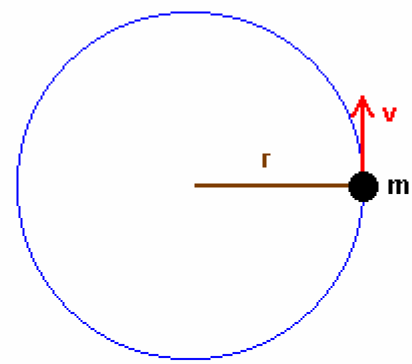
DAY 19

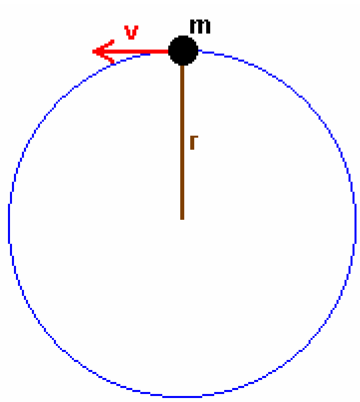
Summary of Primary Topics Covered

Still More Vector Examples

Today we wrap up the discussion of Newton's Laws, Conservation Laws, and Kinematics that we have been working on since the beginning of the semester. These vector problems involve all sorts of concepts, from center of mass to friction to RPM's. Here, at this point in the semester, the problems we are encountering are pretty big, bad and ugly!

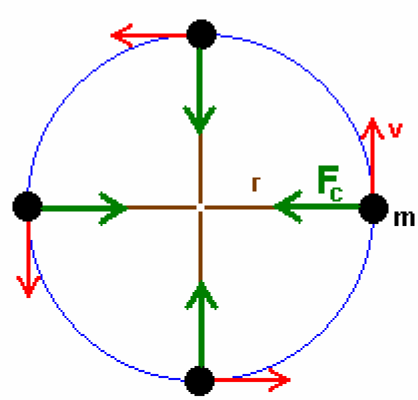
Centripetal force arises in circular motion. If an object of mass m is moving at speed v in a circle of radius r, the object's velocity vector is changing direction even if the object's speed is constant. For example, in the figure shown at right, if the mass is moving at a steady 10 m/s in a circle, the speed (10 m/s – a scalar quantity) is constant. However, the velocity vector is ***v*** *= 10 m/s upward* in the first figure, and ***v*** *= 10 m/s left* in the figure below.





From a vector perspective, the velocity is changing. Therefore the mass is accelerating because **a** = Δ**v**/t. The velocity vector is always being re-directed toward the center of the circle, so the acceleration is centripetal ("center-seeking"). The centripetal acceleration is given by

$$a_c = \frac{v^2}{r}$$



The centripetal acceleration implies the existence of a centripetal force – a force that always points toward the center of the circle. **F = ma** so

$$F_c = m\frac{v^2}{r}$$

A thorough derivation of these equations and proof that the force is directed to the center of the circle requires using calculus. However, you can get a feel for this by trying a metal nut to a string and whirling it in a circle. Remember, strings can only exert pulling forces, right? You can't push an object with a string. So, the string must be pulling on the nut – pulling inward, toward the center of the circle.

Example Problem #1:

What is the acceleration of a person in a carnival ride who is moving at 5 m/s in a circle of radius 15 m? Give your answer in "g's"?

Well, this is pretty simple.

$v = 5$ m/s
$r = 15$ m

$a_C = v^2/r = (5\text{ m/s})^2/15\text{ m} = 25/15\text{ m/s}^2 = 1.67\text{ m/s}^2$

A "g" is the acceleration due to gravity: 9.8 m/s^2

So $1.67\text{ m/s}^2\ [1\text{ g}/9.8\text{ m/s}^2] = 0.17$ g

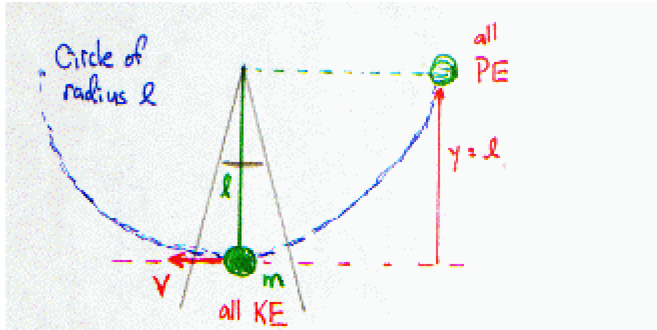
The acceleration of the rider is 0.17 g.

Example Problem #2:

Show that the chains on any swing set must be able to safely support three times the weight of the heaviest person who will ever use the set.

You are responsible for installing and maintaining safe playground equipment in the city parks. You have a very limited budget. What strength chains should you install on the city's playground swings?

The higher someone swings the faster he moves. The faster he moves the greater the centripetal force involved. I know that the highest anyone could ever swing on a swing is to get the chains horizontal. I'll base my solution on that.



$E_{TOP} = E_{BOTTOM}$ Energy Conservation

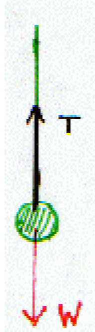
$PE = KE$

$\cancel{m}gy = \frac{1}{2}\cancel{m}v^2 \qquad y = \ell$

$2g\ell = v^2$

$v = \sqrt{2g\ell}$

This gives me speed at the bottom of the swing. At that point the forces acting on the swinger are the Tension in the chains and the swinger's weight. Because the swinger is moving in a circle, the total force on the swinger must be equal to his mass times his centripetal acceleration.



$\Sigma F = ma$ but a is centripetal $a = v^2/r$

$T - W = m\left(\frac{v^2}{r}\right)$

$T - mg = m\left(\frac{v^2}{\ell}\right)$

$T = m\left(\frac{2g\ell}{\ell}\right) + mg$

$T = 3mg = 3W$

So the Tension in the chains is three times the swinger's weight.

For the second part – I've seen some pretty hefty kids swing at the park. Adults swing, too. I'll design my swing to support a 300 lb person. The chains must then support 3x300 lbs = 900 lbs. However, a swing has two chains, so each chain needs to support 450 lbs. So I will find a chain that is rated for a working load of 450 lbs, and maybe a little more if it doesn't drive up the cost of chain too much.

Example Problem #3:

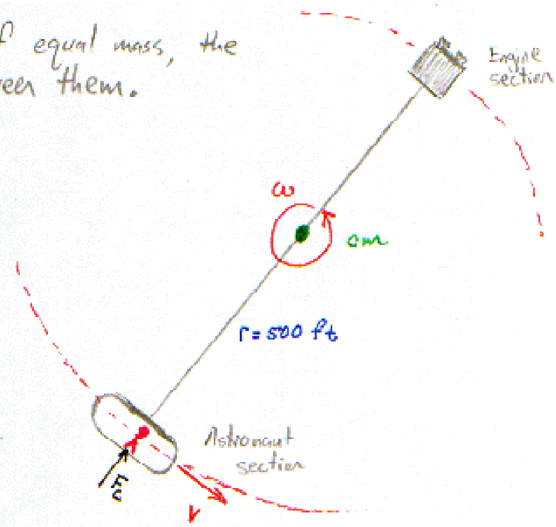
If people are ever to explore other planets (like Mars), they will have to travel on fairly lengthy journeys to get there. The human body is not built to work in a zero-g environment (like on the Space Shuttle where everyone just floats around). Extended time in zero-g tends to cause muscle damage, bone damage, loss of red blood cells, degraded senses of taste and smell, loss of physical fitness, and really bad farting (gas doesn't "rise up" the digestive system to the mouth so a lot more comes out the other end).

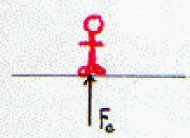
One way to solve this problem would be to have a space ship that separates into two equal-mass sections. One section would hold engines and other mechanicals. The other would hold the astronauts' quarters. The two would be tethered together with a cable and set rotating about their common center of mass with a small burst from a rocket motor. The rotation would produce the illusion of a "centrifugal force" that felt like gravity.

If the tether is 1000 ft long, calculate the rotational speed (in RPM) needed if an astronaut was to experience her usual weight on Earth.

Since the two sections are of equal mass, the C.M. point lies halfway between them.

Therefore $r = 500$ ft.





The astronaut would feel a contact force on her feet just like we feel a contact force on our feet. For it to feel like normal weight

$$F_c = W$$

$$m\frac{v^2}{r} = mg$$

$$\frac{v^2}{r} = g \qquad \text{(the centripetal acceleration would have to be one "g")}$$

$$v = \sqrt{rg} \qquad r = 500\,ft = 152.395\,m \qquad g = 9.8\,m/s^2$$

$$v = 38.645\ m/s$$

To find RPM's: $v = r\omega$

$$\omega = \frac{v}{r} = \frac{38.645\ m/s}{152.395\ m} = .254\ rad/sec$$

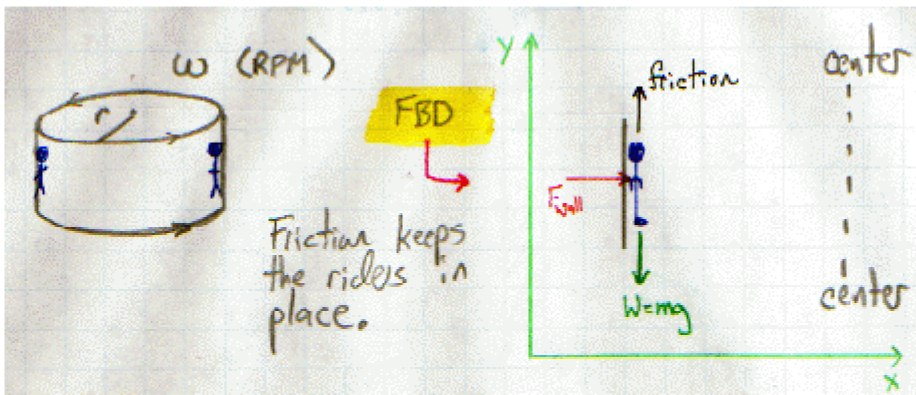
$$.254\ \frac{1}{s}\left(\frac{1\ rev}{2\pi}\right)\left(\frac{60\ s}{1\ min}\right) = 2.42\ RPM$$

Example Problem #4:

In an amusement park ride, people get into a circular room and stand against the wall. The room is then set spinning. At some point, the bottom drops out of the room and the occupants are left hanging on the wall, held in place by friction and centripetal force.

For a ride with diameter 40 ft, how fast will the ride have to spin (in RPM) before the floor can be dropped if the walls are covered with "sure-grip" carpet and the coefficient of friction between the riders and the wall is 0.95.

RPM's imply angular velocity (ω). The only forces acting on a rider are the contact force between the wall and the rider, friction, and gravity.



Now I'll break into x and y:

x-direction

$\Sigma F = ma$

$F_{wall} = m a_c$ — person is accelerating due to circular motion

$F_{wall} = m \frac{v^2}{r}$ — $v = r\omega$

$= m \frac{r^2\omega^2}{r}$

$F_{wall} = m r \omega^2$

y-direction

friction = W — no motion in y direction

$f = mg$

Friction

$f = \mu F_N$

The wall force is the normal force

$mg = \mu (F_{wall})$

$\cancel{m}g = \mu(\cancel{m} r \omega^2)$

$g = \mu r \omega^2$

$\omega = \sqrt{\frac{g}{\mu r}}$

$g = 9.8\ m/s^2$

$\mu = .95$

$r = 20\ ft = 6.096\ m$

$\omega = \sqrt{\frac{9.8\ m/s^2}{.95(6.096\ m)}} = 1.301\ \frac{1}{s}$

$\boxed{\omega = 12.4\ RPM}$