

DAY 4

Summary of Primary Topics Covered

Energy & Power with Math

Kinetic
Energy:

$$KE = \frac{1}{2}mv^2$$

SI energy units are kgm^2/s^2 , also known as Nm, also known as Joules (J).

Gravitational
Potential
Energy:

$$PE_{\text{Grav}} = mgy$$

g is the *gravitational field strength*. At the surface of the Earth this is

$$g = 9.8 \text{ N/kg} = 2.203 \text{ lb/kg}$$

meaning that gravity pulls a 1 kg object to the Earth with a force of 9.8 N or 2.203 lb.

Elastic
Potential
Energy:

$$PE_{\text{Elastic}} = \frac{1}{2}kx^2$$

k is the "spring constant" of the elastic object that is storing the energy. k is the force required to deform the object a certain amount. For example, if it takes 6 N of force to stretch a spring 1 cm, then $k = 6 \text{ N/cm}$ for that spring.

Power

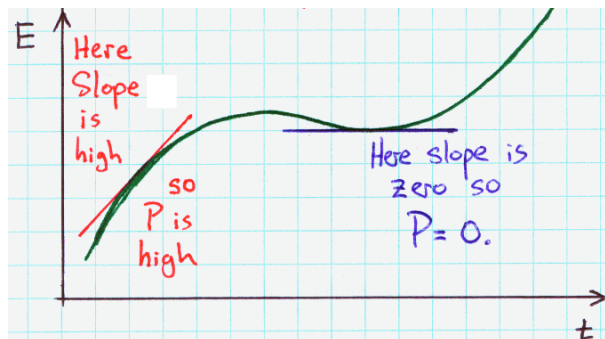
$$P = \frac{E}{t}$$

SI Power units are J/s, or Watts. English power unit is the Horsepower. 1 Hp = 746 W.

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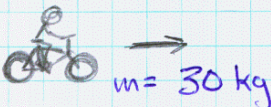
$$P = \frac{dE}{dt}$$

Power is the rate of change of energy - the slope of a tangent line in a plot of Energy vs. Time. On a plot of E vs. t , a steep slope indicates high power because E is increasing rapidly with t .

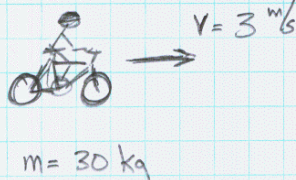


Example Problem #1

A mother who is teaching her child to ride a bike pushes the kid at 3 m/s. If the kid and his bike have a combined mass of 30 kg, what is their kinetic energy?



$m = 30 \text{ kg}$



$v = 3 \text{ m/s}$
 $m = 30 \text{ kg}$

I'll use mass, speed, and the Kinetic Energy formula

$$KE = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(30 \text{ kg})(3 \text{ m/s})^2$$
$$= 15 \text{ kg}(3 \text{ m/s})(3 \text{ m/s})$$
$$= 15(9) \text{ kg m}^2/\text{s}^2$$

I've collected the units together

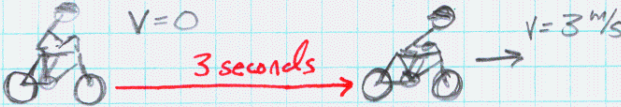
$$= 135 \text{ kg m}^2/\text{s}^2$$
$$KE = 135 \text{ J}$$

$\leftarrow \text{a kg m}^2/\text{s}^2 \text{ is a Joule}$

The KE of kid+bike is 135 Joules

Example Problem #2

In the above problem, it took the mother 3 seconds to get the kid from rest to moving at 3 m/s. What was her power output?



$v = 0$
 3 seconds
 $v = 3 \text{ m/s}$

$KE = 0$ $KE = 135 \text{ J}$

Kinetic Energy is the only energy we have here. So the mother gave the kid 135 J of energy in 3 seconds.

$$P = \frac{E}{t} = \frac{135 \text{ J}}{3 \text{ s}} = 45 \frac{\text{J}}{\text{s}} = 45 \text{ W}$$

\uparrow
 $1 \frac{\text{J}}{\text{s}} = 1 \text{ Watt}$

The mother's power output is 45 Watts

Example Problem #3

It takes 50 lbs of force to stretch a spring 10 cm. How much energy is stored in the spring when it is stretched this much?

Get stuff into SI units.

$$F = 50 \text{ lbs} \left(\frac{4.448 \text{ N}}{1 \text{ lb}} \right) = 222.4 \text{ N}$$

$$x = 10 \text{ cm} = 0.1 \text{ m}$$

$$k = \frac{222.4 \text{ N}}{0.1 \text{ m}} = 2224 \text{ N/m}$$

Now calculate PE_{Elastic} for $k = 2224 \text{ N/m}$ and $x = 0.1 \text{ m}$

$$\begin{aligned} PE &= \frac{1}{2} kx^2 = \frac{1}{2} (2224 \text{ N/m}) (0.1 \text{ m})^2 \\ &= 1112 \text{ N/m} (0.01 \text{ m}^2) = 11.12 \text{ Nm} \end{aligned}$$

ANSWER: 11.12 J

Example Problem #4

A model rocket has a mass of 0.75 kg. It is launched from the ground. The rocket motor fires for 3 seconds. At the end of those 3 seconds the rocket is moving at 300 mph and is 450 yards up in the air. What was the average power output of the motor in Watts?

Kinetic and Gravitational Potential Energy

$$KE = \frac{1}{2}mv^2$$
$$PE = mgy$$

First — Get everything in SI units.

$y = 450 \text{ yards} \left(\frac{3\text{ft}}{1\text{yard}} \right) \left(\frac{1\text{m}}{3.281\text{ft}} \right) = 411.560 \text{ m}$

$m = .75 \text{ kg}$

$v = 300 \text{ mph} \left(\frac{1\text{ m/s}}{2.237\text{ mph}} \right) = 134.108 \text{ m/s}$

$g = 9.8 \text{ m/s}^2$

Now find KE + PE

$$KE = \frac{1}{2} (.75 \text{ kg}) (134.108 \text{ m/s})^2 = 6744.558 \frac{\text{kg m}^2}{\text{s}^2}$$
$$PE = .75 \text{ kg} (9.8 \frac{\text{N}}{\text{kg}}) (411.560 \text{ m}) = 3024.966 \text{ Nm}$$

Both Joules

No Energy

So the engine gave the rocket a total of

$$6744.558 \text{ J} + 3024.966 \text{ J} = 9769.324 \text{ J}$$

of energy in 3 seconds

$$P = \frac{E}{t} = \frac{9769.324 \text{ J}}{3 \text{ sec}} = 3256.44 \text{ W/s}$$

ANSWER: 3256 Watts

$P = \frac{E}{t}$