9319 \text{ kg})\, 8.9405 \text{ m/s} + (79.4371 \text{ kg})\, 8.9405 \text{ m/s} = (49.9319 \text{ kg})\, 6.7054 \text{ m/s} + P_{biker}$

$1156.6234 \frac{\text{kg m}}{\text{s}} = 334.8134 \frac{\text{kg m}}{\text{s}} + P_{biker}$

$821.8100 \frac{\text{kg m}}{\text{s}} = P_{biker} = m_{biker} V$

$$\frac{821.8100 \frac{\text{kg m}}{\text{s}}}{79.4371 \text{ kg}} = V$$

$10.3454 \text{ m/s} = V$

ANSWER: V = 23.1 mph

Example Problem #3:

In the previous example, how much work does the blader do when she pushes the biker?

Before the push-off both blader and biker have KE.

Blader's mass is 49.9319 kg, velocity is 8.9045 m/s.

Biker's mass is 79.4371 kg, velocity is 8.9045 m/s.

$KE_{blader} = ½\ mv^2$
$KE_{blader} = ½\ (49.9319\ kg)(8.9045\ m/s)^2 = 1979.5532\ J$

$KE_{biker} = ½\ mv^2$
$KE_{biker} = ½\ (79.4371\ kg)(8.9045\ m/s)^2 = 3149.2886\ J$

$E_{before} = 1979.5532\ J + 3149.2886\ J = 5128.8418\ J$

After the push-off both blader and biker have KE.

Blader's mass is 49.9319 kg, velocity is 6.7054 m/s.

Biker's mass is 79.4371 kg, velocity is 10.3454 m/s.

$KE_{blader} = ½\ mv^2$
$KE_{blader} = ½\ (49.9319\ kg)(6.7054\ m/s)^2 = 1122.5288\ J$

$KE_{biker} = ½\ mv^2$
$KE_{biker} = ½\ (79.4371\ kg)(10.3454\ m/s)^2 = 4250.9692\ J$

$E_{after} = 1122.5288\ J + 4250.9692\ J = 5373.4980\ J$

There's 244.6562 J more Kinetic Energy
(5373.4980 J - 5128.8418 J = 244.6562 J)
after than there was **before**. The extra energy must come from the blader burning calories and converting food energy into KE when she pushed the biker.

ANSWER: She did 245 J of work.