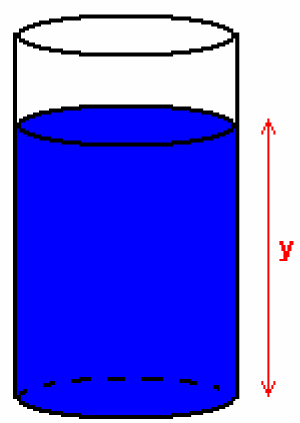
DAY 22

Summary of Primary Topics Covered

Fluid Pressure

Let's apply Bernoulli's equation to a fluid that is not moving, like in the tank of water shown here. The depth of the tank is y.

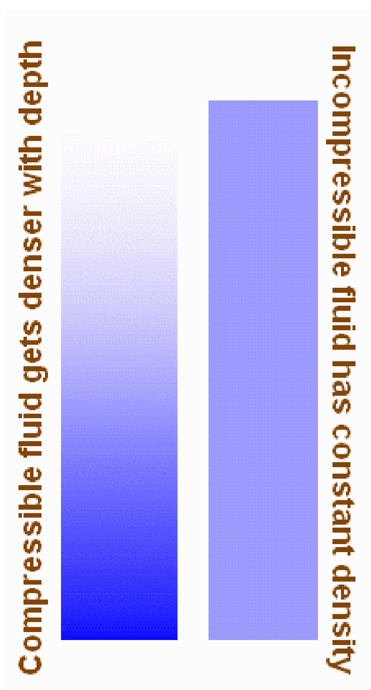


	Top of Tank		Bottom of Tank
	$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2$	=	$P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$
	$P_{TOP} + \rho g y + 0$	=	$P_{BOTTOM} + 0 + 0$
	$P_{TOP} + \rho g y$	=	P_{BOTTOM}

So the pressure at a depth of y is increased by $\rho g y$ over the original pressure in the fluid. This is due to the weight of the fluid and is known as *fluid pressure*. Again, ρ is the density of the *fluid*.

$$P_{fluid} = \rho g y$$

This equation only applies to incompressible fluids such as liquids. If a fluid is incompressible its density is nearly constant. If the fluid is compressible (like a gas) the density is not constant at all -- the fluid above compresses the fluid below. The fluid gets more dense the deeper you go into the fluid. You have to use calculus to solve compressible fluid problems.



PHY 231 Only

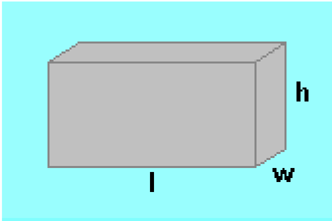
If a fluid is compressible you have to deal with small levels of the fluid over which the density can be considered constant. This infinitesimal distance can be written as dy, and the fluid equation can be written in terms of differentials

$$dP_{fluid} = \rho g \, dy$$

The atmosphere of Earth is this kind of problem -- the weight of the upper atmosphere compresses the lower atmosphere. Therefore the atmosphere is most dense near the surface of the Earth.

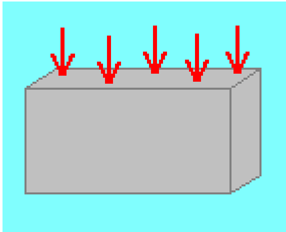
Buoyancy: Fluid Pressure + Pascal's Principle = Archimedes' Principle

Suppose a block of material is immersed in a fluid of density ρ. The top of the block sits at a depth y in the fluid.

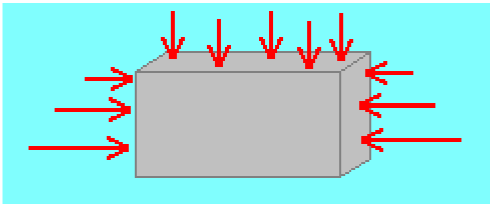


There is fluid pressure on the top of the block, and that pressure exerts a force on top of the block:

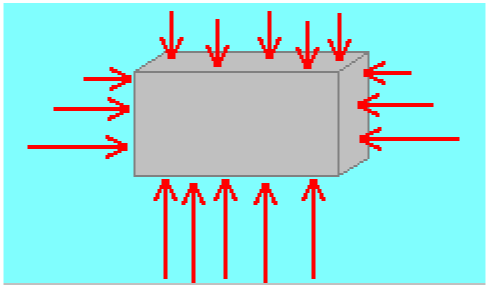
$$F_{TOP} = P\ A$$
$$= \rho g y (lw)$$



However, pressure in a fluid does not just act in one direction. *Pascal's Principle* says that pressure in a fluid is transmitted in <u>all</u> directions within a fluid, to <u>all</u> surfaces in contact with the fluid. The fluid applies pressure to the sides of the block as well as the top, but these forces cancel each other out.



Finally, because of Pascal's principle there is fluid pressure on the bottom of the block, and that pressure exerts a force on the bottom of the block.



$$F_{BOTTOM} = P\ A$$
$$= \rho g (y+h)(lw)$$

The y+h term comes about because the bottom of the block is at a depth of y+h.

The net force on the block is then

The net force on the block is upward and is known as the Buoyant Force.

$$F_{NET} = F_{BOTTOM} - F_{TOP}$$
$$= \rho g(y+h)(lw) - \rho gy(lw)$$
$$= \cancel{\rho gylw} + \rho ghlw - \cancel{\rho gylw}$$

$$F_{BUoY} = \rho ghlw$$

$$F_{BUOT} = \rho gV$$

$V = hlw$
is the volume of the block -- and of the fluid displaced by the block

$$F_{BUOT} = m_{fluid}g$$

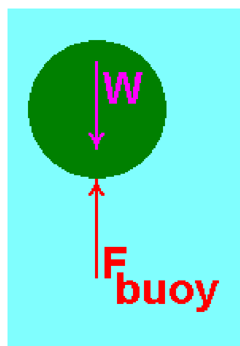
$m_{fluid} = \rho V$
is the mass of the fluid displaced by the block

$$F_{BUOT} = W_{fluid}$$

$W_{fluid} = m_{fluid}g$
is the weight of the fluid displaced by the block.

This is *Archimedes' Principle* -- that the buoyant force on an object immersed in a fluid (any fluid) is equal to the weight of fluid displaced by the object.

Whether an immersed object sinks in the fluid, rises in the fluid, or hovers in a fluid depends on the buoyant force and on the weight of the object.



If the buoyant force is greater than the weight, the object will rise in the fluid. This occurs when the density of the object is less than the density of the fluid:

$$F_{BUOY} > W$$
$$W_{fluid} > W$$
$$m_{fluid}\cancel{g} > m_{object}\cancel{g}$$
$$\rho_{fluid} V > \rho_{object} V$$
$$\rho_{fluid} > \rho_{object}$$

Likewise, if

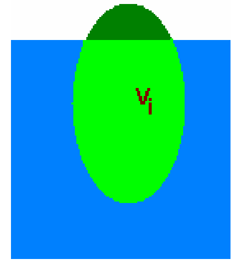
$$\rho_{fluid} < \rho_{object}$$

the object will sink in the fluid, and if

$$\rho_{fluid} = \rho_{object}$$

the object will hover in the fluid, neither rising nor sinking.

For an object floating on the surface of a fluid, the fraction of the object's total volume that is immersed is determined by the ratio of the densities of the object and the fluid.



$V_i/V_{tot} = \rho_{object}/\rho_{fluid}$

Deriving this equation is a homework problem.

Example Problem #1:

In cartoons people are always getting carried away by a bunch of Helium balloons, Determine the lift available from a Helium balloon that measures 0.5 m^3 in volume. The balloon itself has mass 2 g. How many such balloons would be required to lift a 75 lb kid?

The forces on the balloon are the weight of the balloon, the weight of the Helium in the balloon, and the buoyant force of air.

$V = 0.5\ m^3$
$\rho_{air} = 1.29\ kg/m^3$
$\rho_{He} = 0.179\ kg/m^3$

$W_{balloon} = mg = (.002\ kg)(9.8\ N/kg) = 0.0196\ N$

$W_{He} = m_{He}g = \rho_{He}Vg = (0.179\ kg/m^3)(\ 0.5\ m^3)(9.8\ N/kg) = .8771\ N$

$F_{BUOY} = W_{air} = m_{air}g = \rho_{air}Vg = (1.29\ kg/m^3)(\ 0.5\ m^3)(9.8\ N/kg) = 6.321\ N$

The lifting force is

$F_{lift} = F_{BUOY} - W_{balloon} - W_{He} = 6.321\ N - 0.0196\ N - .8771\ N = 5.4243\ N$

75 lb is 333.6 N, so it would take 333.6 N/5.4243 N = 62 balloons to lift the kid.

Example Problem #2:

Hollow glass globes were used in the past by fishermen as floats to support their nets. Glass has a density of about 2500 kg/m^3. What percentage of a globe will float above the surface of the water if the globe is 30 cm in diameter and 1 cm thick?

The globe is not solid glass, so I can't just use the density of glass as the density of the object. Besides, solid glass is more dense than water, and would not float.

I need to determine mass of glass in the globe. First I'll get the volume of the globe:

$V = 4/3\ \pi r^3$ $\quad$ $r = 15\ cm = .15\ m$

$= (4/3)\pi(.15\ m)^3$

$= 0.01414\ m^3$

Now I need to determine the volume of the hollow area within the globe. Since the glass is 1 cm thick, the diameter of the hollow region must be 14 cm.

$V = 4/3\ \pi r^3$ $\quad$ $r = 14\ cm = .14\ m$

$= (4/3)\pi(.04\ m)^3$

$= 0.01149\ m^3$

The glass volume is then

$V_{glass} = .01414\ m^3 - .01149\ m^3 = .00265\ m^3$

The mass of the glass is

$m_{glass} = .00265\ m^3 (2500\ kg/m^3) = 6.625\ kg$

The float will displace 6.625 kg of water, too. How much water is that?

$\rho = m/V$

$1000\ kg/m3 = 6.625\ kg/V$

$V = .006625\ m^3$

This is the volume of the globe that will be under the water. That is 0.006625/0.01414 = 46.9% of the volume of the globe, so the amount that will be above the water is 53.1%.