DAY 17

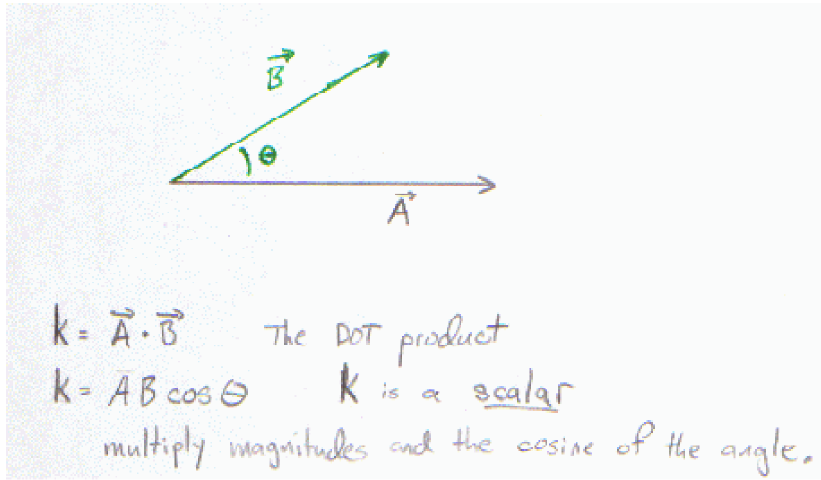
Summary of Primary Topics Covered

The Vector DOT Product

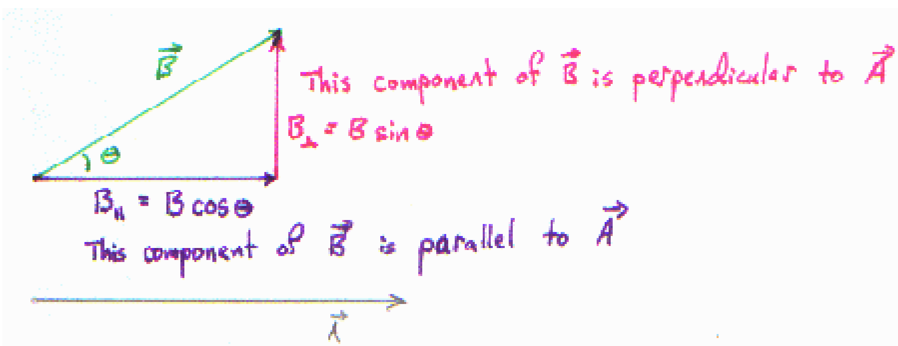
The DOT product k of two vectors **A** & **B** is defined as

k = **A**·**B** = AB cos θ

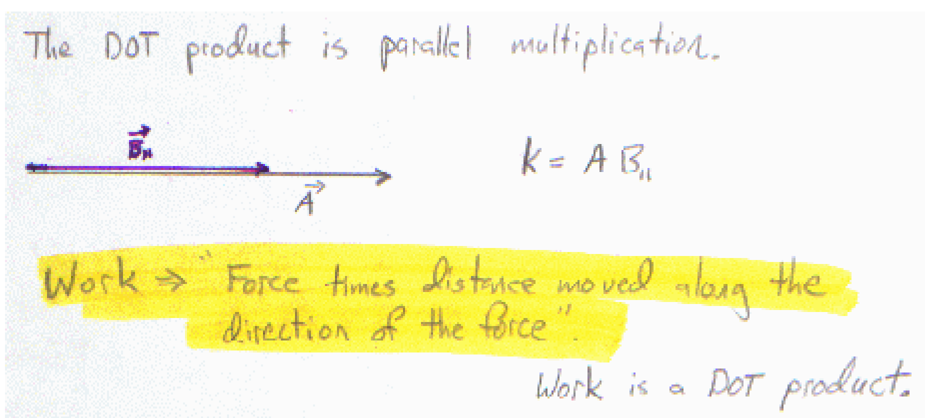
k is a scalar. θ is the angle between **A** & **B** and A & B are the magnitudes of **A** & **B**.



The dot product is a parallel multiplication.
Vector **B** has a component that is parallel to **A** and a component that is perpendicular to **A**.



In the dot product, the component of **B** that is parallel to **A** is multiplied times **A**.



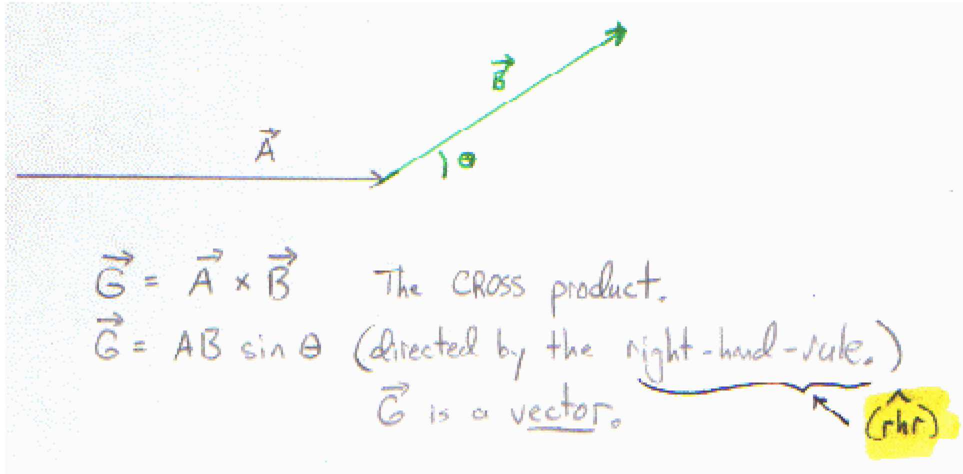
The Vector CROSS Product

The CROSS product **G** of two vectors **A** & **B** is defined as

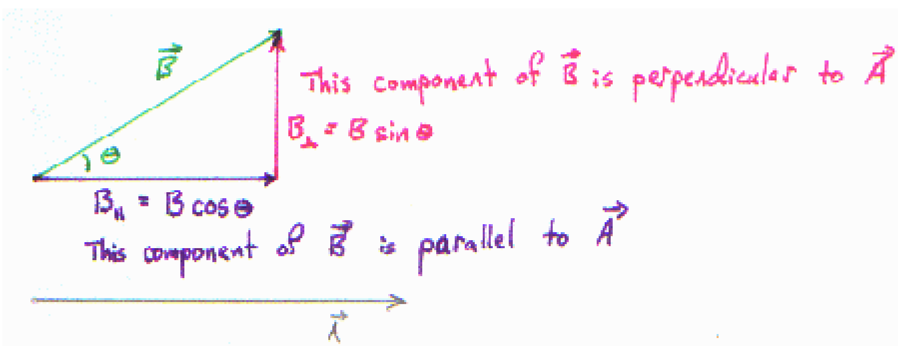
G = **A**×**B** = AB sin θ

G is is vector. θ is the angle between **A** & **B**.

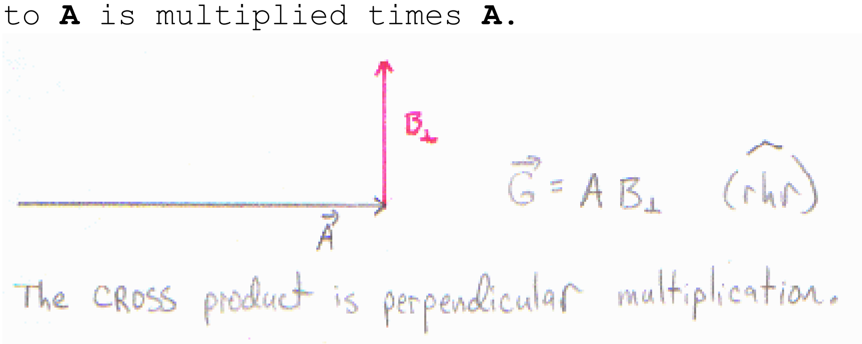
The cross product is a perpendicular multiplication.

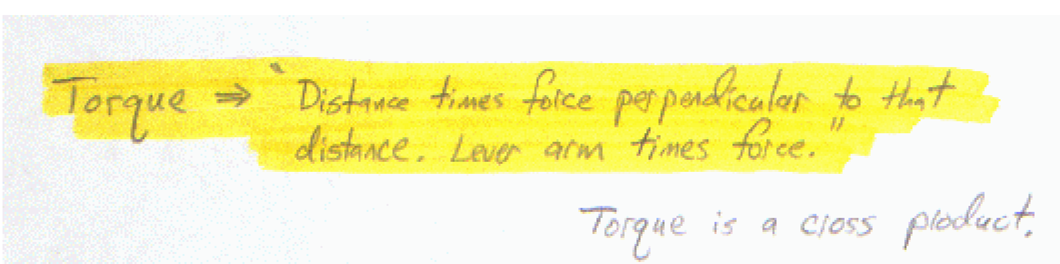


Again, vector **B** has a component that is parallel to **A** and a component that is perpendicular to **A**.



In the cross product, the component of **B** that is perpendicular to **A** is multiplied times **A**.



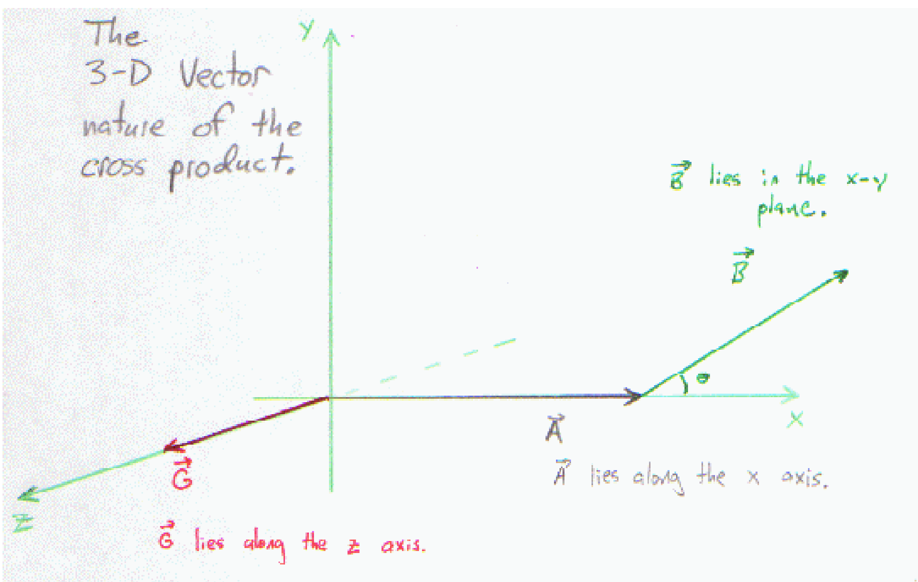


The cross product has a 3-D nature. **G** is perpendicular to the plane that contains vectors **A** & **B**. The direction of G is given by the *right-hand-rule* (rhr).

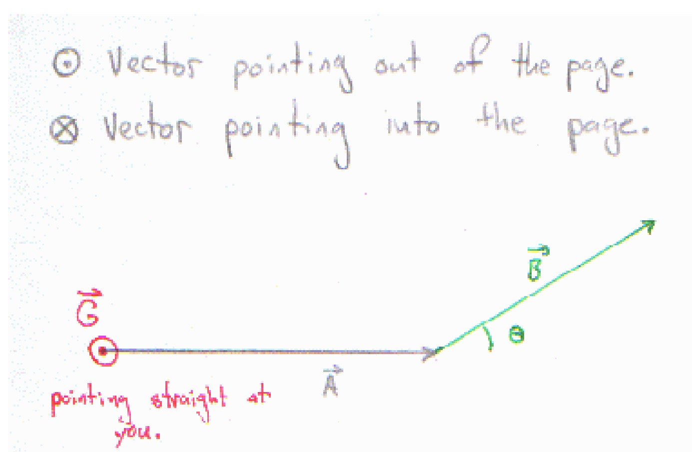
The rhr says:

1. Use your right hand!
2. Point your right index finger (the pointer) in the direction of **A**.
3. Keep that index finger pointing, and at the same time point your right middle finger (the one you see people using when they drive to "communicate" with other drivers) in the direction of **B**.
4. Stick out your thumb – it will point in the direction of **G**.

You can draw 3-D vector problems using a 3-D perspective, like the figure below:



Or you can use symbols for a vector pointing out of or into a piece of paper, like the figure at right.



Many angular quantities that are vectors are simply scalar multiples of torque. If you exert a torque to start a wheel spinning, the angular momentum vector of the wheel points in the same direction as the torque you applied to get it spinning.

$$\boldsymbol{\tau} = I\boldsymbol{\alpha}$$

$$\boldsymbol{\tau} = I\left(\frac{\Delta\boldsymbol{\omega}}{t}\right)$$

$$\boldsymbol{\tau} = I\left(\frac{\boldsymbol{\omega} - \boldsymbol{\omega}_0}{t}\right)$$

$$\boldsymbol{\tau} = \left(\frac{I\boldsymbol{\omega} - I\boldsymbol{\omega}_0}{t}\right)$$

$$\boldsymbol{\tau} = \left(\frac{\mathbf{L} - \mathbf{L}_0}{t}\right)$$

$$\boldsymbol{\tau} = \frac{\Delta\mathbf{L}}{t}$$

By making various substitutions we see that the following are all vectors, too:

- angular acceleration ($\boldsymbol{\alpha}$)
- angular velocity($\boldsymbol{\omega}$)
- angular momentum (**L**)

The correspondence with linear quantities continues! Linear acceleration, velocity, momentum are all vectors.

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$$\boldsymbol{\tau} = I\boldsymbol{\alpha}$$

$$\boldsymbol{\tau} = I\left(\frac{d\boldsymbol{\omega}}{dt}\right)$$

$$\boldsymbol{\tau} = \left(\frac{d(I\boldsymbol{\omega})}{dt}\right)$$

$$\boldsymbol{\tau} = \frac{d\mathbf{L}}{dt}$$

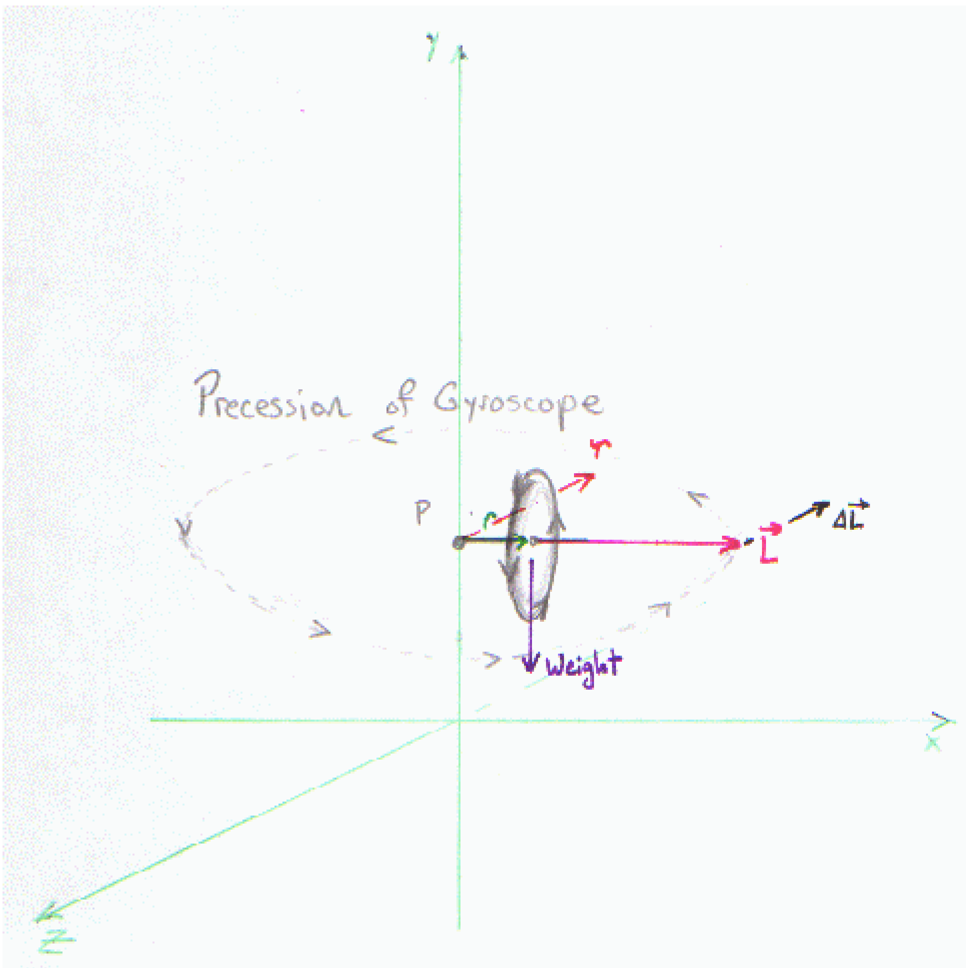
The Gyroscope as an Illustration of the Nature of the Vector CROSS Product

In the gyroscope diagram on the next page, a wheel is spinning and has an angular momentum vector (**L**) that points out along the x axis. The wheel's weight (**W**) is pointed down (in the negative y direction). The axle acts as a lever arm (**r**), so a torque ($\boldsymbol{\tau}$) is created around the pivot point (P). According to the rhr, $\boldsymbol{\tau}$ must point along the negative z axis. The change in angular momentum is given by

$$\Delta \mathbf{L}/t = \boldsymbol{\tau}$$

$$\Delta \mathbf{L} = \boldsymbol{\tau}\ t$$

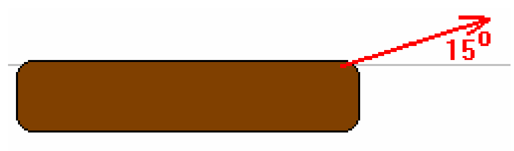
so $\Delta\mathbf{L}$ must point in the same direction as $\boldsymbol{\tau}$ (that is, in the negative z direction). $\Delta\mathbf{L}$ shows the change in **L**, so the **L** vector swings toward the negative z direction. The gyroscope precesses counterclockwise around the positive y axis. It does not "fall" despite the action of the weight acting downward.



Example Problem #1:

To drag a heavy log a 500 lb tension is applied to a rope as shown. The log moves 100 ft under the steady 500 lb pull. How much work is done?

$F = 500$ lb @ $15^\circ = 2224$ N @ 15°.
$d = 100$ ft @ $0^\circ = 30.479$ m @ 0°.
θ (angle between F & d) $= 15^\circ$.



$W = F \cdot d$
$= (2224\ N)(30.479\ m)(\cos(15^\circ))$
$= 65474.52111$ Nm

The work done is 65500 J.

Example Problem #2:

In the figure are shown muscles and bones from a small animal.

The torque that the muscle produces around the joint (P) when it contracts with a tension of 20 lb is 5 in-lbs. What is the distance between the joint and the point of attachment of the muscle (A)?

The first thing I did was to draw in some more angles. In triangle APQ I already have a 10° angle and a 70° angle. The total of the angles of this triangle must be 180°, so I know angle P-A-Q must be 100°. Then the angle between the 20 lb muscle tension and the P-A distance must be 80° since there are 180° in a straight line.

I'll call the P-A distance $r_{P\text{-}A}$.

$\tau = rF \sin(\theta)$

$\tau = 5$ in-lbs
$F = 20$ lbs
$\theta = 100^\circ$
solve for r

5 in-lbs $= (r_{P\text{-}A})(20\ \text{lbs}) \sin(100^\circ)$
5 in $= (r_{P\text{-}A})(19.6962)$
0.25386 in $= r_{P\text{-}A}$

The distance is 0.25 inches.