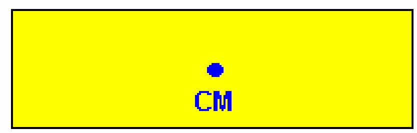
DAY 18

Summary of Primary Topics Covered

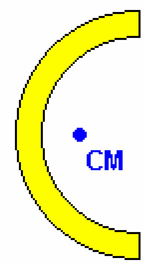
Center of Mass and More Vector Examples

Today we applied our knowledge of vectors to different kinds of problems.

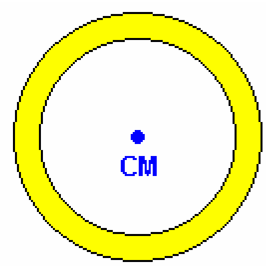
Working these problems is a matter of taking concepts that we already know and then thinking about them in terms of vectors (which typically means breaking a problem up into perpendicular components in order to solve it). Our basics -- Newton's Laws, the Kinematic Equations, the Conservation Laws -- are still there.



One useful new concept that is often involved in vector problems is the concept of *center of mass*. The center of mass of an object is the point at which the object's weight can be considered to act in problems that involve weight. It can be considered the location of the object's mass in problems that involve mass. Practically speaking, the center of mass is the balance point of an object. For a freely moving object (a freefalling object or a projectile or an object floating in space), the center of mass is the point about which the object will naturally rotate. For symmetrical objects the center of mass is located on the axis of symmetry of the object.

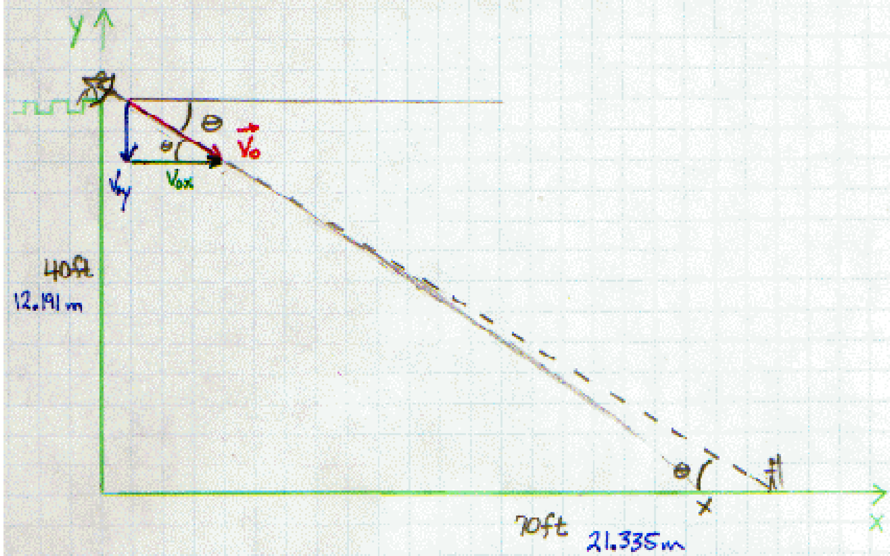


Center of mass is not a fundamental concept like Newton's Laws or Energy Conservation, but it does help in solving all kinds of problems.

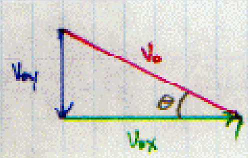


Example Problem #1:

An archer atop a castle wall 40 ft high shoots an arrow directly at a target that is on the ground 70 ft from the base of the wall. The arrow leaves the bow at a speed of 130 m/s. Where does the arrow land? The arrow is very aerodynamic so air resistance is not that significant.



$$\theta = \tan^{-1}\left(\frac{12.191\,m}{21.335\,m}\right) = 29.74^\circ$$



$V_{oy} = V_o \sin\theta = 130\ m/s\ \sin(29.74^\circ) = 64.498\ m/s$ down

$V_{ox} = V_o \cos\theta = 130\ m/s\ \cos(29.74^\circ) = 112.872\ m/s$ right

Vertical

$V_{oy} = -64.498\ m/s$

$y_o = 12.191\ m$

$y = 0$

$a_y = -9.8\ m/s^2$

Horizontal

$V_{ox} = 112.877\ m/s$

$x_o = 0$

$a_x = 0$

$$y = y_o + V_{oy}t + \tfrac{1}{2}at^2$$

$$0 = 12.191 - 64.498t - 4.9t^2$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = -4.9 \quad b = -64.488 \quad c = 12.191$$

$$t = \frac{64.498 \pm \sqrt{-64.498^2 - 4(-4.9)(12.191)}}{2(-4.9)}$$

$$t = \frac{64.498 \pm 66.3147}{-9.8}$$

$t = .1864$ sec (the other solution is negative)

Plug into horizontal info:

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$x = 0 + 122.872 \text{ m/s} (.1864 \text{ s}) + \cancel{\frac{1}{2}(0)(.1864)^2}^{\,0}$$

$$x = 21.039 \text{ m}$$

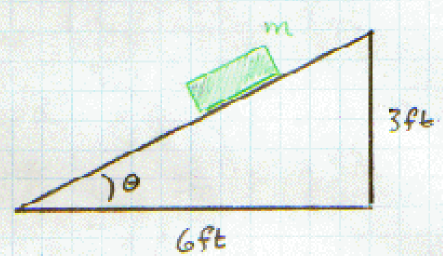
$$x = 21 \text{ m}$$

The arrow hits at 21 m. The target is at 21.3 m So the arrow falls only .3 m or 30 cm short of the mark.

Example Problem #2:

A child slides down a slippery "kiddie slide" at a playground. The slide is 6 ft long and 3 ft high. What is the acceleration of the child?

First I will draw a simple diagram of the child and slide:



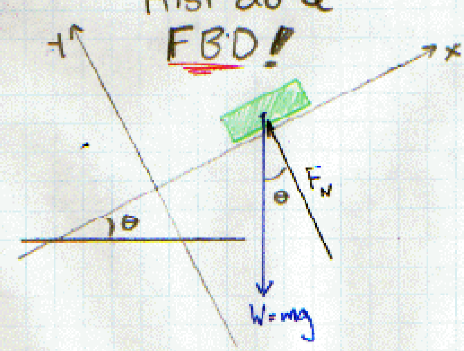
$$\tan\theta = \frac{3\text{ft}}{6\text{ft}}$$

$$\theta = \tan^{-1}(.5)$$

$$\theta = 26.565°$$

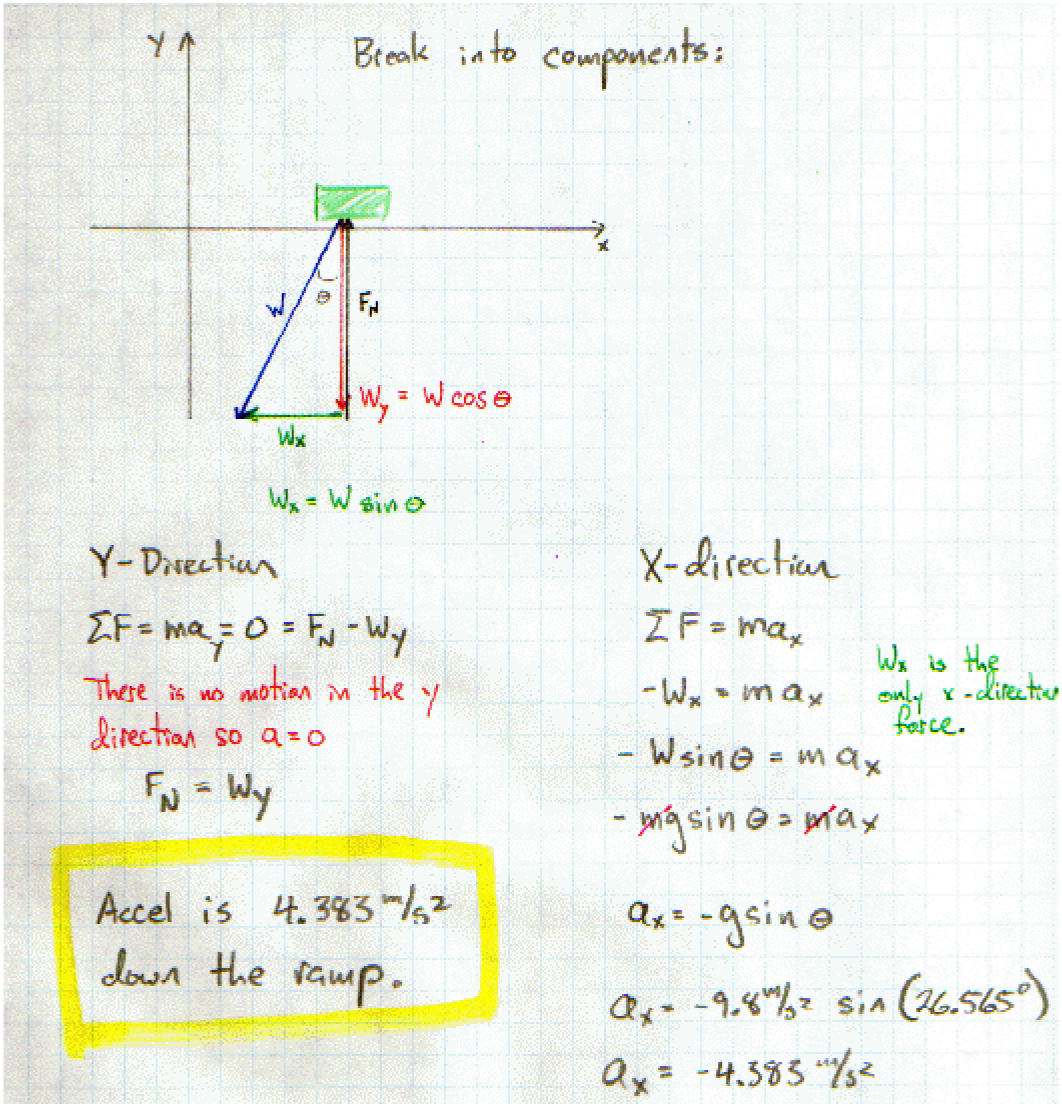
I guess slippery means friction is not significant. So the only forces acting on the child are gravity (weight) and the normal contact force between the child and the slide.

First do a FBD!



Show forces as vectors and add x-y axes, too

Now rotate the axes so I have the usual x-y Cartesian coordinate system and get to work.


Break into components:

$W_y = W\cos\theta$

$W_x = W\sin\theta$

Y-Direction

$\Sigma F = ma_y = 0 = F_N - W_y$

There is no motion in the y direction so $a = 0$

$F_N = W_y$

$\boxed{\text{Accel is } 4.383\ m/s^2 \text{ down the ramp.}}$

X-direction

$\Sigma F = ma_x$

$-W_x = ma_x$

W_x is the only x-direction force.

$-W\sin\theta = ma_x$

$-\cancel{m}g\sin\theta = \cancel{m}a_x$

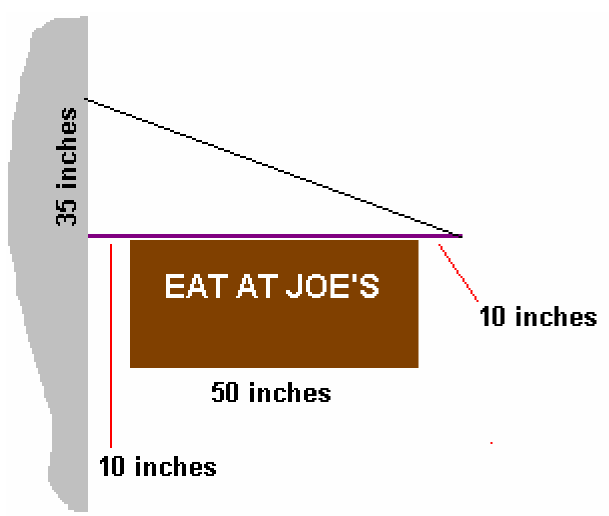
$a_x = -g\sin\theta$

$a_x = -9.8\ m/s^2 \sin(26.565°)$

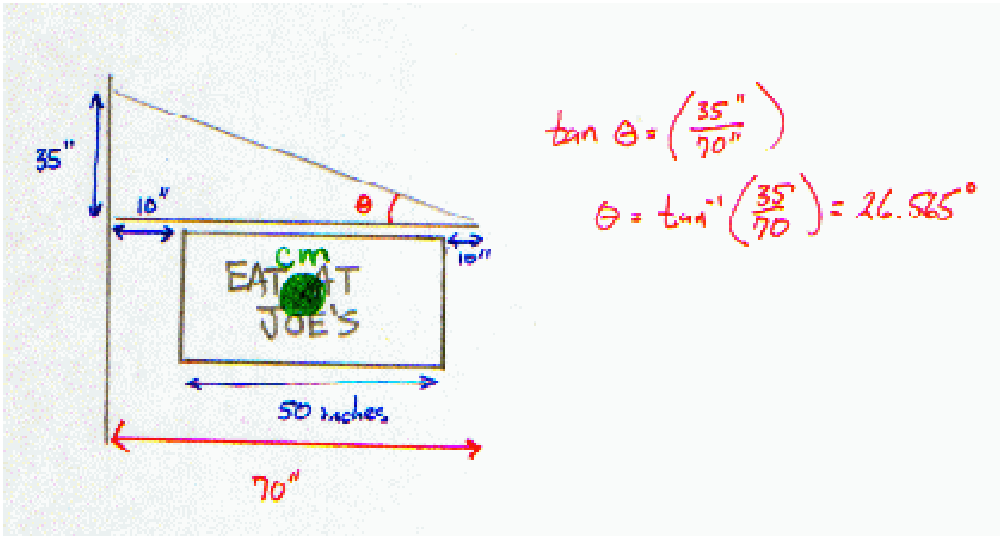
$a_x = -4.383\ m/s^2$

Example Problem #3:

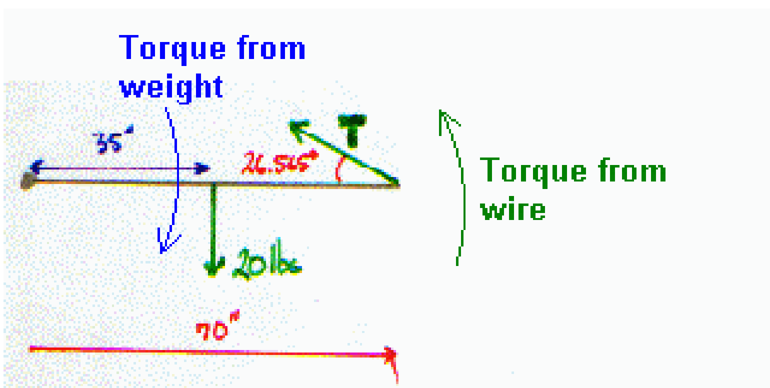
The sign weighs 20 lbs. The rod is very light. The rod would pivot freely were it not for the supporting wire. What is the tension in the supporting wire? What are the vertical and horizontal forces that the wall must exert on the sign to keep it in place?



This is a "**statics**" problem -- there is no motion. All forces and torques must cancel each other out. First I'll work out the geometry of the problem:



The sign is a rectangle -- the center of mass lies at its geometric center. Then once I know the distances I will move on to dealing with forces and torques. They all must cancel out -- all torques and forces must add up to zero. I'll start with torques first to find the tension.



$\Sigma \tau = 0$ Clockwise is positive

$\tau_{weight} - \tau_{wire} = 0$

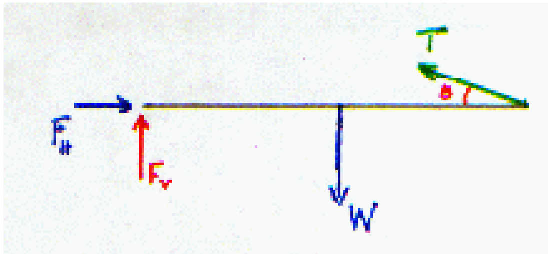
$(35\,in)\,20\,lb - 70\,in\,(T)\sin(26.565°) = 0$

$700\,\cancel{in}\cdot lb = T(31.305\,\cancel{in})$

$22.36\,lb = T$

Tension in the wire is 22 lb

Now I'll use forces to find the horizontal and vertical forces on the end of the pole.



Horizontal

$$\Sigma F = ma_x = 0$$

$$F_H - T_x = 0$$

$$F_H - T\cos\theta = 0$$

$$F_H = T\cos\theta = 22.36\text{ lb}\cos(26.565°)$$

$$F_H = 20\text{ lb}$$

Vertical $\Sigma F = ma_y = 0$

$$F_V + T_y - W = 0$$

$$F_V = W - T\sin\theta$$

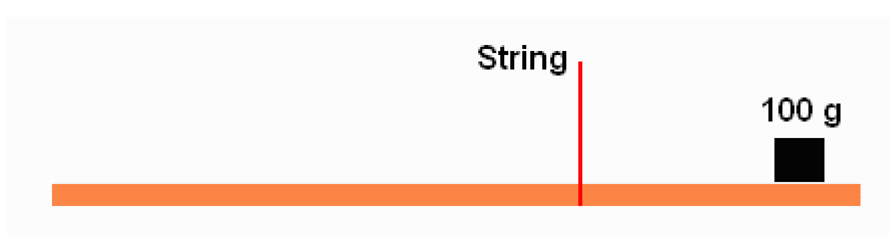
$$= 20\text{ lb} - 22.36\text{ lb}\,(\sin 26.565°)$$

$$F_V = 10\text{ lb}$$

So the horizontal force on the end of the pole is 20 lb and the vertical force is 10 lb.

Example Problem #4:

A meter stick is hung from a string. When a 100 g mass is placed at the 95 cm mark on the stick, the stick will balance if the string is moved to the 70 cm mark. Calculate the mass of the stick.



Assuming the stick is made of uniform material, the cm of the stick should be located at the middle of the stick (the 50 cm) mark. We can consider this to be the point at which the stick's weight acts. The string is our pivot or balance point. So the distance from the cm to the pivot point is 70 cm - 50 cm = 20 cm. The distance from the 100 g mass to the pivot point is 95 cm - 70 cm = 25 cm.

There's no motion, so this must be another **statics** problem! The counterclockwise torque created by the stick's weight acting at the cm must equal the clockwise torque created by the 100 g mass's weight acting at the 95 cm mark.

$\tau_{ccl} = \tau_{cl}$

$r_{stick}W_{stick} = r_{100}W_{100}$ $\quad r_{stick} = 20$ cm; $r_{100} = 25$ cm

$r_{stick}m_{stick}g = r_{100}m_{100}g$ $\quad W = mg$ for both the stick & the 100 g mass.

$r_{stick}m_{stick} = r_{100}m_{100}$ $\quad g$ cancels out on both sides.

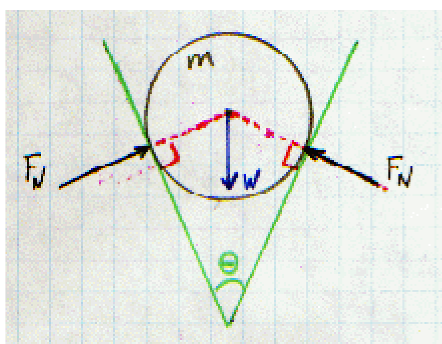
$m_{stick} = (r_{100}m_{100})/r_{stick}$

$= (25\text{ cm} * 100\text{g})/20\text{ cm} = 125\text{ g}$

The stick's mass is 125 g.

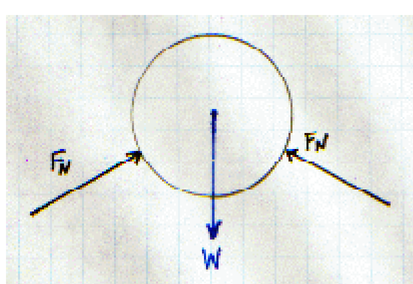
Example Problem #5:

A glass ball is sitting in a ceramic "V" with vertex angle θ as shown. The walls of the "V" are slippery. Find all forces acting on the ball. Give your answer in terms of m, θ, and g.

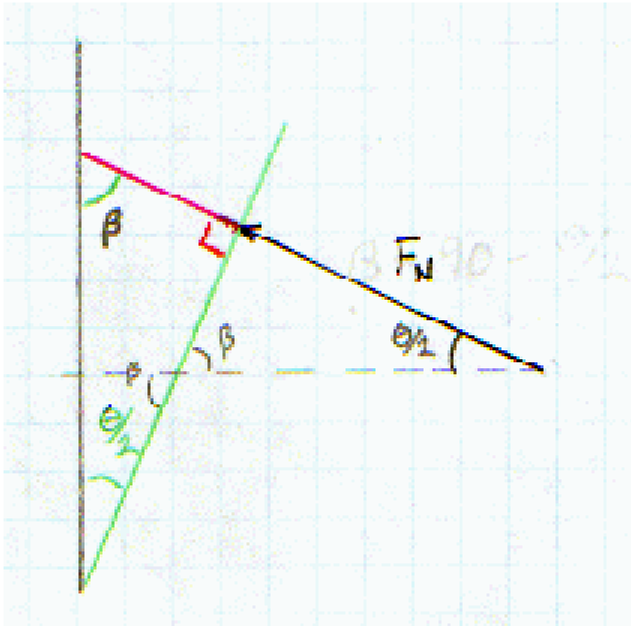


This is another **statics** problem. First I'm going to sketch out the problem. If there is no friction ("slippery sides"), then the only forces that act on the ball are its weight and the normal contact forces between the ball and the walls of the "V"

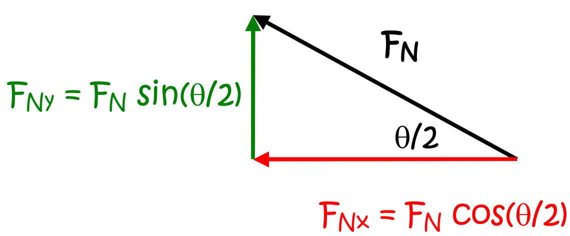
Now I'll do a FBD:



Let me try to figure out the angles of those F_N vectors:



A little high school geometry in action! So F_N is directed at and angle $\theta/2$ above the horizontal. The problem is symmetrical so F_N should be the same on both sides. Now split into components and do the usual vector stuff:



Vertical (Y)

$\Sigma F = ma_y = 0$

$F_{NY} + F_{NY} - W = 0$

$W = 2F_{NY}$

$mg = 2F_N \sin(\theta/2)$

$F_N = \dfrac{mg}{2\sin(\theta/2)}$

Horizontal (X)

$\Sigma F = ma_x = 0$

$F_{Nx} - F_{Nx} = 0$

$F_{Nx} = F_{Nx}$

(Big deal!)

a is zero in both directions.

So my answer is that the forces acting on the ball are its weight **$W=mg$** and two contact forces, both **$F_N = mg/(2\sin(\theta/2))$**.

Note that if $\theta = 180^{\circ}$, which means the "V" is flattened out to a straight line, then $\sin(\theta/2) = \sin 90^{\circ} = 1$. Each F_N becomes equal to half the weight, which makes sense for a ball resting on a flat surface.

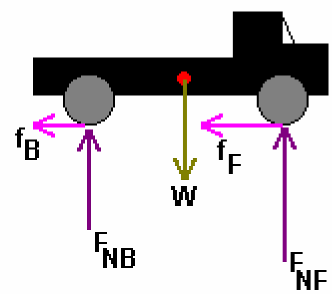
Example Problem #6 (the example problem from HELL):

A vehicle has mass 2000 kg and a wheelbase (distance between wheels) of 2.5 meters. Its center of mass sits at a height 0.5 meters off the ground and is half way between the wheels. The vehicle is moving over flat ground.

The driver slams on the brakes. The coefficient of static friction between the tires and road is 0.9 (the vehicle has anti-lock brakes to make sure the friction is always static -- the tires never skid). Find the stopping force on the front and back tires of the vehicle. Compare the two and comment on which tires do the most to stop the car.

A "lift kit" is then added to the vehicle, raising the c.m. height to 0.7 meters. Again compare stopping forces.

First, I make an FBD for the stopping vehicle.



There are five forces acting on it:

- the frictional forces **f** on the front and back tires
- the normal forces **F_N** on these tires
- the weight **W** acting at the center of mass of the truck

the red dot is the c.m. point in this diagram.

Breaking this down into x & y directions:

X – direction

$\Sigma F = m a$

$-2f_B - 2f_F = m a$ — 2 front tires, 2 back tires

$f_B + f_F = - \frac{1}{2} (m a)$

$\mu F_{NF} + \mu F_{NB} = - \frac{1}{2} (m a)$ — $f = \mu F_N$

$\mu(F_{NF} + F_{NB}) = - \frac{1}{2} (m a)$

$F_{NF} + F_{NB} = - ma/2\mu$

$9800 \text{ N} = - 2000 \text{ kg } (a)/(2(0.9))$

$9800 \text{ N} = - 1111.111 \text{ kg } (a)$

Y – direction

$\Sigma F = 0$ — No motion in y direction

$2F_{NF} + 2F_{NB} - W = 0$ — 2 front tires, 2 back tires

$F_{NF} + F_{NB} = W/2$

$F_{NF} + F_{NB} = mg/2$

$F_{NF} + F_{NB} = 2000 \text{ kg}(9.8 \text{ N/kg})/2$

$F_{NF} + F_{NB} = 9800 \text{ N}$

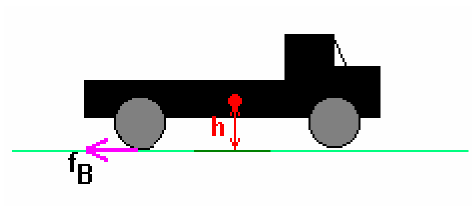
$a = -8.82\ m/s^2$

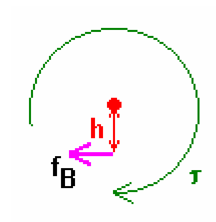
$f_B + f_F = -\tfrac{1}{2}(2000\ kg)(-8.82\ m/s^2)$

$f_B + f_F = 8820\ N$

Now let's do torques. The total torque about the c.m. must be zero since the vehicle is not flipping end over end (rotating).

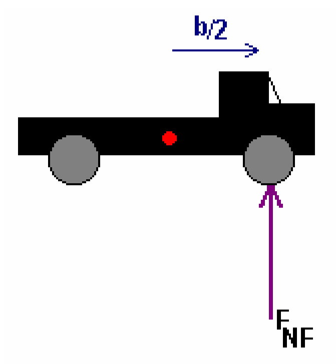
The lever arm for f_B and f_F is h.

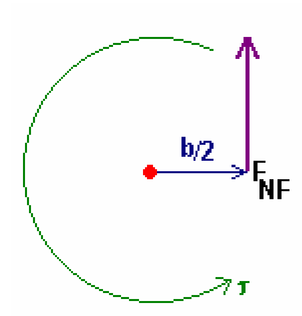




$\tau = rF = hf_B$

The lever arm for F_{NF} and F_{NB} is b/2.





$\tau = rF = (b/2)F_{NF}$

The torque caused by the normal force on the front tire is counter-clockwise. The rest are clockwise. I'll call clockwise positive

	Torque from frictional force on back tire	$+\ hf_B$
+	Torque from frictional force on front tire	$+\ hf_F$
+	Torque from normal force on back tire	$+\ (b/2)F_{NB}$
-	Torque from normal force on front tire	$-\ (b/2)F_{NF}$
	Zero	0

$hf_B + hf_F + (b/2)F_{NB} = (b/2)F_{NF}$

$h(f_B + f_F) = (b/2)F_{NF} - (b/2)F_{NB}$

$h(f_B + f_F) = (b/2)(F_{NF} - F_{NB})$

Now I'll sub in for the values for h, b, and $f_B + f_F$

$0.5\ m\ (f_B + f_F) = (2.5\ m/2)(F_{NF} - F_{NB})$

$0.5\ m\ (8820\ N) = (1.25\ m)(F_{NF} - F_{NB})$

$3528\ N = F_{NF} - F_{NB}$

$$\begin{array}{rl} & 9800\ N = F_{NF} + F_{NB} \\ + & 3528\ N = F_{NF} - F_{NB} \\ \hline & 13328\ N = 2F_{NF} \end{array}$$

The highlighted items are from the x and y forces stuff I just did.

$6664\ N = F_{NF}$

$$f_F = \mu F_{NF}$$
$$f_F = 0.9\ (6664\ N) = 5997.6\ N$$

$$f_B + f_F = 8820\ N$$
$$f_B + 5997.6\ N = 8820\ N$$
$$f_B = 2822.4\ N$$

So the frictional stopping force at each front tire is 5998 N and the stopping force at each back tire is 2822 N. The front tires exert almost twice the stopping force as the back tires – they do the most to stop the car.

With the lift kit, h becomes 0.7 m and the last part of the problem is as follows:

$$0.7\ m\ (f_B + f_F) = (2.5\ m/2)(F_{NF} - F_{NB})$$
$$0.7\ m\ (8820\ N) = (1.25\ m)(F_{NF} - F_{NB})$$
$$4939.2\ N = F_{NF} - F_{NB}$$

$$
\begin{array}{rl}
 & 9800\ N = F_{NF} + F_{NB} \\
+ & 4939.2\ N = F_{NF} - F_{NB} \\
\hline
 & 14739.2\ N = 2F_{NF}
\end{array}
$$

$$7369.6\ N = F_{NF}$$
$$f_F = \mu F_{NF}$$
$$f_F = 0.9\ (7369.6\ N) = 6632.64\ N$$

$$f_B + f_F = 8820\ N$$
$$f_B + 6632.64\ N = 8820\ N$$
$$f_B = 2187.36\ N$$

So the frictional stopping force at each front tire is 6633 N and the stopping force at each back tire is 2187 N. The lift kit puts even more of the stopping load at the front tires.

Geesh! Is that a horrible problem or what?