DAY 24

Summary of Topics Covered in Today's Lecture

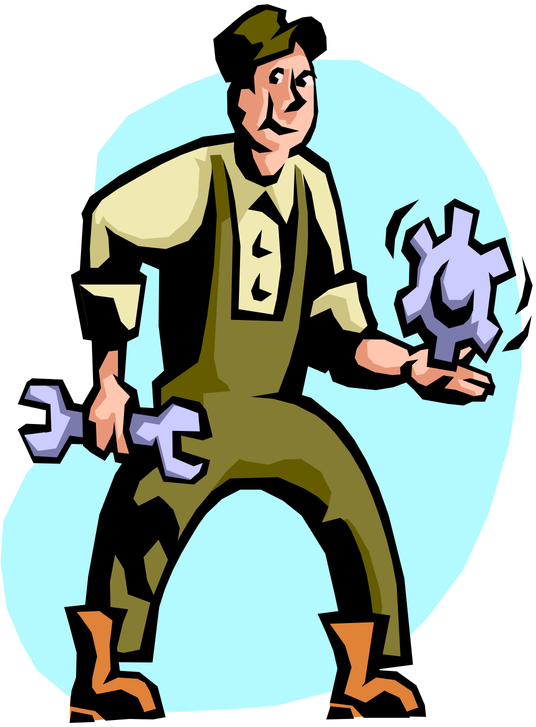
How Not To Measure Something

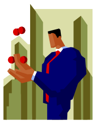
Imagine the following method for measuring length:

The bar at right is defined as having a length of zero. Why? Because it is convenient!

The bar above is defined as having length of 100 (we'll call the units of measure *froogles*). Again, it is defined this way because it is convenient to do so.

Using this system of measure, the man at right is perhaps 90 froogles tall, while the gentleman below is over 100 froogles tall.





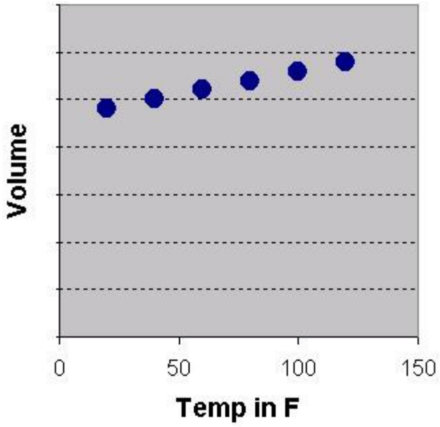
On the other hand, this little guy actually has a height of less than zero (because he is shorter than zero froogles).

There are real problems with this system of measurement. For instance, suppose a ball drops the length of the longer bar in 2 seconds. Is its average speed then 100 froogles/2 sec = 50 froogles/sec? What if it drops half the length of the red bar in 2 seconds? Does that mean it has a negative speed? What if it drops exactly the length of the red bar in 2 seconds? Since the red bar has length zero froogles is the ball’s speed zero froogles per second? *A froogle is really not a regular unit of measure because zero froogles is not zero length.*

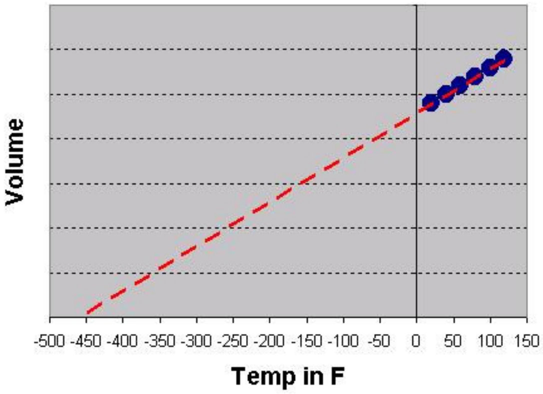
This is a goofy way of making measurements. Nonetheless that is how the two most common systems of temperature measurement were created.

Temperature (which is an entirely new quantity for us, whose units cannot be derived from kg, m, and s) is commonly measured using either Fahrenheit or Celsius degrees. Celsius has its zero point set at the freezing point of water and its 100 point set at the boiling point of water. Fahrenheit has its zero point set at something like the temperature of a 50/50 ice-salt mixture (at one time that was the lowest temperature that could be achieved in a lab) and its 100 point set near human body temperature. Neither zero °F nor zero °C represents true zero temperature. To be fair, at the time these systems were created no one knew what zero temperature was.

One way to figure out what zero temperature truly is is to look at a gas such as air. Air expands when its temperature is raised. Air contracts when its temperature is lowered.



A “Ziplock” bag with air in it will be bigger when warmer and smaller when cooler. If you measure the bag’s volume and its temperature and plot volume vs. temperature you get a plot that looks like the plot at right. This plot is linear. If you project backwards to see at what temperature the volume of the gas in the bag would be zero you get an answer of *460 degrees below zero*. This works no matter what gas is in the bag -- air, helium, natural gas, CO_2, you name it. If you do the same experiment in Celsius temperature your answer comes out to 273 degrees below zero Celcius. This special point is true zero or *absolute zero*.



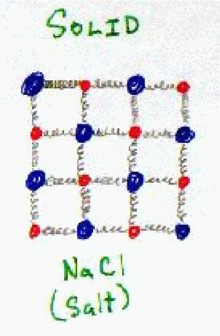
Real temperature units are ones that have their zero point at absolute zero. There are two of these “absolute” temperature scales -- one that has units based on the Celsius system (the Kelvin scale), and one that has units based on the Fahrenheit system (the Rankine scale).

	Fahrenheit	Celsius	Rankine	Kelvin
Water boils	212°F	100°C	672 R	373 K
Water freezes	32°F	0°C	492 R	273 K
Absolute zero	-460°F	-273°C	0 R	0 K

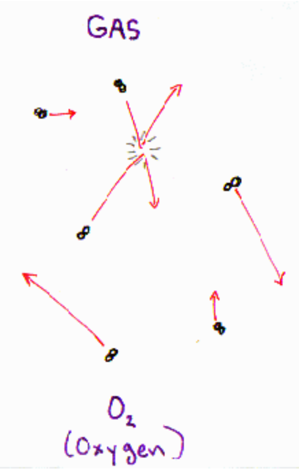
Note that between freezing and boiling of water there are 100 K and 180 R. Therefore **100 K = 180 R.** To go from Celsius to Kelvin add 273. To go from Fahrenheit to Rankine add 460.

What is Temperature?

To understand what temperature is we need to look at matter on a microscopic level. The three common phases or states of matter are solids, liquids, and gasses. Each of these has certain microscopic characteristics which result in macroscopic properties which we can see easily:



PHASE	Example	Macroscopic Properties	Microscopic characteristics
SOLID	**salt, rock, ice**	Rigid (holds it own shape), incompressible	Atoms/molecules are held in fixed positions by elastic bonds. They can vibrate around those fixed positions -- much as if they were connected together by springs -- but cannot move around among each other.
LIQUID	**water, mercury**	Takes shape of container but has a defined top surface; flows; incompressible	Atoms/molecules are bound together but can move freely around one another. They are like magnetic marbles.
GAS	**oxygen, methane, helium**	Takes shape of container completely -- has no defined surface; flows; compressible	Atoms/molecules are not bound together. They fly freely at high speed and only interact when they collide. Their collisions are "elastic", meaning no Kinetic Energy is lost in the collision. Atoms/molecules also collide elastically with the walls of the gas's container.

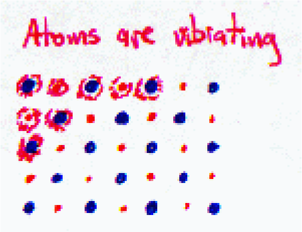


In each state, greater temperature corresponds to greater motion of atoms/molecules and more energy per molecule. In ice at -40°F the molecules vibrate around their fixed positions more violently than in ice at 25°F. When liquid water is at 180°F the molecules move around one another more rapidly than when the liquid water is at 45°F. And the molecules in steam fly around faster and collide more violently in steam whose temperature is 800°F than in steam whose temperature is 300°F. Temperature is a measure of the motion energy of molecules in any substance.

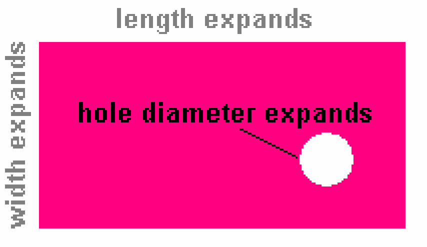
Thermal Expansion of Solids & Liquids

The fact that greater temperature means greater violence of movement in atoms/molecules explains why most materials expand when heated. In the case of solids, the greater violence of vibration means that the atoms/molecules that make up the solid must "spread out" -- 100 people standing and shuffling their feet can be packed pretty close together. If those same 100 people start dancing jigs they will have to spread out. The result is that solids expand when heated. The expansion is in all directions.

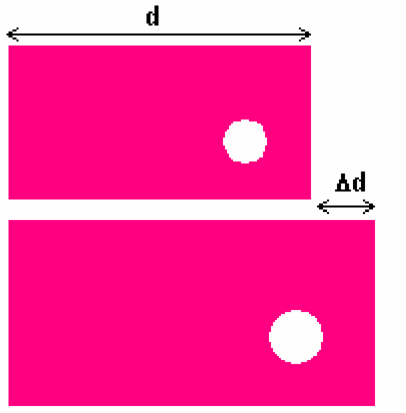
Object at lower temperature.



Same object at higher temperature.



The amount of expansion depends on how much the object's temperature increases, on the original size of the object, and on the material the object is made of. The amount of expansion is given approximately by the equation



$$\Delta d = \alpha\ d\ \Delta T$$

Here Δd is the amount of expansion, ΔT is the amount of temperature increase, d is the original size of the object, and α is the *thermal expansion coefficient* which depends on the material the object is made of. The equation is approximate because expansion is only linear over a limited range in temperatures -- and obviously at some point the object will melt!

Liquids also expand like this. They expand in volume according the equation

$$\Delta V = \beta\ V\ \Delta T$$

where β is called the *volume expansion coefficient*. Expansion coefficients for different materials can be looked up in commonly available tables. A table of expansion coefficients has been added to the class web page. Volume expansion for a liquid depends only on the initial volume of the liquid, the temperature change, and the material (i.e. water, gasoline, etc.).

Example Problem #1:

Human body temperature is nominally 98.6 °F. Calculate this in R, K, and °F.

Solution:

First, get into true temperature units of R.

F = R – 460
98.6 = R – 460
558.6 = R

Now, convert to K.

558.6 R (100 K/180 R) = 310.33 K
K = 310.3

Now, get into °C.

C = K – 273
C = 310.33 – 273
C = 37.3

Example Problem #2:

A steel plate has a precision ½ inch hole drilled into it when the plate is at 95°F. How large will the hole be at 32°F?

Solution:

Expansion coefficient for steel is 12×10^{-6} 1/K according to the table of expansion coefficients that is on the class web page, so I'll have to convert temperatures to Kelvin.

$\Delta T = 95^{\circ}F - 32^{\circ}F = 63^{\circ}F = 63\ R$ (because degree sizes are the same for F and R).
$\Delta T = 63\ R\ (100\ K/180\ R) = 35\ K$

$\Delta d = \alpha\ d\ \Delta T$
$= 12\times10^{-6}\ 1/K\ (.5\ \text{inch})(35\ K)$
$= 420\times10^{-6}\ (.5\ \text{inch})$ Kelvins cancel out.
$= 0.00021$ inch

The hole shrinks by that much, so the hole will be
0.5 in - 0.00021 inch = 0.49979 inch in diameter at 32°F -- not a huge change.

Example Problem #3:

Dentists today use resin and ceramic fillings instead of the silver amalgam fillings that had been in use for over a century. These fillings, along with looking better than silver fillings, have a coefficient of thermal expansion that matches that of natural teeth much more closely than silver fillings. Why would this be important?

http://www.yourdentistrytoday.com/resin-bonded-fillings.html

Solution:

If the expansion coefficient of the filling is larger than that of the tooth, then when the tooth is warmed (such as when you drink a hot drink), the filling will expand more than the cavity in the tooth which it fills. This will put the tooth under stress. Furthermore, if the tooth is cooled (such as when you eat ice cream), the filling will contract more than the cavity and might fall out.

Conversely, if the expansion coefficient of the filling is smaller than that of the tooth, then when the tooth is cooled, the filling will contract less than the cavity in the tooth which it fills. This will put the tooth under stress. If the tooth is warmed, the filling will expand less than the cavity and might fall out.

The bottom line is that the more the filling's expansion coefficient differs from that of the natural tooth, the more likely it is that the tooth will become weakened by stress or that the filling will fall out.