DAY 25

Summary of Primary Topics Covered

Thermal Expansion in Gasses

While the expansion of solids and liquids depends on the type of solid or liquid material involved (aluminum vs. steel, gasoline vs. water), expansion in a gas does not depend on the type of gas. Air, methane, helium all expand identically – there is no "expansion coefficient" that is different for different gasses.

Because gasses are compressible, the volume of a gas depends on its pressure (the greater the pressure the smaller the volume of the gas) as well as on the amount of gas present, and on its temperature. The volume of a gas is given by

$$V = \frac{NkT}{P}$$

Where V is the volume in m^3, N is the number of gas molecules, T is the absolute temperature (K or R), and P is the pressure in Pa. k is a constant known as Boltzman's constant. $k = 1.381 \times 10^{-23}$ J/K. This equation is known as the "Ideal Gas Law" and can be used to determine the volume expansion of any gas with change in temperature.

The number of gas molecules is sometimes stated in moles – a term from chemistry. A "mole" is just a number: a "dozen" is twelve; a "gross" is 144; a "mole" is 6.022×10^{23} (known as Avogadro's number – N_A). So $N = n\,N_A$.

The gas law equation can be re-written as

$$\frac{VP}{NT} = k$$

Since VT/NP is a constant, that means that whatever happens to a gas, that total quantity must be the same. Hence, if we measure V, T, N, & P at time 1, and then again at time 2, we will find that

$$\frac{V_1 P_1}{N_1 T_1} = \frac{V_2 P_2}{N_2 T_2}$$

This is another form of the Ideal Gas Law. Again, recall that T must be in absolute units (R or K).

Heat Energy

Throughout the semester we've learned that heat is a form of energy. There are two ways to put heat into an object. These are

A) Do work on it involving friction. We've been talking about this all semester. "When friction is present, where does the energy go, class? **Heat.**"
B) Expose it to an object that has a higher temperature.

So if you want to warm your cold hands you can either (A) rub them together (work via friction) or (B) hold them over a warm heater (expose them to a hot object). Energy conservation applies to heat just as it applies to any other form of energy. When applied to thermodynamics the Law of Conservation of Energy is often referred to as the *1st Law of Thermodynamics*.

There are also thermodynamics energy units. The *calorie* is the amount of energy required to raise the temperature of 1 gram of liquid water by 1 ^{o}C. The kcal or the Calorie (note the capital "C") is 1000 calories.

1 kcal = 1 Cal = 1000 cal

1 cal = 4.186 J

The British Thermal Unit or BTU is the amount of energy required to raise the temperature of 1 lb (at Earth's surface) of water by 1 ^{o}F.

1 BTU = 252 cal

The reason there are different energy units for heat and a different name for the Law of Conservation of Energy in heat is because at one time heat was not understood to be energy, but rather was thought to be a type of fluid called "Caloric". It was only after some concepts in thermodynamics had been developed that it was realized that heat was energy.

Latent Heat

You would think that when you add heat to a substance it would get warmer. Often it does. However, if the substance is going through a phase change, adding heat does not cause the object to get warmer – rather it simply causes it to change phase. For example, adding heat to boiling water doesn't make the water hotter – it makes the water boil faster. This is because temperature is a measure of the motion of molecules. When a substance changes phase, the heat energy flowing in does not act to make its molecules move faster – it acts to change their structure.

The heat per unit mass required to change the phase of a substance is call the Latent Heat (L). Latent heats can be found in commonly available tables. The heat required to change the phase of a mass m of material is given by

$$\mathbf{Q = m\ L}$$

This equation assumes that the material is already at the phase change temperature. Latent heats involved in melting/freezing are called *latent heats of fusion*. Latent heats *involved in boiling/condensing are called latent* heats of vaporization.

Specific Heat

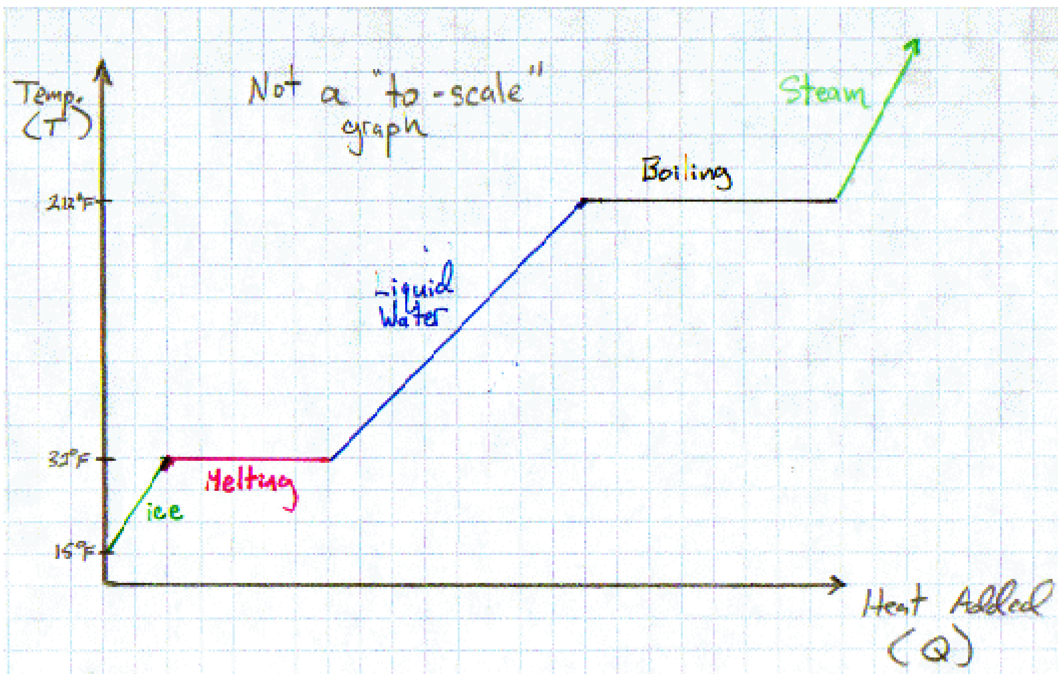
Imagine that you put a block of ice straight from the deep-freeze (15°F) into a really low-power microwave oven. The oven would let you slowly add heat to the ice, and the heat would be delivered throughout the ice, not just to the surface.

As the oven ran, here is what would happen to the ice:

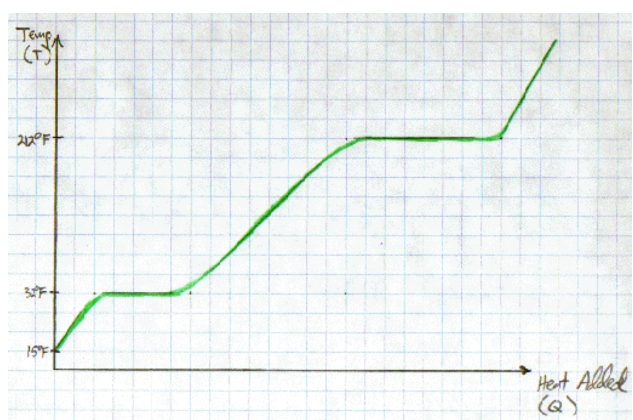
- First the ice would increase in temperature, until it reached 32°F and started to melt.
- Once the ice reached 32°F melting would commence and the temperature of the ice would remain constant throughout the phase change from solid to liquid.

- Once the ice was fully melted, the temperature of the liquid water would begin to rise, until it reached 212°F and started to boil.
- Once the ice reached 212°F boiling would commence and the temperature of the water would remain constant throughout the phase change from liquid to gas.
- If the steam produced by boiling the water was sealed within the oven, then once the water was fully boiled into steam, the temperature of the steam would begin to rise again.

A graph of temperature vs. heat added for this system would look like this:



Usually heat is not added to a material in such a controlled fashion, and the plot of temperature vs. heat added is a little less crisp looking – more like the graph at right.



When the material is not undergoing a phase change, its temperature goes up when heat is added. The amount of temperature change depends on the amount of heat added, the mass of material being heated up (more mass means less temperature change per calorie of heat), and the material itself.

This last factor is described by “specific heat”. This is the amount of heat energy required to raise the temperature of a certain amount of substance by a certain amount. For example, the specific heat of water is 1 cal/g°C – meaning it takes one calorie of heat to raise the temperature of 1 gram of water by 1°C.

By contrast, the specific heat of aluminum is .215 cal/g°C. In other words, it takes roughly 5 times as much heat to raise the temperature of a gram of water by 1°C as it takes to raise the temperature of an equal mass of aluminum by 1°C.

Specific heats for various materials can be found in commonly available tables.

The equation

$$\mathbf{Q = m\ C\ \Delta T}$$

describes this. Here Q is the heat added to the substance in question, m is the mass of the substance being heated, c is the specific heat, and ΔT is the temperature change of the substance.

Example Problem #1:

A sealed balloon has volume of 2 ft^3 at 50°F. What will its volume be at 100°F? Assume the balloon is subject to atmospheric pressure at all times.

Solution:

$V_1 = 2\ ft^3$
$T_1 = 50^\circ F = 510\ R$

$T_2 = 100^\circ F = 560\ R$

The balloon is sealed. That means no molecules enter or escape. So $N_1 = N_2$.
The balloon's pressure is $P_1 = P_2 = 1$ atm.

$$\frac{V_1 P_1}{N_1 T_1} = \frac{V_2 P_2}{N_2 T_2}$$

Using the Ideal Gas Law.

$$\frac{(2\,ft^3)(1atm)}{N_1(510R)} = \frac{V_2(1atm)}{N_2(560R)}$$

$$\frac{(2\,ft^3)}{(510R)} = \frac{V_2}{(560R)}$$

The 1 atm and the N values are the same on both sides so they cancel out.

$$\frac{(2\,ft^3)}{(510R)}(560R) = V_2$$

$V_2 = 2.2\ ft^3$

Example Problem #2:

How much heat is required to heat up a 1.5 liter kettle of water from faucet temperature (15°C) to boiling? Give your answer in calories and Calories, and compare to the calorie content of a bottle of soft drink.

Water has a density of 1000 g/l so m = 1500 g of water
c = 1 cal/g°C
ΔT = 100°C - 15°C = 85°C

$Q = m\,c\,\Delta T$

Q = 1500 g (1 cal/g°C) 85°C = 127,500 cal = 127.5 kcal = 127.5 Cal

By comparison, by 20 oz bottle of Pepsi contains 2.5 servings at 100 Cal per serving, or 250 Cal. So I guess if you could use the energy in the Pepsi it would more than boil itself!

Example Problem #3:

A historical re-enactor (a gunsmith) is making lead shot for his musket. He drops 4 lbs of lead into a 500 W electric crucible. The lead was at room temperature (68°F) when it was dropped into the crucible. The gunsmith needs the lead to be fully melted. How long will he have to wait?

First I'll find how much mass I have since mass factors into much of this heat stuff.

$W = 4\text{ lb}$

$W = mg \qquad g = 2.203\ \frac{\text{lb}}{\text{kg}}$

$4\text{ lb} = m\left(2.203\ \frac{\text{lb}}{\text{kg}}\right)$

$m = 1.8157\text{ kg}$

Now get other values into metric units.

$68°F = 528R = 293K = 20°C$

Find data for lead. - Values from tables.

$c = .0305\ \frac{\text{cal}}{g\,°C}$ Spec. Heat

$L_f = 5.9\ \frac{\text{cal}}{g}$ Latent heat - melting

$T_{melt} = 600K$ Melting Temp.

So the crock has to first raise the temp of the lead to the melting point, and then melt it.

I - Raise temp to melting point:

$\Delta T = 600K - 293K = 307\ K = 307\ °C$

Change in Kelvin is same as Change in Celsius.

$m = 1815.7\ g$

$Q = mc\Delta T$

$= 1815.7g\left(.0305\ \frac{\text{cal}}{g\,°C}\right)(307°C)$

$= 17001.30695\text{ cal}$

II - Melt lead.

$Q = mL_f$

$= (1815.7g)\left(5.9\ \frac{\text{cal}}{g}\right)$

$= 10712.63\text{ cal}$

Total Heat Needed to melt lead:

$$Q_{TOT} = 17001.30695 \text{ cal} + 10712.63 \text{ cal}$$
$$= 27713.93695 \text{ cal}$$

Using 1 cal = 4.186 J that's

$$Q_{TOT} = 116010.5401 \text{ J}$$

The crucible has $P = 500 \text{ W} = 500 \text{ J/s}$

$$P = \frac{\text{Energy}}{t} = \frac{Q}{t}$$

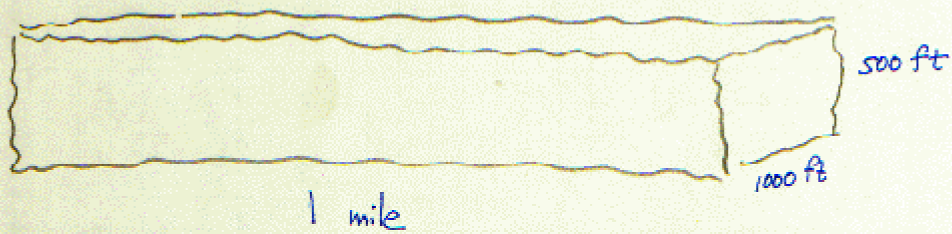
The energy here is heat.

$$t = \frac{Q}{P} = \frac{116010.5401 \text{ J}}{500 \text{ J/s}} = 232.0210802 \text{ sec}$$

It will take 3.9 minutes to melt the lead.

Example Problem #4:

A small iceberg measures 1 mile long by 1000 feet wide by 500 feet deep. The berg is in water at 32oF and has begun to melt. How much heat will be required to melt the berg? LG&E + Kentucky Utilities set a new record for power output a few summers ago – 6513 MW (http://www.lgeenergy.com/itn/releases/080702_peaks.html). If LG&E/KU could sustain that output nonstop and all that energy went into melting the berg, how long would it take to melt it? Give answer in days.



1000 ft = 304.8 m

500 ft = 152.4 m

1 mile = 1609 m

$Vol = 7.47 \times 10^7 \, m^3$

$m = \rho V \qquad \rho = .92 \, g/cm^3 = 920 \, kg/m^3$

$m = 6.875 \times 10^{10} \, kg$

From table on web.

It's melting so use Heat of Fusion.

$L_v = 79.7 \frac{cal}{g} \left(\frac{4186 \, J/kg}{1 \, cal/g} \right)$

$L_v = 333624.2 \, J/kg$

$Q = mL$

$= (6.875 \times 10^{10} \, kg)(333624.6 \, \frac{J}{kg})$

$= 2.294 \times 10^{16} \, J$

$P_{LG\&E} = 6513 \, MW = 6513 \times 10^6 \, W = 6513 \times 10^6 \, J/s$

$P = \frac{E}{t} = \frac{Q}{t} \qquad t = \frac{Q}{P} = \frac{2.294 \times 10^{16} \, J}{6513 \times 10^6 \, J/s} = 3.522 \times 10^6 \, sec$

$3.522 \times 10^6 \, sec \left(\frac{1 \, hr}{3600 \, sec} \right) \left(\frac{1 \, DAY}{24 \, hr} \right)$

$\hookrightarrow \boxed{41 \text{ Days}}$ to melt away this ice berg.