DAY 27

Summary of Primary Topics Covered

Heat Transfer Overview – Conduction, Convection, Radiation

There are three principle methods by which heat may be transferred from a hotter object to a colder object. These are all similar in that all three depend on

(a) the temperature difference between the two objects (the greater the temperature difference, the greater the rate of heat transfer).

(b) the surface area through which heat can flow (the greater the surface area the greater the rate of heat transfer – this is why spreading your hot noodles out on a plate causes them to cool down more rapidly).

In general, the rate of heat flow between two objects -- a hot object with temperature T_{hot} and a cold object with temperature T_{cold} -- is given by the general heat transfer equation

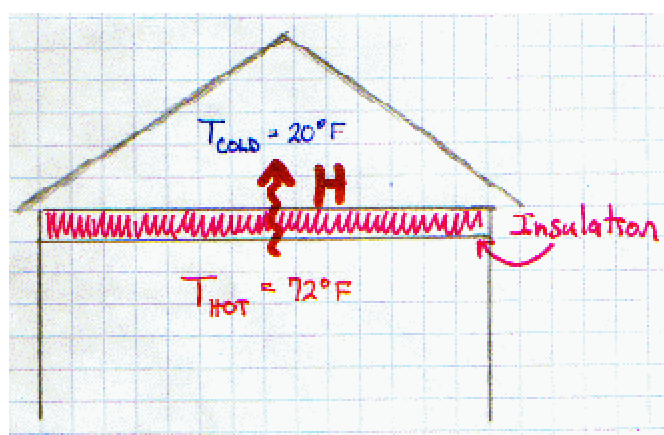
$$H = Q/t = C\ A\ (T_{hot}^{\ n} - T_{cold}^{\ n})$$

H = dQ/dt PHY 231 ONLY

A is the surface area through which heat flows. The temperatures are measured in absolute units. C is a constant and n is an integer. C & n depend on which type of heat flow is occurring.

Conduction

Conduction occurs through the direct interaction between molecules. This is what causes heat to flow from a warm room (T_{hot}) into the cold outdoors (T_{cold}) through the attic of a house.



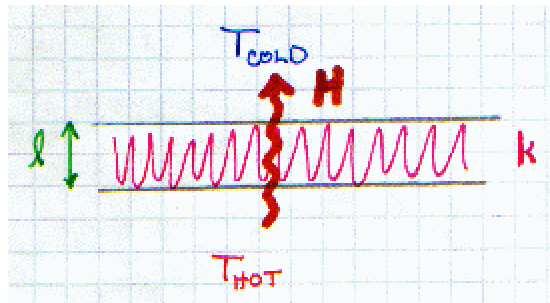
The in the general heat transfer equation

$$H = Q/t = C\ A\ (T_{hot}^{\ n} - T_{cold}^{\ n})$$

the values for the case of conduction are n = 1, and $C = k_t/L$, so the equation becomes

$$H = Q/t = k_t\ A\ (T_{hot} - T_{cold})/L$$

k_t is a constant called the *thermal conductivity*. Thermal conductivities vary from material to material and can be looked up in commonly available tables (there is one on the class web page). Materials like copper or silver have high k_t values and are called good *thermal conductors*. Materials like pine or gasses are poor conductors (also called good *thermal insulators*). L is the thickness of the insulator or conductor.



In English units k_t & L are often combined into one value known as the R-value (n = 1, C = 1/R). The conduction equation is then

$$H = Q/t = A\ (T_{hot} - T_{cold})/R$$

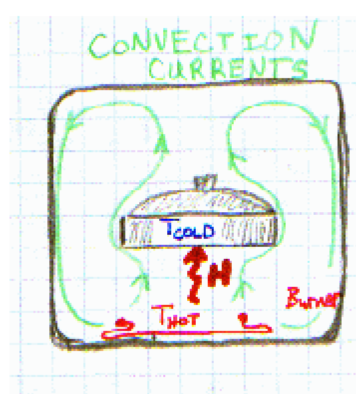
Here H is in BTU/hr, A is in ft^2, the temperatures are in °F, and R is the R-value, which has units of

$$\frac{hr\ ft^2\ {}^\circ F}{BTU}$$

How 'bout them units, huh? This is the "R-value" quoted at places like Home Depot and ACE hardware for home insulation. Next time you are there, ask someone what the units are for the R-value.

Convection

Convection occurs when the movement of a fluid moves heat. For instance, in an electric oven, a hot burner is able to heat a cold casserole dish even though the two are separated by air – which is an insulator. The burner heats the air, which expands and rises via the Ideal Gas Law and Archimedes' Principle. The rising hot air then heats the casserole. The rise and fall of air is an example of a *convection current*.



For the general heat transfer equation in the case of convection, n = 1. However, C depends on many factors, such as shape, orientation of surfaces (vertical vs. horizontal), etc. Convection is a very complex process. We can't write a simple equation for it.

There are two types of convection – *natural convection* (where the movement of fluid occurs via the Ideal Gas Law and Archimedes' Principle) and *forced convection* (where the movement of fluid is caused by a pump or blower).

Newton's Law of Cooling

If an object is placed in thermal contact with a *thermal reservoir* (something that is large enough to give up or accept a large amount of heat with no appreciable change in temperature), the temperature of that object will gradually change to match the temperature of the reservoir. For example, if a hot bolt is dropped into a 55-gallon drum of water, the temperature of the water will change little. The bolt, however, will eventually come into *thermal equilibrium* with the water where its temperature will match the water's temperature.

If the heat flow between hot bolt and cool water is mainly in the form of conduction or convection, then n = 1 in the general heat transfer equation and

$$Q/t = C\ A\ (T_{hot}^{1} - T_{cold}^{1}) = C\ A\ (T_{bolt} - T_{water})$$

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$$dQ/dt = C\ A\ (T_{hot}^{1} - T_{cold}^{1}) = C\ A\ (T_{bolt} - T_{water})$$

However, the temperature of the bolt is given by

$$Q = m\ c\ \Delta T = m\ c\ (T_{bolt} - T_{water}).$$

Thus the heat flow depends on the temperature difference and the temperature difference depends on the heat flow. We can't solve a problem like this using algebra. However, use of calculus will allow us to solve it. What we find out is that --

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$$d(m\ c\ (T_{bolt} - T_{water}))/dt = C\ A\ (T_{bolt} - T_{water})$$

where T_{bolt} is a variable and T_{water} is constant.

$$d(m\ c\ (T_{bolt} - T_{water})) = C\ A\ (T_{bolt} - T_{water})\ dt$$
$$m\ c\ d(T_{bolt} - T_{water}) = C\ A\ (T_{bolt} - T_{water})\ dt$$
$$m\ c\ d(\Delta T) = C\ A\ (\Delta T)\ dt$$
$$-\ d(\Delta T)/\ \Delta T = \{(C\ A)/m\ c\}\ dt$$

(The negative is there because we know ΔT decreases.)

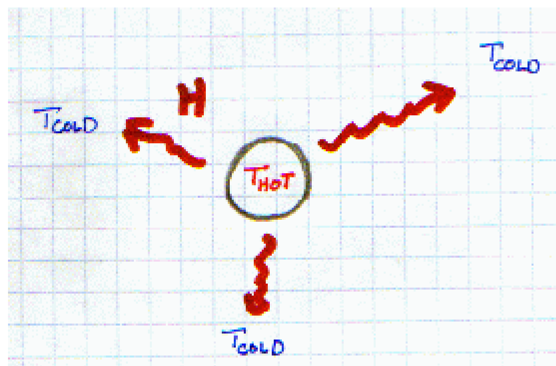
If we integrate this (see example #3 below) we get the solution that --

-- if an object is placed in a fixed-temperature environment that object's temperature will exponentially approach the temperature of the environment:

$$\Delta T = \Delta T_0 e^{-k_{cool}t}$$

ΔT is the temperature difference between the object (the bolt) and its environment (the water) at time t, and ΔT_0 is the initial temperature difference between the two. k_{cool} is a constant that depends on the heat transfer and the mass and specific heat of the object. This is *Newton's Law of Cooling*. A larger k_{cool} value indicates more rapid cooling -- a smaller k_{cool} value indicates less rapid cooling. This *exponential behavior* is a very important concept in science.

Radiation



Radiation is the movement of heat via electromagnetic waves (light, infrared, etc.). Radiation is the only way that heat can move through a vacuum, and is the means by which the Sun warms the Earth.

For the general heat transfer equation in the case of radiation, n = 4, and C = e σ:

$$H = Q/t = e\, \sigma\, A\, (T_{hot}^4 - T_{cold}^4)$$

Here σ is Stefan's constant (σ = 5.6703 x 10^{-8} W/m^2K^4). e is the emissivity of the object – a measure of how well it radiates and absorbs energy.

e = 0	Perfect reflector
e < 0.5	Light-colored objects
e > 0.5	Dark-colored objects
e = 1	Perfect absorber (perfect "blackbody")

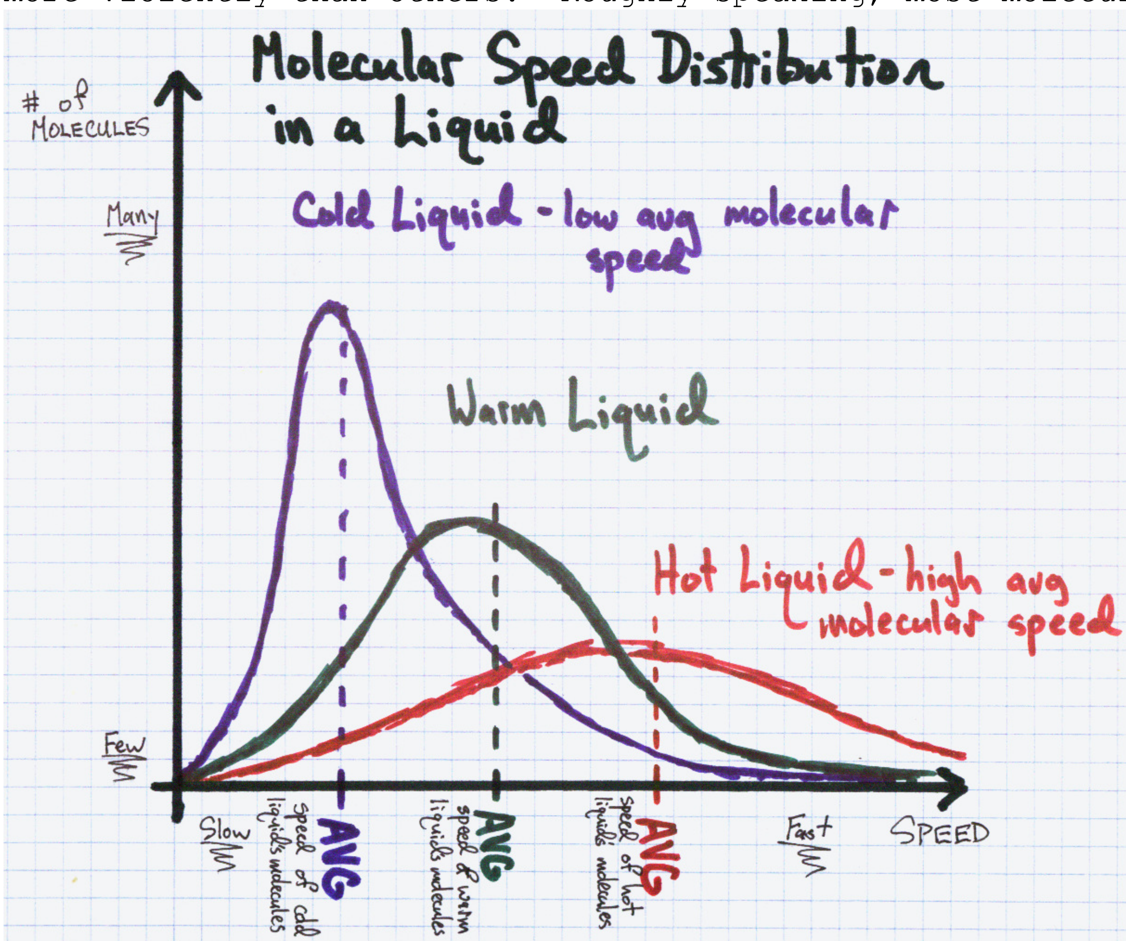
Black objects tend to absorb and radiate heat very efficiently, as anyone who has ever owned a black car with a black interior, or who has ever walked across a black parking lot on a hot day, can attest to.

If radiation is an important heat transfer mechanism then Newton's Law of Cooling will not hold because $n \neq 1$. The exponential nature of the law requires that the heat flow depend on the temperature difference and the temperature difference depend on the heat flow. Differences in temperature to the 4^{th} power do not produce exponential functions.

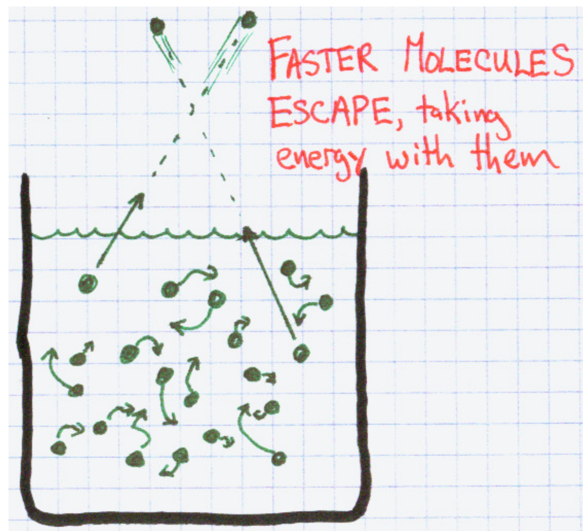
Evaporative Cooling

Evaporative cooling is not a case of true heat transfer. Unlike the case of conduction/convection/radiation, something that cools by evaporation is actually losing mass. Evaporative cooling occurs because while it is true that in a hot material molecules move more violently than in a cooler material, that statement is only true in the average.

In fact, molecules in any material in random directions, at random speeds. Some move faster and some move slower than the average for the whole. Even in a solid some molecules vibrate more violently than others. Roughly speaking, most molecules in



the material move near the average speed, with relatively few molecules moving at exceptionally high or low speeds. The profile of molecular speeds is known as the *molecular speed distribution*. Note that even in a hot material there will be some slow-moving molecules, and even in a cold material there will be some fast-moving molecules.



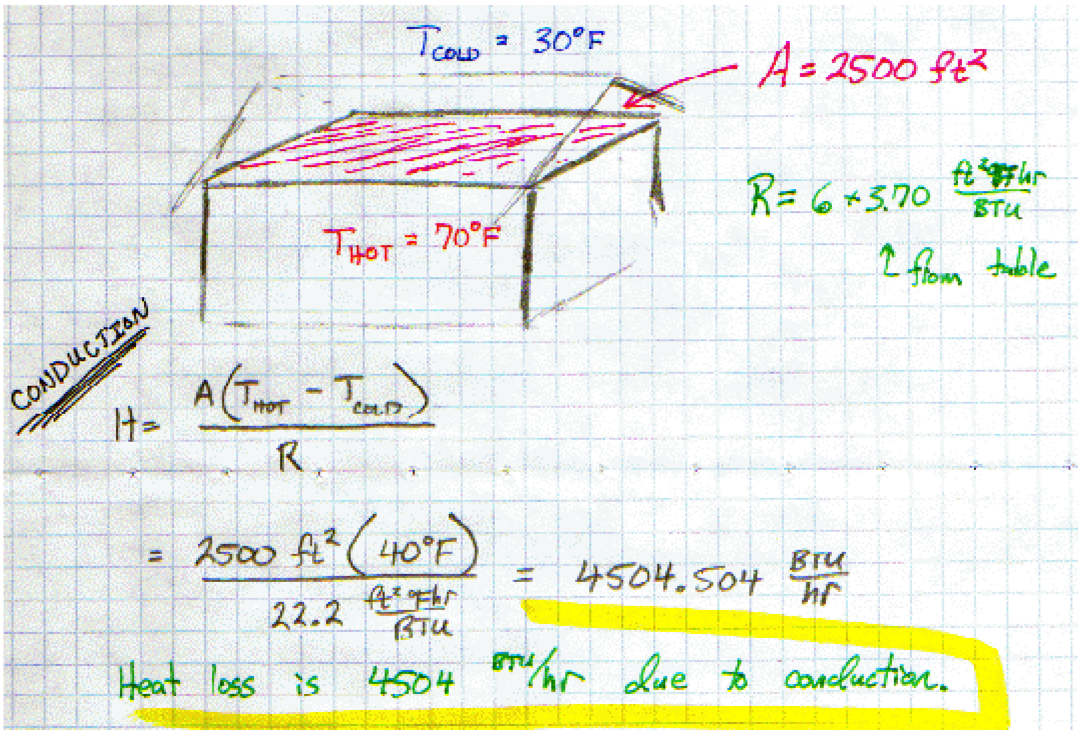
In a liquid, the faster-moving molecules are more likely to break free of the rest of the molecules and fly free -- going into a gaseous state (evaporation). When the faster molecules leave, the slower ones are left behind and the average speed of molecules for the liquid is lowered. Since temperature depends on molecular speed, a lower average speed means a colder liquid.

The rate of evaporation depends on things like temperature difference and surface area, but it also depends on other factors. For instance, our bodies cool themselves via evaporative cooling by sweating. Evaporation of water depends on the humidity level in the air. If we are doing strenuous activity on a day where the temperature is 90°F, we will sweat. If the air outside our bodies is 90°F and dry, evaporation occurs rapidly, our bodies cool themselves efficiently by sweating, the sweat goes away (evaporates), and we feel comfortable and dry. If the air outside our bodies is 90°F and very humid, evaporation does not occur rapidly, our bodies do not cool themselves efficiently by sweating, the sweat runs off our brows and noses and down our backs (because it does not evaporate quickly), and we feel hot and sweaty.

If evaporation is an important cooling mechanism Newton's Law of Cooling also will not hold. But if evaporation and radiation are not major cooling mechanisms, Newton's Law of Cooling works very well.

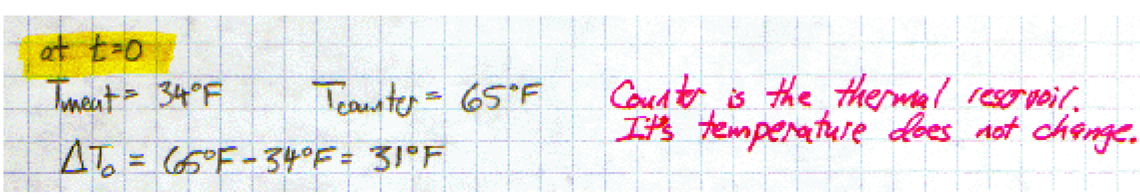
Example Problem #1:

The roof of a 2500 ft^2 1-story home is insulated with 6 inches of cellulose fiber. The average temperature inside the home in the winter is 70°F. The average temperature outside the home in the winter is 30°F. What is the average rate of heat loss through the attic due to conduction? Give answers in BTU/hr.



Example Problem #2:

Thawed meat comes out of the refrigerator (temperature 34°F) and is placed on the counter (temperature 65°F). Bacteria begin to grow on meat when the temperature of the meat rises above 40°F (see http://www.ces.ncsu.edu/depts/fcs/food/pubs/fcsw502.pdf). If a thin steak has a temperature of 35°F after 2 minutes of sitting on the counter, how much longer can the meat remain on the counter and still be safe?



at $t = 2$ min

$T_{meat} = 35°F$ $\Delta T = 30°F$

$$\Delta T = \Delta T_0 e^{-k_{cool} t}$$

$$30°F = 31°F\, e^{-k_{cool}(2\ min)}$$

Solve for k_{cool}

$$\frac{30°F}{31°F} = e^{-k_{cool}(2\ min)}$$

$$\ln\left(\frac{30}{31}\right) = \ln\left(e^{-k_{cool}(2\ min)}\right)$$

$$\ln\left(\frac{30}{31}\right) = -k_{cool}(2\ min)$$

$$k_{cool} = \frac{\ln(30/31)}{-2\ min} = 0.016395\ \frac{1}{min}$$

Now use k_{cool} to find time until $T_{meat} = 40°F$

When $T_{meat} = 40°F$ $\Delta T = 65 - 40 = 25°F$

$\Delta T_0 = 31°F$

$k_{cool} = 0.016395\ \frac{1}{min}$

$$\Delta T = \Delta T_0 e^{-k_{cool} t}$$

$$25°F = 31°F\, e^{-0.016395 \frac{1}{min}(t)}$$

$$\ln\left(\frac{25}{31}\right) = -0.016395 \frac{1}{min}(t)$$

$$t = \frac{\ln(25/31)}{-0.016395 \frac{1}{min}} = 13.1206$$

THE MEAT WILL BE SAFE FOR ABOUT 13 minutes.

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Example Problem #3:

Do the integral and derive the equation for Newton's Law of Cooling. Show that

$k_{cool} = CA/mc$

Solution:

Start with the last line in the PHY 231 section above:

$$-\frac{d\Delta T}{\Delta T} = \frac{CA}{mc}\,dt$$

$$-\int \frac{d\Delta T}{\Delta T} = \int \frac{CA}{mc}\,dt$$

A is area

m is mass

c is specific heat

C is the general heat transfer equation constant.

Now for limits of integration:

I know that at t = 0 the temperature difference between the bolt and the water is ΔT_0. And at time t the temperature difference will be ΔT.

So upper limits are ΔT on left side integral and t on the right side integral. Lower limits are ΔT_0 on left side integral and 0 on the right side integral.

$$-\int_{\Delta T_0}^{\Delta T} \frac{d\Delta T}{\Delta T} = \frac{CA}{mc}\int_0^t dt$$

$$-\ln \Delta T \Big|_{\Delta T_0}^{\Delta T} = \frac{CA}{mc}\, t \Big|_0^t$$

$$\ln \Delta T - \ln \Delta T_0 = -\frac{CA}{mc}(t - 0)$$

$$\ln \frac{\Delta T}{\Delta T_0} = -\frac{CA}{mc}\, t$$

$$\frac{\Delta T}{\Delta T_0} = e^{-\frac{CA}{mc} t}$$

$$\Delta T = \Delta T_0\, e^{-k_{cool} t}$$

$$k_{cool} = \frac{CA}{mc}$$

Example Problem #4:

The roof of a 2500 ft^2 1-story home is insulated with 6 inches of cellulose fiber. The average temperature inside the home in the winter is 70°F. The average temperature outside the home in the winter is 30°F. What is the average rate of heat loss through the attic due to radiation, if the emissivity of the ceiling is 0.05? Give answers in BTU/hr.

RADIATION

$$H = e\sigma A\left(T_{HOT}^4 - T_{COLD}^4\right)$$

Powers of 4 and metric units mean no shortcuts. I have to get everything into SI units.

$T_{HOT} = 70°F = 530\,R = 294.4\,K$

$T_{COLD} = 30°F = 490\,R = 272.2\,K$

$$A = 2500\,ft^2\left(\frac{1\,m}{3.281\,ft}\right)^2 = 232.2\,m^2$$

$e = .05$

$$H = (.05)\left(5.6703\times10^{-8}\,\frac{W}{m^2K^4}\right)\left(232.2\,m^2\right)\left((294.4\,K)^4 - (272.2\,K)^4\right)$$

$$H = 6.5832183\times10^{-7}\,\frac{W}{K^4}\left(2.02218\times10^{9}\,K^4\right)$$

$H = 1331.242411$ Watts

$= 1331.242411$ J/s

$$1331.242411\,\frac{J}{s}\left(\frac{1\,BTU}{1054\,J}\right)\left(\frac{3600\,s}{1\,hr}\right) = 4547\,BTU/hr$$

Due to Radiation

Heat loss is 45469 BTU/hr due to radiation.