DAY 21

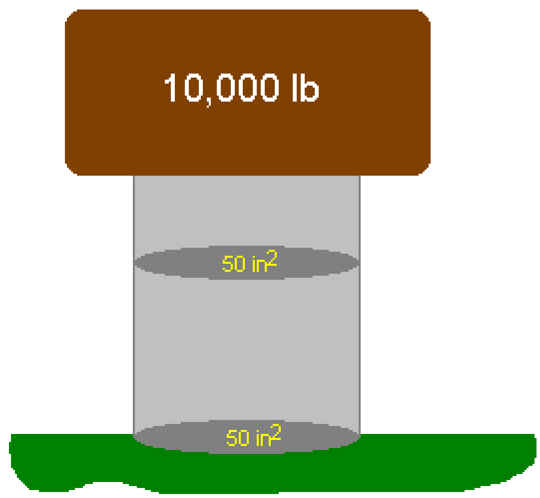
Summary of Primary Topics Covered

Force per Unit Area – Pressure and Stress

When a force is applied to an object that force acts over a certain area. The quantity of force per unit area is known as pressure or stress. "Pressure" is the term usually used when the force/area is applied to the surface of a solid object, or if the force/area is within a fluid. "Stress" is usually used when the force/area is within a solid object. Units of pressure are usually lb/in^2 (or psi) in the English system, and N/m^2 (or Pascals – Pa) in the SI metric system.

For example, in the figure at right, a 10,000 lb weight is applied to a column with cross-sectional area of 50 in^2. The <u>pressure</u> *on the ground* is

$$\begin{aligned} P &= F/A \\ &= 10000\ lb/50\ in^2 \\ &= 200\ lb/in^2 \\ &= 200\ psi \end{aligned}$$



Likewise the <u>stress</u> *in the column* is also 200 psi.

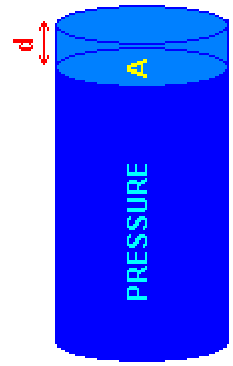
The pressure atmosphere exerts at sea level is 14.7 psi. This is known as one <u>atmosphere</u> (1 atm) of pressure.

Pressure as Energy Density

Pressure, especially in a fluid (anything that can flow – including liquids, gasses, and granulated solids like salt, grain, and gravel), can be visualized as a form of potential energy. After all, if something is under pressure we know it has the ability to do work (think of a can of Coke that has been given a really good shake). However, pressure is not energy – it is energy per unit volume, as illustrated by the following rough calculations.

$$P = \frac{F}{A}$$

$$P = \frac{F\,(d)}{A\,(d)} = \frac{\text{Work}}{\text{Volume}} = \frac{\text{Energy}}{\text{Volume}} \quad \frac{[\text{Joules}]}{[m^3]}$$

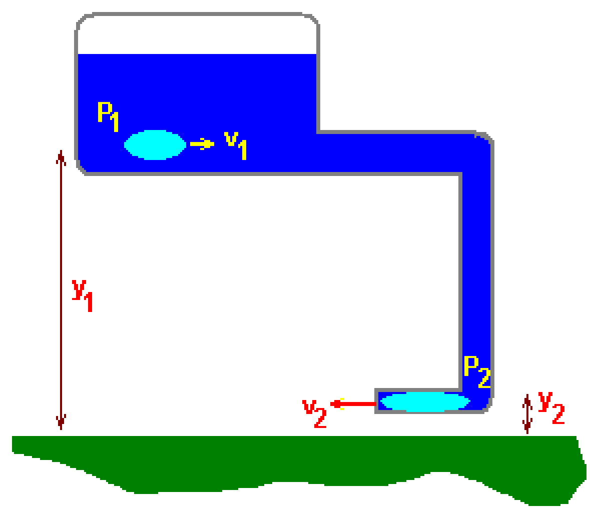


The units work out, too:

$$P = \frac{[J]}{[m^3]} = \frac{[Nm]}{[m^3]} = \frac{[N]}{[m^2]} = \text{Pascals} = \text{Pa}$$

One shaken can of Coke is under pressure – and can do work if opened (the work being to geyser coke all over the place). Two such cans can do more work, but not because they are under greater pressure. Each can is under the same pressure, but more work can be done because there is more volume of Coke.

Pressure in Fluids – Energy Density and Bernoulli's Principle



A fluid flowing through a system can have energy due to pressure in the fluid, but it can also have energy due to gravitational potential and kinetic energies as well. Energy must be conserved as always.

$$E_{before} = E_{after} = \text{Constant}$$

If friction is not significant in the fluid, then

Energy from pressure + gravitational potential + kinetic = Constant

Put in terms of energy density:

$$P + PE_{grav}/\text{Volume} + KE/\text{Volume} = \text{Constant}$$

This means that the sum of these quantities is the same before and after any event. This is Bernoulli's equation:

Before **After**

$$P + PE_{grav}/\text{Volume} + KE/\text{Volume} = P + PE_{grav}/\text{Volume} + KE/\text{Volume}$$

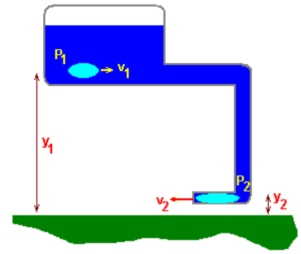
$$P + mgy/Volume + \tfrac{1}{2}mv^2/Volume \quad = \quad P + mgy/Volume + \tfrac{1}{2}mv^2/Volume$$

$$P + \rho gy + \tfrac{1}{2}\rho v^2 \quad = \quad P + \rho gy + \tfrac{1}{2}\rho v^2$$

ρ = m/Volume.

Here ρ is density of fluid.

In the figure at right and on the previous page, a blob of water is shown before and after descending through a pipe. At the top it is under pressure P_1, it is moving at speed v_1, and it is at height y_1. After flowing down the pipe, the pressure is P_2, the speed is v_2, and the height is y_2. In these terms, Bernoulli's equation is



$$\mathbf{P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 \quad = \quad P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2}$$

If the height of a moving fluid does not change ($y_1 = y_2$), then Bernoulli's equation is just:

$$\mathbf{P_1 + \tfrac{1}{2}\rho v_1^2 \quad = \quad P_2 + \tfrac{1}{2}\rho v_2^2}$$

This says that the faster a fluid moves, the lower the pressure in the fluid. This Bernoulli effect is behind all kinds of phenomena, from why cigarette smoke flows out a "cracked open" car window to why the spin of a ball affects its flight to why airplanes stay aloft.

If significant friction is present in the flow of the fluid then you have to add a term for frictional heat generated (Q) per unit volume:

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 \quad = \quad P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2 + Q/Vol$$

Note that the <u>shape</u> of the fluid (i.e. whether it is in a round container, a twisted container, etc.) plays no role in Bernoulli's equation. Only heights and speeds matter.

Example Problem #1:

A kid sticks a plunger that measures 5 inches in diameter to a smooth tile floor. If the kid expelled all the air from under the plunger, and if no air leaks back in, what force will be required to pull the plunger off the floor?

The force required to remove the plunger will be the same as the force that air pressure exerts on the plunger. Air pressure is 14.7 psi.

$F = PA$

$A = \pi r^2 = 3.14159\ (2.5\ \text{in})^2 = 19.635\ \text{in}^2$

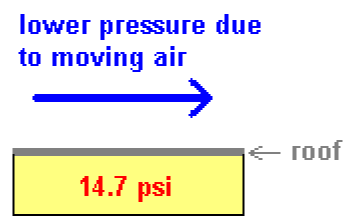
$F = P A = 14.7\ \text{lb/in}^2\ (19.635\ \text{in}^2) = 289\ \text{lbs}$

In all likelihood, however, not all the air will be expelled, and some may leak back under the plunger, so the force will be less than this. Also, if the lip of the plunger is pried off the floor the plunger will be much easier to remove from the floor.

Example Problem #2:

A tornado blows across the roof of a 10,000 ft^2 single-story business with a flat roof. The wind in the tornado is moving at 110 mph. Calculate the force on the roof due to the Bernoulli effect (in pounds).

Presumably the air inside the building is at rest, more or less, and the air pressure is 14.7 psi.



$V_1 = 0$
$P_1 = 14.7\ \text{lb/in}^2$

$V_2 = 110\ \text{mph}$

First convert to SI metric units:

$V_1 = 0$

$$P_1 = 14.7\ \frac{\text{lb}}{\text{in}^2} \times \frac{4.448\ \text{N}}{1\ \text{lb}} \times \frac{39.37\ \text{in}}{1\ \text{m}} \times \frac{39.37\ \text{in}}{1\ \text{m}} = 101347.5\ \text{N/m}^2$$

$$V_2 = 110\ \text{mph}\ \frac{1\ \text{m/s}}{2.237\ \text{mph}} = 49.173\ \text{m/s}$$

$$A = 10{,}000\ \text{ft}^2 \times \frac{1\ \text{m}}{3.281\ \text{ft}} \times \frac{1\ \text{m}}{3.281\ \text{ft}} = 928.94\ \text{m}^2$$

Now use Bernoulli's principle to find the pressure on the roof (the density of air is $\rho = 1.29\ \text{kg/m}^3$):

$$P_1 + \cancel{\rho g y_1} + \tfrac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g y_2} + \tfrac{1}{2}\rho v_2^2$$

I cancel the ρgy terms since the top and bottom of the roof should be just a few inches different in height.

$$101347.5\ N/m^2 + 0.5\,(1.29 kg/m^3)(0^2) = P_2 + 0.5(1.29\ kg/m^3)(49.173\ m/s)^2$$

$$101347.5\ N/m^2 = P_2 + 1559.600\ kgm^2/(m^3s^2)$$

Now I'm going to work out those units.

$$101347.5\ N/m^2 = P_2 + 1559.600\ kgm/(m^2s^2)$$

$$101347.5\ N/m^2 = P_2 + 1559.600\ kgm/s^2/m^2$$

$$101347.5\ N/m^2 = P_2 + 1559.600\ N/m^2$$

$$99787.9\ N/m^2 = P_2$$

That's the pressure on the top of the roof.

The downward force on the top of the roof caused by the pressure is

$F_{down} = PA = 99787.9\ N/m^2\ (928.94\ m^2) = 92{,}696{,}945\ N$

The upward force on the bottom the roof caused by the pressure under the roof is

$F_{up} = PA = 101347.5\ N/m^2\ (167.209\ m^2) = 94{,}145{,}720\ N$

Therefore the net force on the roof is upward (a lift force). The lift on the roof is

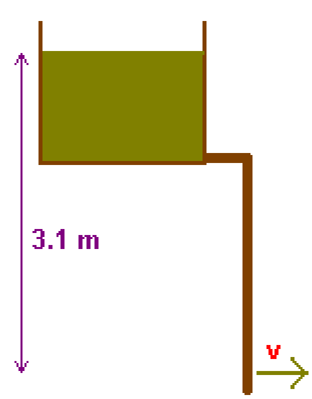
$F_{lift} = 94{,}145{,}720\ N - 92{,}696{,}945\ N = 1{,}448{,}775\ N$

Or 326,000 lbs.

That just might lift off a roof if the roof isn't anchored down securely.

Example Problem #3:

Someone opened a small valve on a pipe leading from a large vat of distiller's beer at one of our Kentucky distilleries. The density of distiller's beer is about 900 g/l. The distiller's beer is spurting out the bottom of the tank, a distance of 3.1 meters below the level of the fluid. How fast is the beer squirting out? If the hole is half a meter off the ground, where will the beer hit the ground? What will happen as the beer leaks out?



Since the tank is large and the valve is small and no other information is given, I'm presuming that the rate at which beer is leaking out is not sufficient to cause the level in the tank to drop much in a short time. Based on that, I have the following information to use:

$P_1 = P_2 = 1$ atm (since both the top of the vat and the valve are exposed to air)

$v_1 = 0$ (since the level in the tank is not dropping significantly)

$y_1 = 3.1$ m
$y_2 = 0$ m
$\rho = 900$ g/l $= 900$ kg/m^3

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

$$\cancel{1\text{ atm}} + 900\text{ kg/m}^3(9.8\text{ m/s}^2)(3.1\text{ m}) + 0 = \cancel{1\text{ atm}} + 0 + 0.5(900\text{ kg/m}^3)(v_2)^2$$

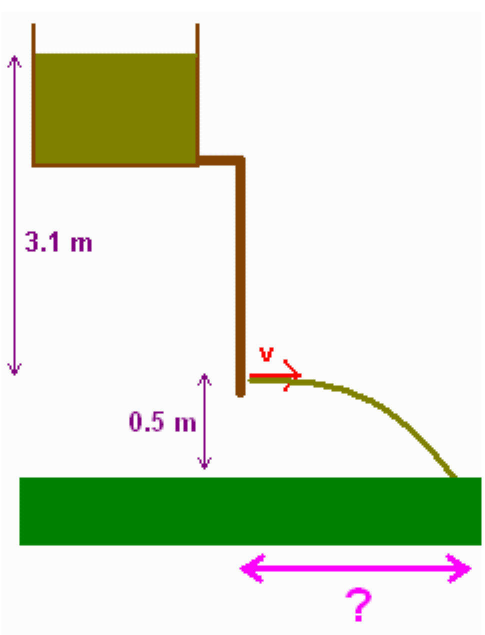
$$\cancel{900\text{ kg/m}^3}(9.8\text{ m/s}^2)(3.1\text{ m}) = 0.5(\cancel{900\text{ kg/m}^3})(v_2)^2$$

$$(9.8\text{ m/s}^2)(3.1\text{ m}) = .5(v_2)^2$$

$$60.76\text{ m}^2/\text{s}^2 = v_2^2$$

$$7.8\text{ m/s} = v_2$$

Note that neither the beer density nor the atmospheric pressure have an impact on the final answer.



Now for the question of where the beer hits the ground.

The beer becomes a projectile. It emerges from the hole at 7.8 m/s moving horizontally. I'll use projectile motion -- break this down into vertical and horizontal motions:

Vertical

$y_0 = 0.5$ m
$v_0 = 0$
$a = -9.8$ m/s^2
$y = 0$

$y = y_0 + v_0 t + \tfrac{1}{2} a t^2$
$0 = 0.5\text{m} + (0)t + \tfrac{1}{2}(-9.8\text{ m/s}^2)\, t^2$
$(4.9\text{m/s}^2)\, t^2 = 0.5\text{m}$
$t = 0.3194$ sec

Horizontal

$x_0 = 0$
$v_0 = 7.8$ m/s (actually 7.7949)
$a = 0$
$t = 0.3194$ sec

$x = x_0 + v_0t + ½ a t^2$
$x = 0 + (7.7949 \text{ m/s})(0.3194 \text{ sec}) + ½ (0) (0.3194 \text{ sec})^2$
$x = (7.7949 \text{ m/s})(0.3194 \text{ sec})$
$x = 2.5$ m

The beer hits the ground 2.5 m away from the leak. As the beer leaks out, the level of beer in the tank will drop, the speed at which the beer emerges from the leak will drop, and the distance that the beer travels will decrease.