DAY 5

Summary of Primary Topics Covered

Kinematic Equations

The Kinematic Equations represent no new knowledge. They are simply a re-arranging of the equations for average acceleration, and average velocity, that we have already learned.

First I write starting and ending values a certain way:

	Starting	Ending	Change
Velocity	V_0	V	$\Delta V = V - V_0$
position	X_0	X	$\Delta X = X - X_0$

Now I use these in the equations that I've learned so far...

$$a_{avg} = \frac{\Delta V}{t} = \frac{V - V_0}{t}$$ Acceleration (equation A)

$$V_{avg} = \frac{\Delta X}{t} = \frac{X - X_0}{t}$$ Velocity (equation B)

$$V_{avg} = \frac{V_0 + V}{2}$$ Averaging velocity (equation C)

... and I labelled them A, B, + C

Equations B + C are both for V_{avg}, so they must be equal...

$$V_{avg} = V_{avg}$$

$$\frac{X - X_0}{t} = \frac{V_0 + V}{2}$$ multiply through by "t"

$$X - X_0 = \frac{1}{2}(V_0 + V)\,t$$

$$X = X_0 + \frac{1}{2}(V_0 + V)\,t$$ ← I'll call this Equation 1

Now re-arrange equation A...

$$a_{avg} = \frac{V - V_0}{t}$$

$$a_{avg}\,t = V - V_0$$ multiplying through by "t"

$$V_0 + a_{avg}\,t = V$$ bringing V_0 over

$$V = V_0 + a_{avg}\,t$$ ← Equation 2

Now plug equation 2 into equation 1

$$x = x_0 + \frac{1}{2}(v_0 + v)\,t$$ equation 1

$$v = v_0 + at$$ equation 2 goes in for "v"

$$x = x_0 + \frac{1}{2}(v_0 + v_0 + a_{avg}t)\,t$$

$$x = x_0 + \frac{1}{2}v_0 t + \frac{1}{2}v_0 t + \frac{1}{2}a_{avg}t(t)$$

$$x = x_0 + v_0 t + \frac{1}{2}a_{avg}t^2$$ ← Equation 3

Finally, plug equation A into equation 1

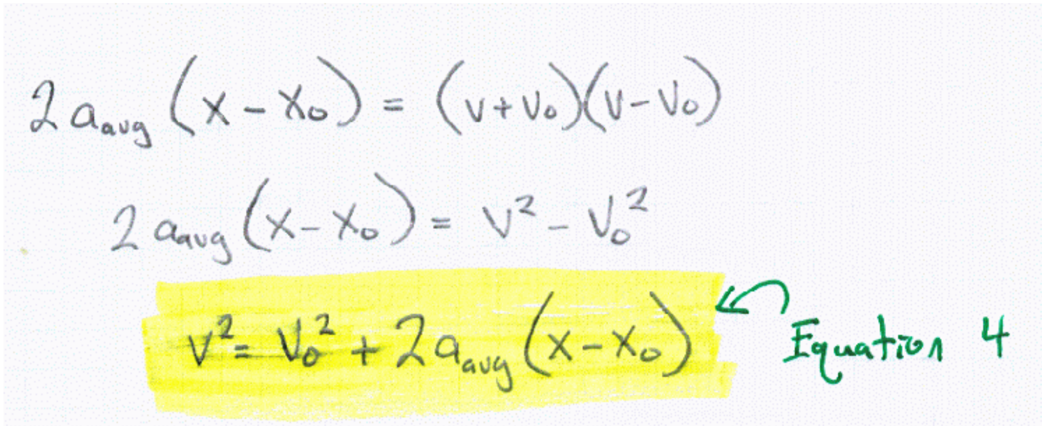
$$x = x_0 + \frac{1}{2}(v_0 + v)\,t$$ equation 1

$$a_{avg} = \frac{v - v_0}{t}$$ equation A

$$t = \frac{v - v_0}{a_{avg}}$$ this goes in for "t"

$$x = x_0 + \frac{1}{2}(v_0 + v)\left(\frac{v - v_0}{a_{avg}}\right)$$

$$x - x_0 = \frac{(v + v_0)(v - v_0)}{2\,a_{avg}}$$



The four equations derived here are the *kinematic equations*:

$$\mathbf{x = x_0 + \tfrac{1}{2} (v + v_0)\ t}$$

$$\mathbf{v = v_0 + a_{avg}\ t}$$

$$\mathbf{x = x_0 + v_0 t + \tfrac{1}{2}\ a_{avg}\ t^2}$$

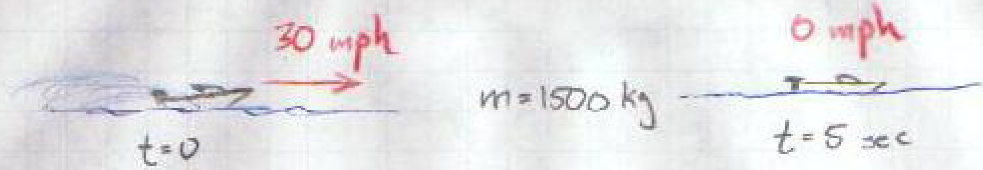
$$\mathbf{v^2 = v_0^2 + 2\ a_{avg}\ (x - x_0)}$$

While we could solve problems just using $a_{avg} = \Delta v/t$ and $v_{avg} = \Delta v/t$, with the kinematic equations we can solve problems more easily. They are very convenient.

Example Problem #1:

A boat of mass 1500 kg is moving at a steady 30 mph across the water.
(a) What is the net force on the boat?

The engine is then cut, and the boat comes to rest in 5 seconds.
(b) What is the boat's acceleration?
(c) How far does the boat move in coming to a halt?
(d) What is the net force on the boat?



FIRST –
Metric units: $30\ mph \left(\frac{1\ m/s}{2.237\ mph}\right) = 13.411\ m/s$

* Boat is moving at a constant 30 mph so it is <u>not</u> accelerating! Therefore net force on boat is zero.

SECOND –
I'll identify as many variables as I can:

$V_o = 13.411\ m/s$
$V = 0$
$X_o = 0$
$X = ?$
$a_{avg} = ?$
$t = 5\ sec$

Use $V = V_o + a_{avg} t$ to find a_{avg}:

$0 = 13.411\ m/s + a_{avg}(5\ s)$

$\frac{-13.411\ m/s}{5\ s} = a_{avg}$

$a_{avg} = -2.682\ m/s^2$

Now use

$X = X_o + \frac{1}{2}(V + V_o)t$ to find X:

$X = 0 + \frac{1}{2}(0 + 13.411\ m/s)\ 5\ s$

$X = 33.5275\ m$

Use Newton's 2nd Law to find net force:

$\Sigma F = ma$

$\Sigma F = 1500\ kg(-2.682\ m/s^2)$

$\Sigma F = -4023.25\ kg\ m/s^2$

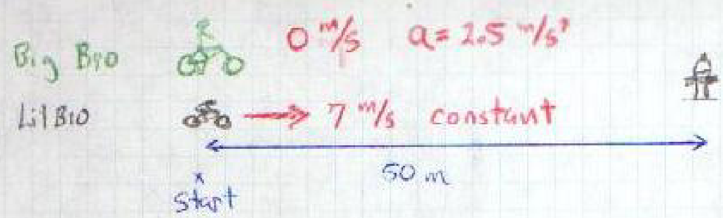
$\Sigma F = -4023.25\ N$

Answers:

* $-2.68\ m/s^2$ is the boat's acceleration
* The boat moves 33.5 m (110 ft) while stopping.
* The net force on the boat when stopping is −4023 N (−905 lbs). That force is to the rear of the boat

Example Problem #2:

Big brother challenges little brother to a bike race. They are going to race to a fire hydrant 50 m away. Big Bro tells Li'l Bro that he can have a running start. So Li'l Bro gets going as fast as he can (7 m/s) and, as he races past Big Bro, Big Bro takes off from rest with a constant acceleration of 2.5 m/s^2. Li'l Bro stays at 7 m/s the whole way, pedaling as hard as he can. Who gets to the hydrant first? What is the gap between them when the first one reaches the hydrant? Where does Big Bro overtake Li'l Bro?



For Li'L Bro

$x_o = 0$

$v_o = 7\,m/s$

$a = 0$

$x = x_o + v_o t + \frac{1}{2}at^2$

$x = 0 + 7\,m/s\, t + \frac{1}{2}(0)t^2$

$x = 7\,m/s\, t$

The problem asks for times (who gets there first) so I'll use these equations that have time in them.

For Big Bro

$x_o = 0$

$v_o = 0$

$a = 2.5\,m/s^2$

$x = x_o + v_o t + \frac{1}{2}at^2$

$x = 0 + 0(t) + \frac{1}{2}(2.5\,m/s^2)t^2$

$x = 1.25\,m/s^2\, t^2$

The hydrant is at $x = 50\,m$. Plug this in for x in each bro's equation to find time to get to hydrant.

$50\,m = 7\,m/s\; t$

$t = \frac{50\,m}{7\,m/s} = 7.1429\,s$

$50\,m = 1.25\,m/s^2\; t^2$

$t = \sqrt{\frac{50\,m}{1.25\,m/s^2}} = 6.3246\,s$

So Big Bro gets to hydrant first, at $t = 6.3$ sec.

When Big Bro is at hydrant at $t = 6.3246\,s$, where is Li'l Bro?

$x = 7\,m/s\,(6.3246\,s) = 44.2719\,m$

Where does Big Bro catch up? He catches up when their positions are equal:

$7\,m/s\; t = 1.25\,m/s^2\; t^2$

$\frac{7\,m/s}{1.25\,m/s^2} = t = 5.6\,sec$

So what are the positions?

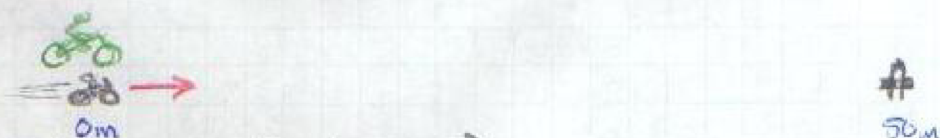
$x = 7\,\text{m/s}\,(5.6\,\text{s}) = 39.2\,\text{m}$ Li'l Bro

$x = 1.25\,\text{m/s}^2\,(5.6\,\text{s})^2 = 39.2\,\text{m}$ Big Bro

ANSWERS:

* Big Bro reaches hydrant first (at 6.3 sec vs. 7.1 s)
* When Big Bro is at hydrant (50 m), Li'l Bro is at 44.3 m. There's a 5.7 m gap between them.
* Big Bro catches up to Li'l Bro after 5.6 sec, at the 39.2 m mark.

Start (t=0)



Early in race (0 < t < 5.6 s)

Catches Li'l Bro (t = 5.6 sec)

Big Bro wins (t = 6.3 sec)