DAY 16

Summary of Primary Topics Covered

More Vector Addition Examples

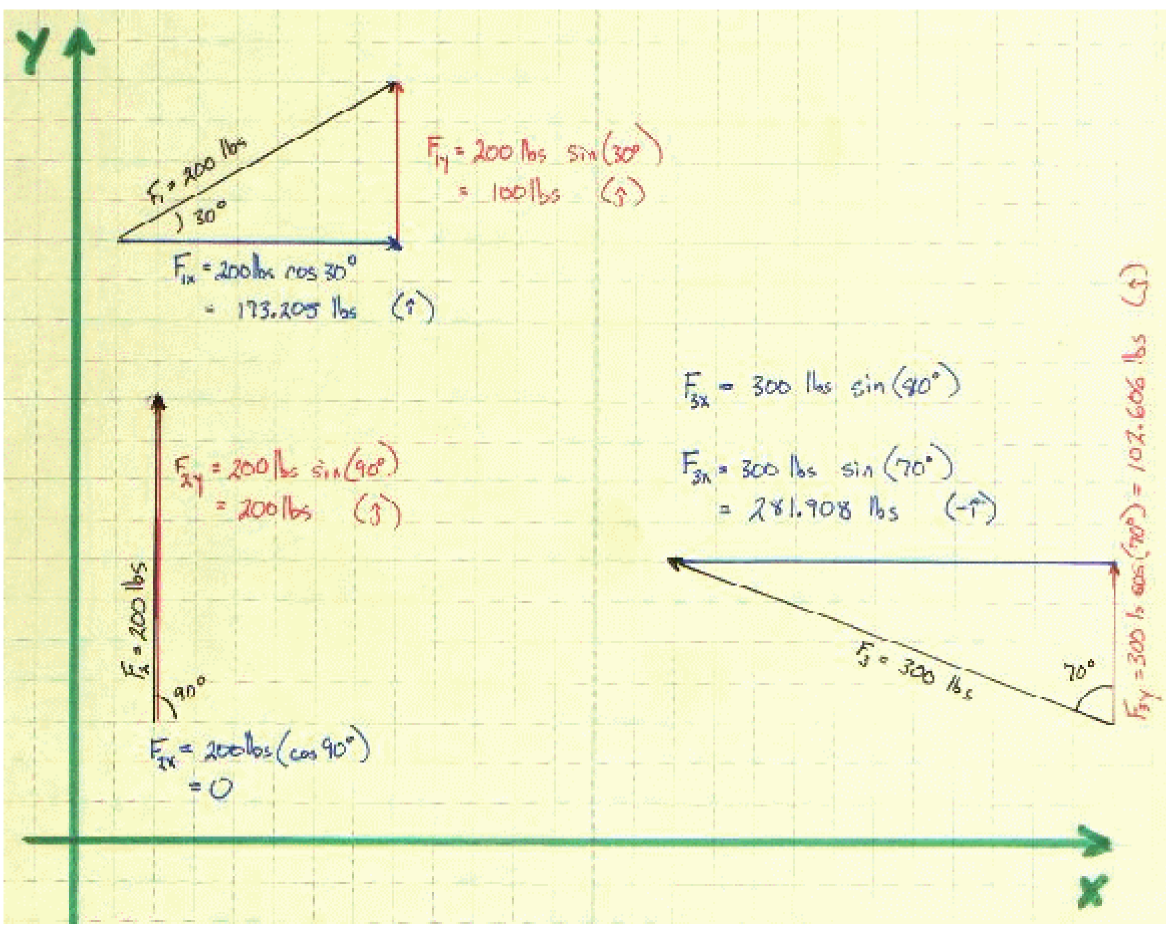
We did some more in-depth vector problems today. A little note -- the TAN^{-1} (inverse tangent) function is also known as the arctangent. You might see it written as ATAN or ARCTAN.

Example Problem #1:

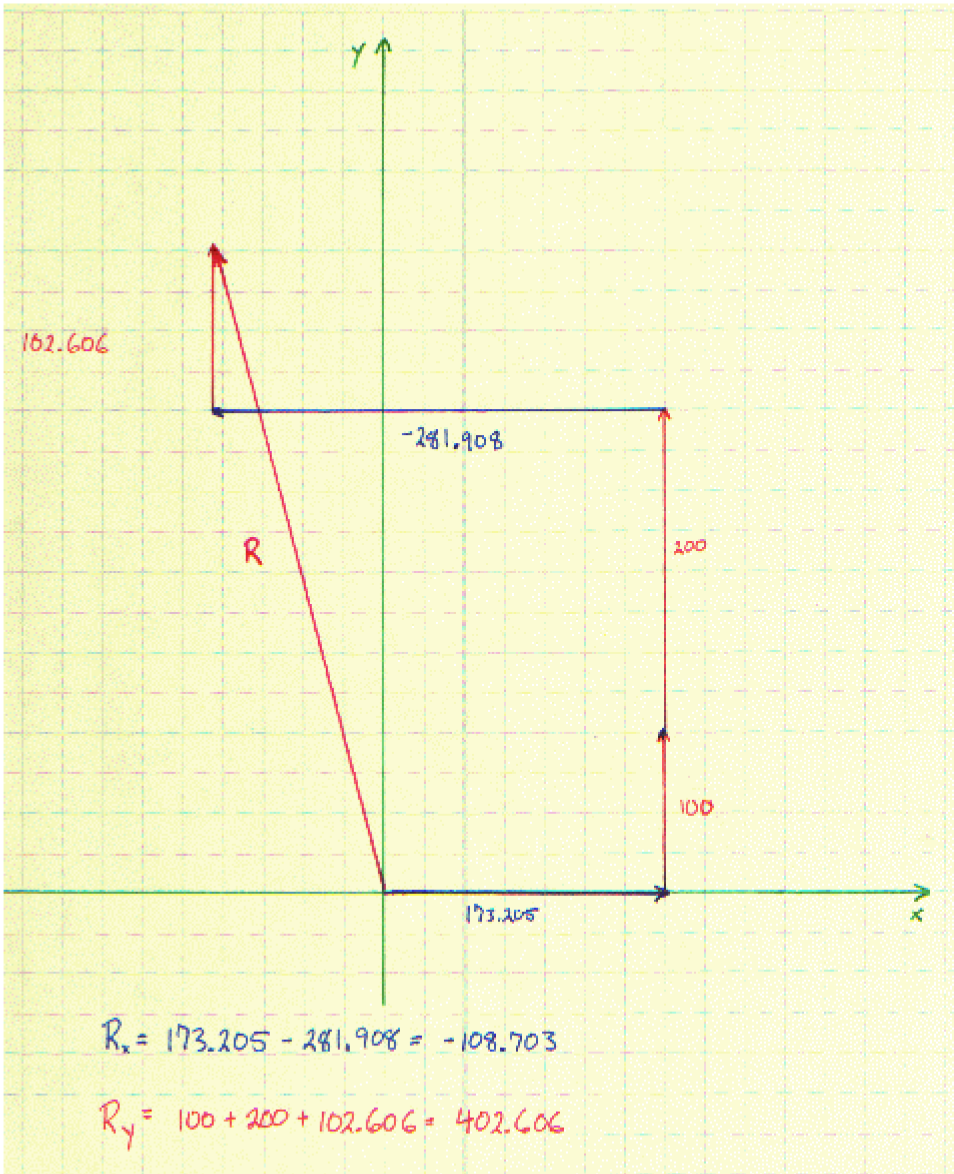
Calculate the resultant of the following three force vectors:

$\mathbf{F_1}$ = 200 lbs @ 30° measured counter-clockwise from the positive x axis.
$\mathbf{F_2}$ = 200 lbs directed along the positive y axis.
$\mathbf{F_3}$ = 300 lbs @ 70° measured counter-clockwise from the positive y axis.

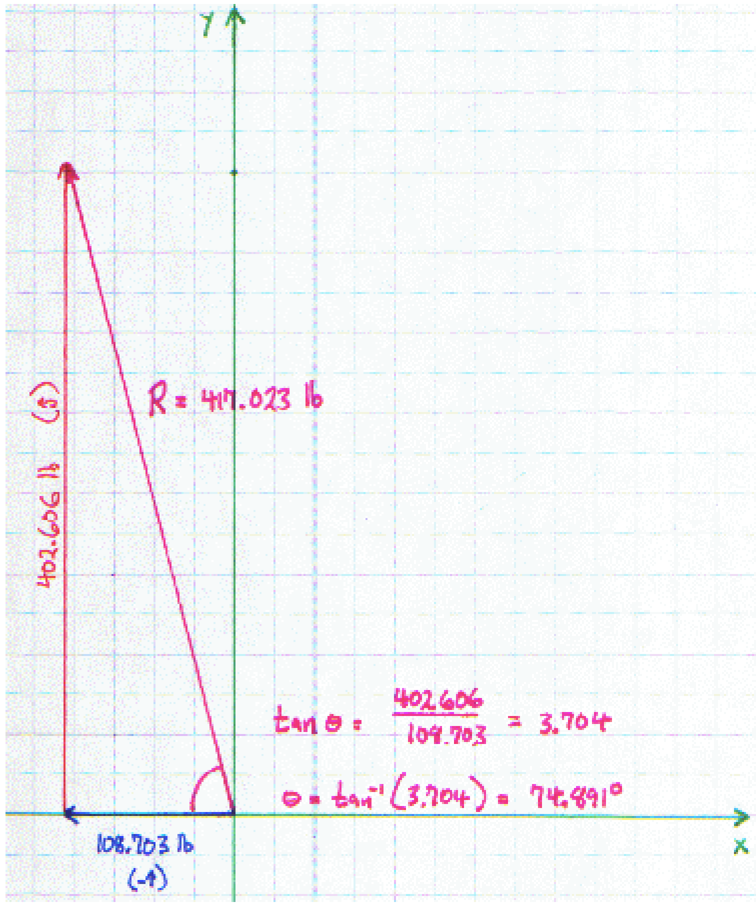
First I'm going to sketch my three vectors and break them down into x & y components:



Now I add up the x & y components to get the components of a resultant R:



I use R_x and R_y to find the magnitude and direction of **R**:



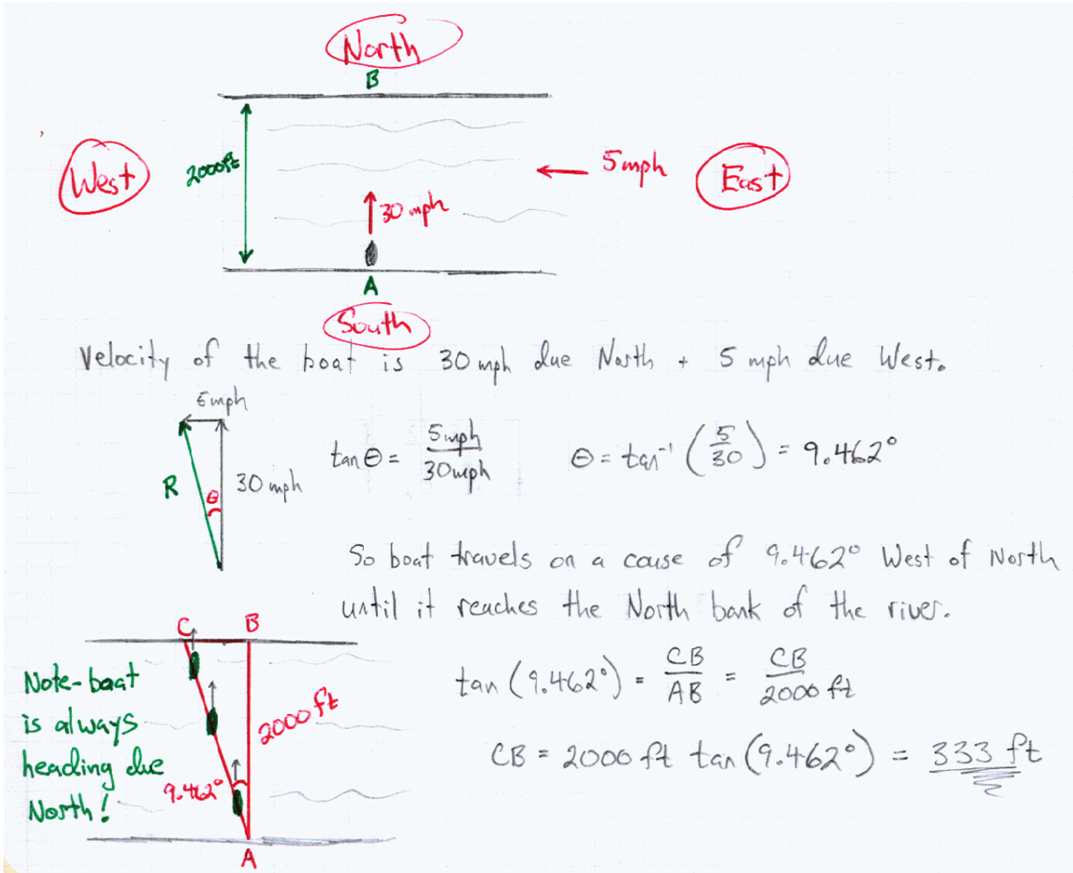
My final answer is that the resultant of these three forces is a force of **R** = 417 lb @ 75° measured clockwise from the negative x axis.

$F_1 + F_2 + F_3 = R$

Example Problem #2:

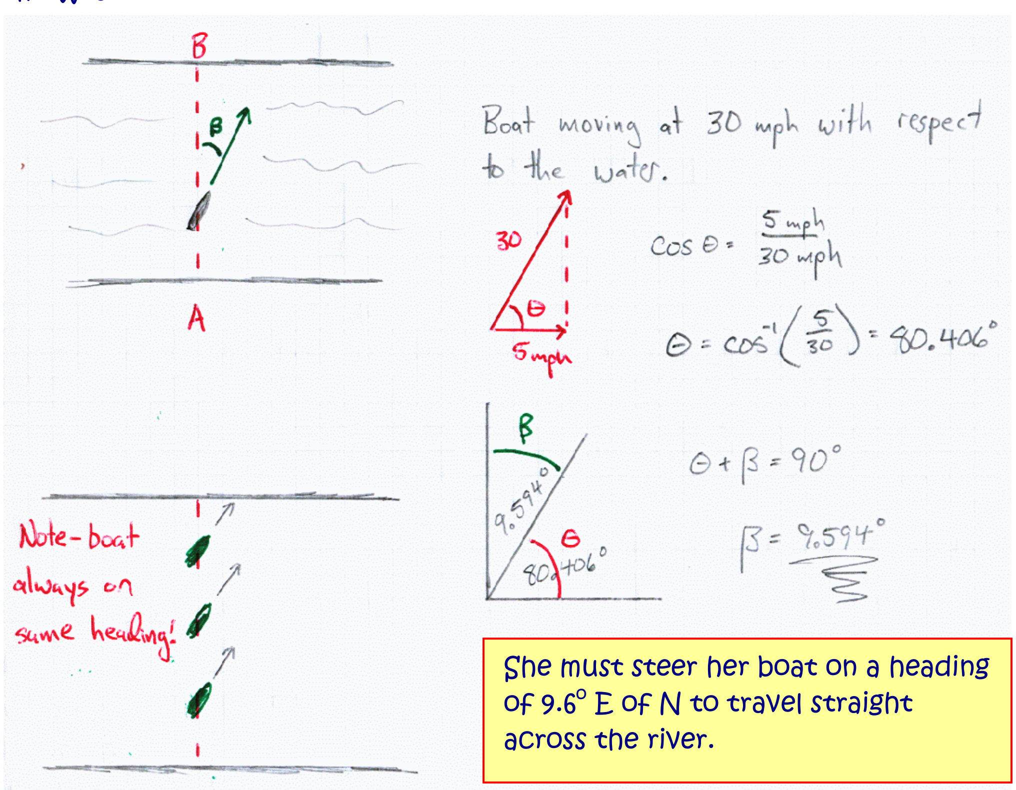
A heroine in a speedboat must race across a rain-swollen river to rescue a kitty cat in danger of being swept off the far bank! The river measures 2000 ft across, flows from E to W, and has a current of 5 mph. Heroine's boat will travel at 30 mph (measured with respect to water). Our heroine is on the south bank of the river. If she aims her boat straight for the North bank, where will she come ashore? On what heading must she steer her boat if she wants to travel straight across the river to the cat?

Let me call the location of the cat point B, and the location of our heroine on the south bank point A. Point B is directly north of point A.



So if steers her boat due North, she will end up 333 ft downstream from the cat.

To travel directly across the river to the cat, she must steer her boat so that it has a 5 mph component to the East, which will cancel out the river's 5 mph current to the West.



She must steer her boat on a heading of 9.6° E of N to travel straight across the river.

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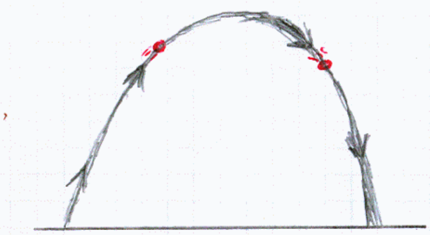
Example Problem #3:

A heavy iron ball is launched into the air such that it leaves the ground with a vertical speed of 98 m/s and a horizontal speed of 4.9 m/s. Determine the ball's x & y positions as functions of time. Determine how long the ball is in the air. The ball is tracked with a telescope. Determine an equation for the angle the telescope makes with the ground as a function of time as it tracks the ball. Graph this equation. Determine an equation for the rate at which the telescope must slew in order to track the ball. Graph this equation.

The ball will travel in an arc. I can treat the horizontal and vertical motions of the ball separately:



Vertical: (Free falling object)

$V_o = 98\ m/s$

$a = -9.8\ m/s^2$

$y_o = 0$

$y = y_o + v_o t + \frac{1}{2}at^2$

$y = 0 + 98t + \frac{1}{2}(-9.8)t^2$

$y = 98t - 4.9t^2$

Horizontal: (Constant velocity)

$V_o = 4.9\ m/s$ $\quad x = x_o + V_o t + \frac{1}{2}at^2$

$x_o = 0$ $\quad x = 0 + 4.9t + \frac{1}{2}(0)t^2$

$a = 0$ $\quad x = 4.9t$

Now – how long is ball in the air?

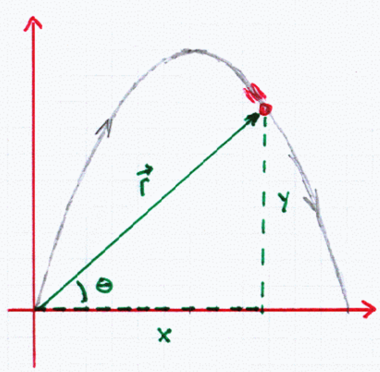
Ball hits ground when $y = 0$

$0 = 98t - 4.9t^2$

$4.9t^2 = 98t$

$t = 20\ sec$

Now I will figure the angle the tracking telescope will make with the ground. I imagine taking a snapshot of the ball as it is in mid-air. At that point it will have an x and y position, given by the equations I derived above:



Telescope aimed along the position vector of the ball.

$\tan\theta = \frac{y}{x}$

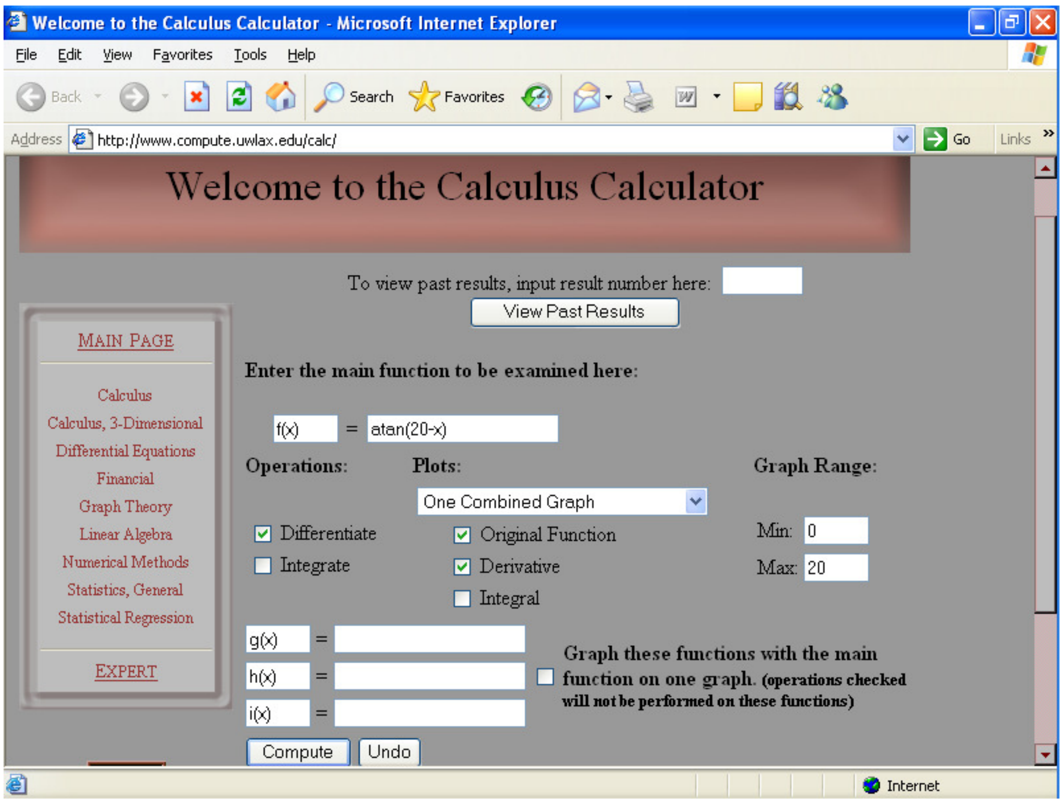
$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{98t - 4.9t^2}{4.9t}\right)$

$\theta = \tan^{-1}(20 - t)$ graph this from $0 \le t \le 20$ sec

Well, now to calculate an equation for how fast the telescope must slew to track the ball, I use

$\omega = d\theta/dt = d/dt\{\tan^{-1}(20\text{-}t)\}$

No point in doing this by hand. I am going to use a free web-based calculator to do this derivative (AND plot the functions, too)!



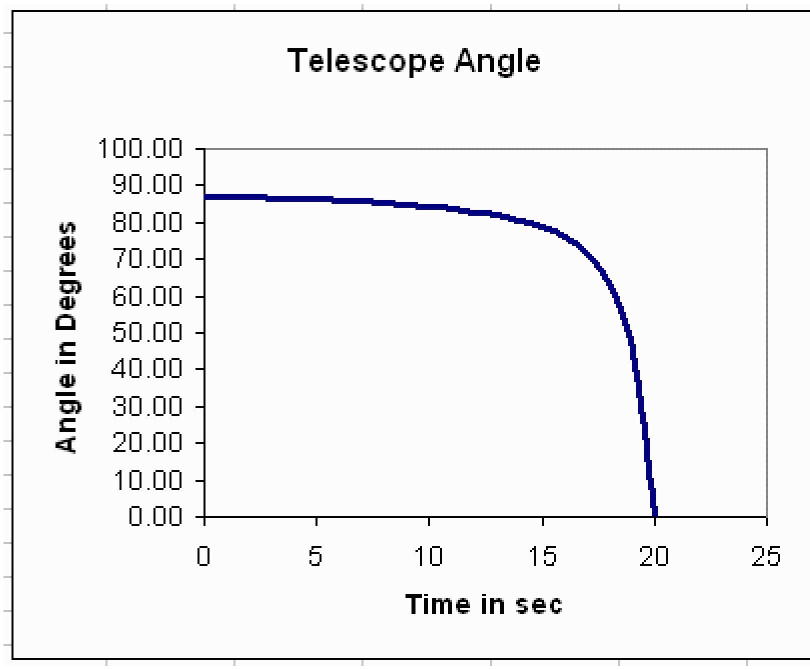
The program required me to write the TAN^{-1} function as ATAN.

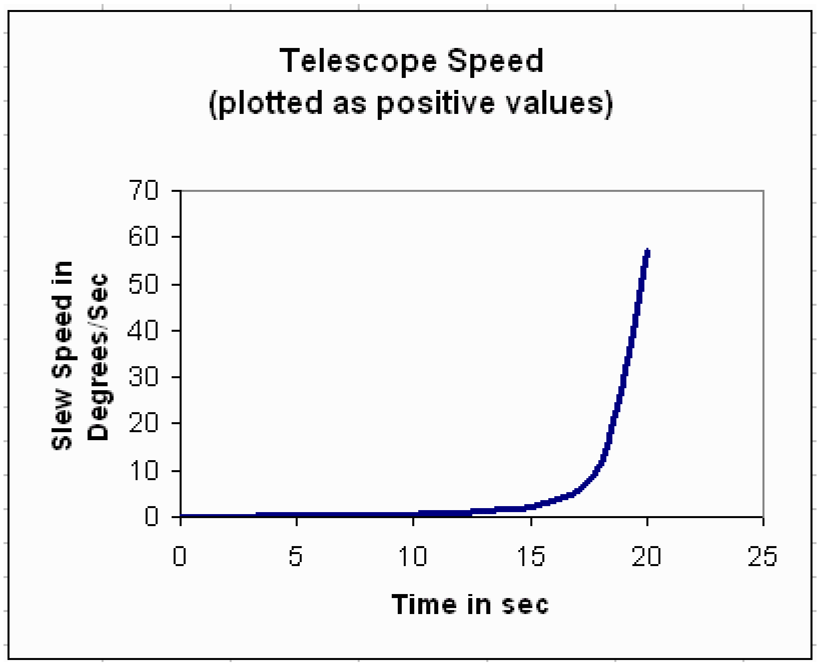
$$\omega = \frac{-1}{(t-20)^2+1}\left(\frac{180}{\pi}\right)$$

A lot of calculators will do this, too.

Now I graph these (graphs on next page). I see the telescope's angle changes slowly at first but after about 15 seconds the telescope must be slewed more and more rapidly. The negative speeds make sense since the angle is getting smaller as time passes.

t	θ (degrees)	ω (deg/sec)
0	87.14	-0.14
2.5	86.73	-0.19
5	86.19	-0.25
7.5	85.43	-0.36
10	84.29	-0.57
12.5	82.41	-1.00
15	78.69	-2.20
17.5	68.20	-7.90
20	0.00	-57.30


Telescope Angle
100.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
Angle in Degrees
0
5
10
15
20
25
Time in sec


Telescope Speed
(plotted as positive values)
70
60
50
40
30
20
10
0
Slew Speed in Degrees/Sec
0
5
10
15
20
25
Time in sec