DAY 15

Summary of Primary Topics Covered

Vectors

Vectors are quantities than have magnitude (size), with appropriate units, and direction. Quantities that do not have a direction associated with them are called scalars.

Scalar Examples		Vector Example	
Time	*t = 10 years* (10 years North doesn't make much sense).	Velocity	**v =** 10 mph from the Southwest ("The wind is blowing at 10 mph out of the Southwest") **NOTE – vectors are represented with boldface letters when typed; with an arrow over them when hand-written.**
Volume	*V = 5 gallons* (What would be a 5 gallon South tank?)		
Mass	*m = 2 kg* (2 kg to the left? That doesn't make sense.)		

Vectors convey more information than scalars. Consider the statement that a certain place is located "400 meters from the lobby of the JCC-SW Science Building." That is a scalar – no direction is given. It only specifies that something lies somewhere on a circle of radius 400 m that is centered on the Science Building Lobby. Refer to the photo of the JCC-SW campus shown below. The radius extends from the Science Building Lobby.

D = 400 m

http://terraserver.homeadvisor.msn.com/image.aspx?t=1&s=11&x=1503&y=10547&z=16&w=0

However, if I say that something is located "400 meters from the lobby of the JCC-SW Science Building at 61° South of due East", that specifies an exact location – in this case the entrance to the JCPS bus parking lot over by the middle school:

$$\mathbf{D} = 400 \text{ m @ } 61^\circ \text{ S of E}$$

This is an example of a **position vector**. Note that my D is now **D** (with boldface meaning it's a vector).

Multiplying a Vector by a Scalar

There are three ways to do multiplication with vectors.

Scalar x Vector → Vector
Vector x Vector → Scalar (DOT product)
Vector x Vector → Vector (CROSS product)

The simplest is multiplying a scalar times a vector. The result is a vector -- the scalar only multiplies the magnitude portion of the vector.

For example, in the above picture

D = 400 m @ 61° S of E

If I multiply **D** by the scalar k = 0.5 to get a new vector **Z**:

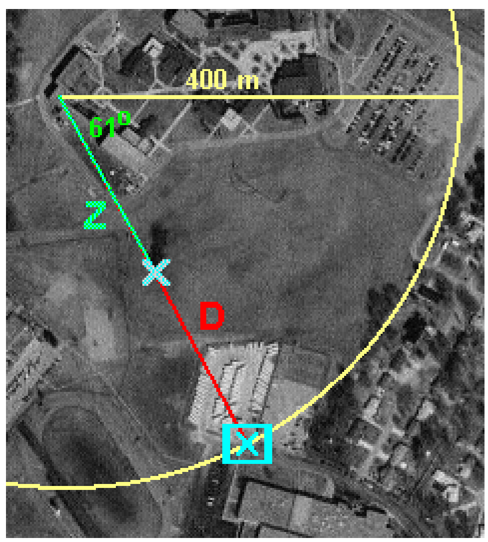
Z = k **D**

Z = (0.5)(400 m @ 61° S of E)

Z = (0.5*400) m @ 61° S of E

Z = 200 m @ 61° S of E

Z = 200 m @ 61° S of E



If I multiply **D** by the scalar k = -0.75 to get a new vector **Z**:

Z = k **D**

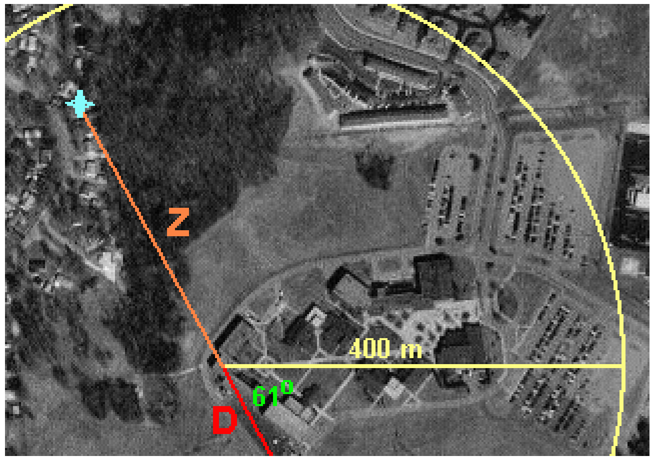
Z = (-0.75)(400 m @ 61° S of E)

Z = (-0.75*300) m @ 61° S of E

Z = -300 m @ 61° S of E

Z = 300 m @ 61° N of W

Z = 300 m @ 61° N of W



Since position is a vector, so are a lot of other quantities that are simply scalar multiples of position:

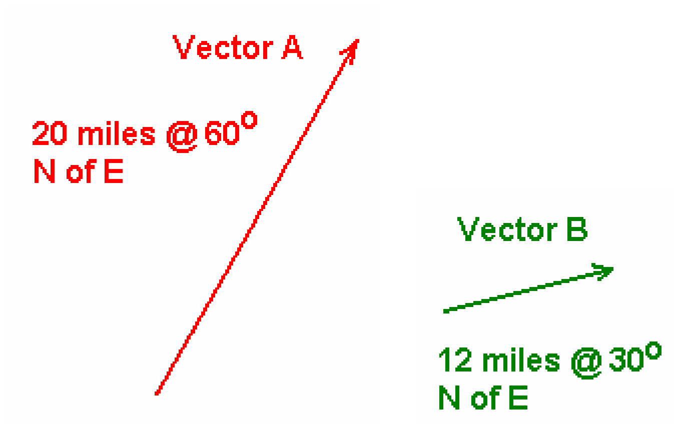
position	**r**	Position is the vector version of *distance*.	
velocity	**v**	Velocity is distance (position) over time, and time is a scalar.	**v** = **r**/t = (1/t) **r**
acceleration	**a**	Velocity is acceleration over time, and time is a scalar.	**a** = **v**/t = (1/t) **v**
force	**F**	Force is mass times acceleration, and mass is a scalar.	**F** = m **a**
momentum	**p**	Momentum is mass times velocity, and mass is a scalar.	**P** = m **v**

Vector Addition

Adding vectors hinges on using trigonometry to break a vector into perpendicular components. Once that is done, like components can be added normally.

I want to add two vectors, **A** and **B**.

R = **A** + **B**



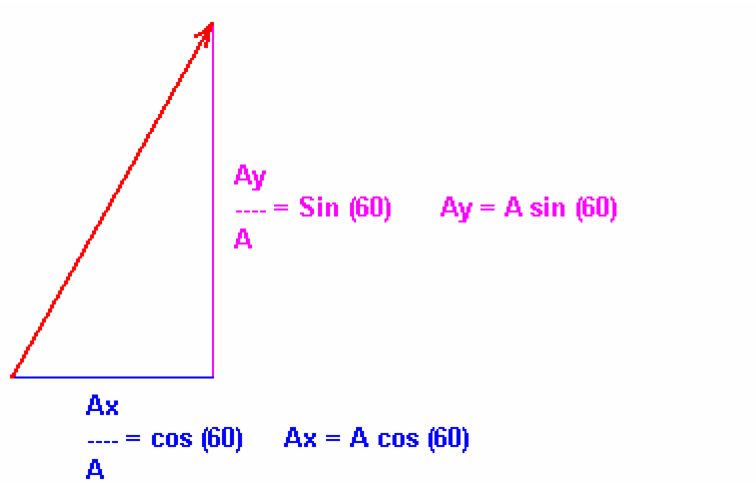
Using trig, break **A** into components

Ay = 20 mi sin(60)
   = 20 mi (0.8660)
   = 17.32 mi

Ax = 20 mi cos(60)
   = 20 mi (0.5)
   = 10 mi

Ax = 10 mi (**i**)

The **i** notation indicates that **Ax** is in the positive x (or **i**) direction.



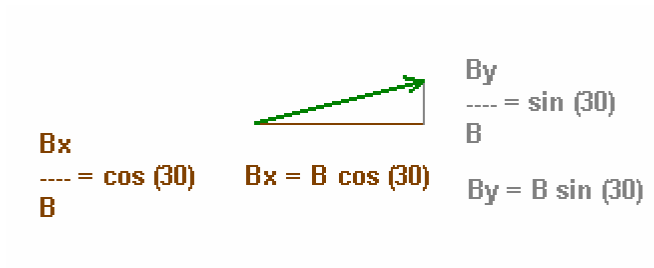
Ay = 17.32 mi (**j**)

The **j** notation indicates that **Ay** is in the positive y (or **j**) direction.

Using trig, break **B** into components

By = 12 mi sin(30)
= 12 mi (0.5)
= 6 mi

Bx = 12 mi cos(30)
= 12 mi (0.5)
= 10.39 mi



Bx = 10.39 mi (**i**)

The **i** notation indicates that **Bx** is in the positive x (or **i**) direction.

By = 6 mi (**j**)

The **j** notation indicates that **By** is in the positive y (or **j**) direction.

Add like components.

Ry = 17.32 mi + 6 mi
= 23.32 mi

Rx = 10 mi + 10.39 mi
= 20.39 mi

$R^2 = Rx^2 + Ry^2$

$R = (Rx^2 + Ry^2)^{0.5}$

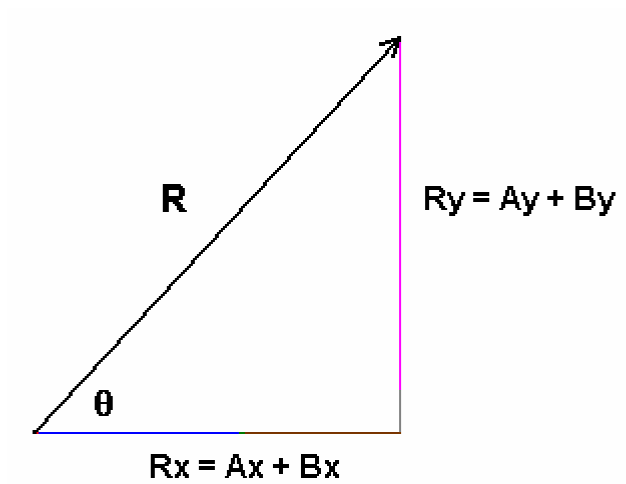
R = 30.98 mi

$\tan(\theta) = Ry/Rx$

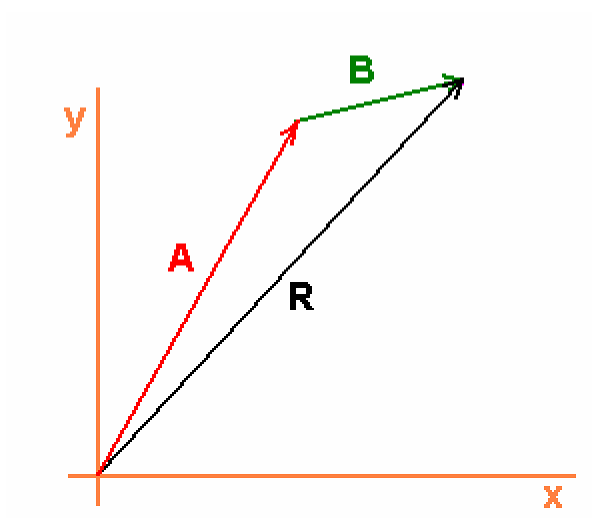
$\tan(\theta) = 23.32/20.39$
$= 1.14$

$\theta = \tan^{-1}(1.14)$

$\theta = 48.7°$



R = 31 mi @ 48.7° N of E



Example Problem #1:

A freighter leaves the port of Los Angeles and steams due west at 25 mph for 8 hours. The captain then adjusts the ship's heading to 30° North of West and continues on that heading for 6 hours with no change in speed.

What is the ship's position after 14 hours?

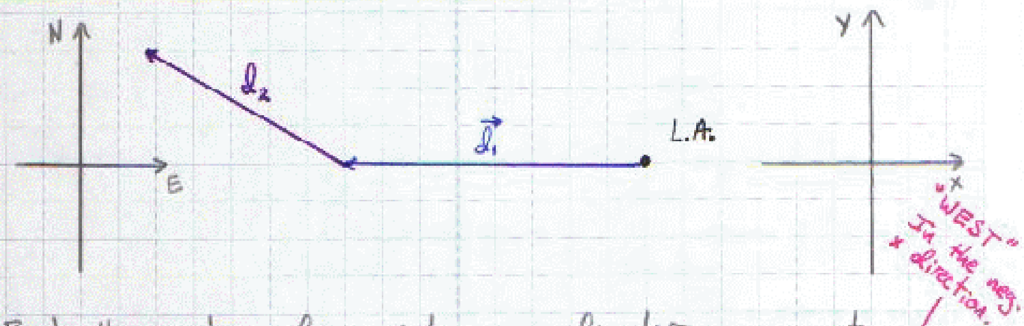
8 hours: $\vec{V}_1$ = 25 mph due W

6 hours: $\vec{V}_2$ = 25 mph @ 30° N of W

$\vec{d}_1 = \vec{V}_1 t$ = (25 mph W)(8 hr) = 200 mi due W

$\vec{d}_2 = \vec{V}_2 t$ = (25 mph @ 30° N of W)(6 hr) = 150 mi @ 30° N of W

Multiplying a vector times a scalar does not affect direction.

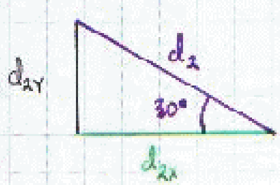


Break the vectors down into perpendicular components:

d_1 only has an x component: d_{1x} = 200 mi (-↑)

$d_{1y} = 0$

d_2 has both x & y components:



$\sin 30° = \frac{d_{2y}}{d_2}$

$d_{2y} = d_2 \sin 30°$

= (150 mi)(.5)

= 75 mi (-↑)

WEST in the neg. x direction

$\frac{d_{2x}}{d_2} = \cos 30°$ $\quad d_{2x} = d_2 \cos 30°$

$= 150\text{ mi}\,(.8660)$

$= 129.904\text{ mi}$ (↑)

NORTH in the positive Y direction

Now add x components + y components.

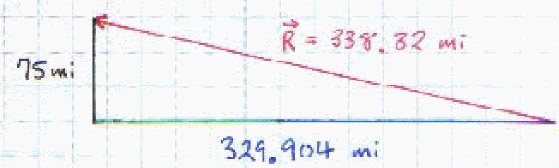
X:

d_{1x}	200	(-↑)
$+d_{2x}$	129.904	(-↑)
	329.904	(-↑)

Y:

d_{1y}	0	
$+d_{2y}$	75	(↑)
	75	(↑)

These totals now become the x and y components of the resultant vector.



$R^2 = (75\text{ mi})^2 + (329.904\text{ mi})^2$

$R = \sqrt{114461.649\text{ mi}^2} = 338.32\text{ mi}$

$\tan\theta = \frac{75\text{ mi}}{329.904\text{ mi}} = .2273$

$\theta = \tan^{-1}(.2273) = 12.808°$

After 14 hours the ship is 338 miles from and 12.8° N of W of L.A.