DAY 3

Summary of Primary Topics Covered

Newton's Laws (with Mathematics)

Newton's 1st Law:	$\sum F = 0$	Deals with cases where the net force on an object is zero.
Newton's 2nd Law:	$a = \dfrac{\sum F}{m}$	Deals with cases where the net force on an object is not zero and the object accelerates.
Newton's 3rd Law:	$F_{12} = -F_{21}$	Deals with where forces come from.

Acceleration and Velocity

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Acceleration is change in velocity over elapsed time.

Usual units are the *mph per second* (*mph/s*) or the *meter per second per second* (*m/s/s* or *m/s*^{2}).

average

$$a_{avg} = \frac{\Delta v}{t}$$

instantaneous

$$a = \frac{dv}{dt}$$

Velocity is change in position over elapsed time.

Usual units are *mph* or *m/s*.

average

$$v_{avg} = \frac{\Delta x}{t}$$

instantaneous

$$v = \frac{dx}{dt}$$

Average of two velocities.

$$v_{avg} = \frac{v_1 + v_2}{2}$$

Mass and Force

Mass is a measure of inertia. Mass unit in the SI metric system is the *kilogram* (kg) -- the mass of 1 liter of water. One kg weighs about 2.203 lb at Earth's surface. There is no common mass measure in the English system of units.

Force is measured in *Newtons* in the SI metric system. 1 Newton = 1 kgm/s^2 (the force required to accelerate 1 kg at a rate of 1 m/s^2). The common English force unit is the pound (lb). 1 lb = 4.448 N.

Example Problem #1

A woman weights 130 lbs. Determine her weight in N and her mass in kg.

Solution:

To find Newtons I need to do a conversion. I know 1 lb = 4.448 N:

$$130lbs\left(\frac{4.448N}{1lb}\right) = 578.24N$$

To find her mass in kg, I remember that 1 kg weighs about 2.2 lb on Earth's surface:

$$130lbs\left(\frac{1kg}{2.2lb}\right) = 59.0909kg$$

So my final answer is that her weight is 578 N and her mass is 59.1 kg.

Example Problem #2

Near the Jefferson Downtown Campus (along 1^{st} street) there are a couple of lights where you can accelerate from a stop light right up an on-ramp and get onto I-65 South.

A driver is waiting at a light. The light turns green, and she steps on the gas. In 15 seconds she is speeding on I-65 going 75 mph. Calculate her acceleration in English and Metric units.

Solution:

0 - 75 mph in 15 seconds.

Change in speed is $\Delta V = +75$ mph

(speed increased by 75 mph)

Elapsed time is $t = 15$ sec

Calculating acceleration: $a_{avg} = \dfrac{\Delta V}{t} = \dfrac{+75 \text{ mph}}{15 \text{ sec}} = 5 \text{ mph/sec}$

$$a_{avg} = 5 \text{ mph/sec}$$

To do the problem in metric units we must convert 75 mph to "meters per second". I look up in the unit conversion table on this web page that 1 meter/sec = 2.237 mph. I set up a ratio to do this, eliminating the mph units

$$75 \text{ mph} \left(\frac{1 \text{ m/s}}{2.237 \text{ mph}} \right) = 33.527 \text{ m/s}$$

Now $\Delta V = +33.527$ m/s and $t = 15$ sec

$$a_{avg} = \frac{+33.527 \text{ m/s}}{15 \text{ sec}} = 2.2351 \frac{\text{m/s}}{\text{sec}}$$

Since $\dfrac{\text{m/s}}{\text{sec}} = \dfrac{\text{m/s}}{\text{s}} = \dfrac{\text{m}}{\text{s}} \cdot \left(\dfrac{1}{\text{s}}\right) = \text{m/s}^2$ the answer can be written

Dividing is the same as multiplying by an inverse.

$$a_{avg} = 2.2351 \text{ m/s}^2$$

Final Answer: $a_{avg} = 5 \text{ mph/sec}$ or $2.24 \dfrac{\text{m/s}}{\text{sec}}$

or 2.24 m/s^2

Example Problem #3

A worker has to load bottles of water into a cooler in a convenience store. Suppose he has a cart containing 50 1-liter bottles of water. The mass of the cart itself is 20 kg. By giving the cart a good shove he accelerates it from rest to 2 m/s in 0.75 seconds. Calculate the force of his push, in Newtons and pounds.

Solution:

Force? That sounds like a Newton's Laws Problem.

Newton's 2nd Law is $\Sigma F = ma$, so I need m and a to find force.

Find mass from mass of cart & knowing that each 1 ℓ of water has mass 1 kg

$m = 20\ kg + 50\ (1\ kg) = 70\ kg$

Find acceleration using speed and time info:

$\Delta v = +2\ m/s$

$t = 0.75\ s$

$a = \frac{\Delta v}{t} = \frac{2\ m/s}{0.75\ s} = 2.6667\ m/s^2$

Now use $\Sigma F = ma$

$\Sigma F = (70\ kg)\ 2.6667\ m/s^2 = 186.6667\ kg\ m/s^2$ $\quad 1\ kg\ m/s^2 = 1\ N$

$= 186.6667\ N$

Convert to pounds

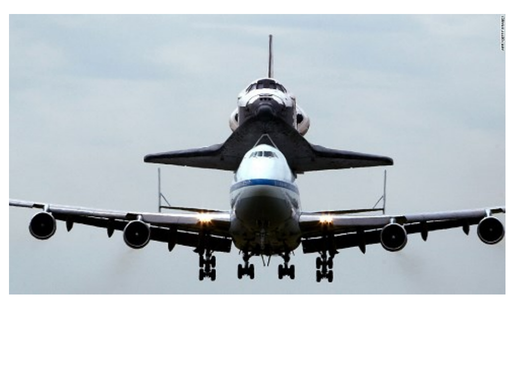
$1\ lb = 4.448\ N$

$186.6667\ N \left(\frac{1\ lb}{4.448\ N}\right) = 41.9664\ lb$

ANSWER:
187 N
42.0 lb

Example Problem #4

The 747 jet that carried the Space Shuttle weighed 710,000 lbs when fully loaded. It had four engines, each of which generated 50,000 lbs of forward force (thrust). [Data from D. Jenkins, *Space Shuttle -- The History of Developing the National Space Transportation System.*]



(a) Determine the mass of the fully loaded jet. If the jet must be moving at 150 mph to take off, determine (b) how long it takes the jet to go from being at rest on the runway to taking off when given full throttle, and (c) how far the jet moves in doing this (in both meters and feet).

First, lets get this into metric SI units.

Forward force is

$4 \times 50{,}000 \text{ lb} = 200{,}000 \text{ lb}$

$200{,}000 \text{ lb}\left(\frac{4.448 N}{1 \text{ lb}}\right) = 889{,}600 N$ Force

Weight is 710,000 lb

1 kg weighs 2.203 lb

$710{,}000 \text{ lb}\left(\frac{1 \text{ kg}}{2.2081 \text{ lb}}\right) = 322{,}287.8 \text{ kg}$ mass

So, now I have net force and mass, both in SI units.

$\Sigma F = 889{,}600 N$ $\quad m = 322{,}287.8 \text{ kg}$

Now I can find acceleration using Newton's 2nd Law

$a = \frac{\Sigma F}{m} = \frac{889{,}600 N}{322{,}287.8 \text{ kg}} = 2.7603 \frac{N}{kg}$ $\quad N = kg\,m/s^2$

$a = 2.7603 \frac{kg\,m/s^2}{kg} = 2.7603 \, m/s^2$ acceleration

OK! With acceleration and speed information I can find time.

$a = \frac{\Delta V}{t}$

$\Delta V = +150 \text{ mph}\left(\frac{1 \, m/s}{2.237 \text{ mph}}\right) = +67.0541 \, m/s$

The plane starts at rest + accelerates to 150 mph so $\Delta V = +150$ mph

$2.7603 \, m/s^2 = \frac{67.0541 \, m/s}{t}$

$t = \frac{67.0541 \, m/s}{2.7603 \, m/s^2}$

$t = 24.2923$ sec time to taking off

Now with time and average speed I can find distance (change in position.

$V_{avg} = \frac{0 + 150}{2} = 75 \text{ mph}$ or $33.5270 \, m/s$

$V_{avg} = \frac{\Delta X}{t}$

$$33.5270 \text{ m/s} = \frac{\Delta x}{27.2923 \text{ s}}$$

$$\left(33.5270 \frac{\text{m}}{\cancel{\text{s}}}\right) 27.2923 \cancel{\text{s}} = \Delta x$$

$$915.0302 \text{ m} = \Delta x$$ Distance to taking off

Convert to feet. 1 m = 3.281 ft.

$$915.0302 \text{ m} \left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = 3002.214 \text{ ft}$$

ANSWERS:
(a) mass is 323,000 kg
(b) time down runway is 24.3 sec
(c) distance down runway is 915 m or 3000 ft

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Example Problem #5

The position of an object that is propelled from rest by an engine that puts out a fixed amount of power is given by

$x = 10 + 5\ t^{3/2}$

Where x is in meters.

(a) What is the position of the object at t = 0? t = 10 s?
(b) Obtain an equation for velocity (v) and acceleration (a) as functions of time.
(c) Discuss whether the object's acceleration is constant, increasing, or decreasing. Use graphs in your discussion.

$$X = 10 + 5t^{3/2}$$

at $t=0$ $\quad X = 10 + 5(0)^{3/2} = 10\ m$ ← Answers

$t = 10s$ $\quad X = 10 + 5(10)^{3/2} = 168\ m$ ←

Use derivatives to get equations for v and a:

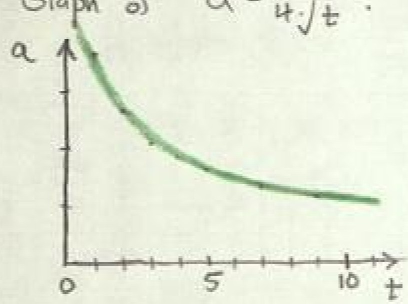
$$V = \frac{dx}{dt} = \frac{d}{dt}\left(10 + 5t^{3/2}\right) = 0 + \frac{3}{2}(5)t^{\frac{1}{2}}$$

$$\boxed{V = \frac{15}{2}t^{\frac{1}{2}}}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{15}{2}t^{\frac{1}{2}}\right) = \frac{1}{2}\frac{15}{2}t^{-\frac{1}{2}}$$

$$\boxed{a = \frac{15}{4\sqrt{t}}}$$

Graph of $a = \frac{15}{4\sqrt{t}}$.



So acceleration is getting less and less as time goes by. Acceleration is decreasing.