DAY 14

Summary of Primary Topics Covered

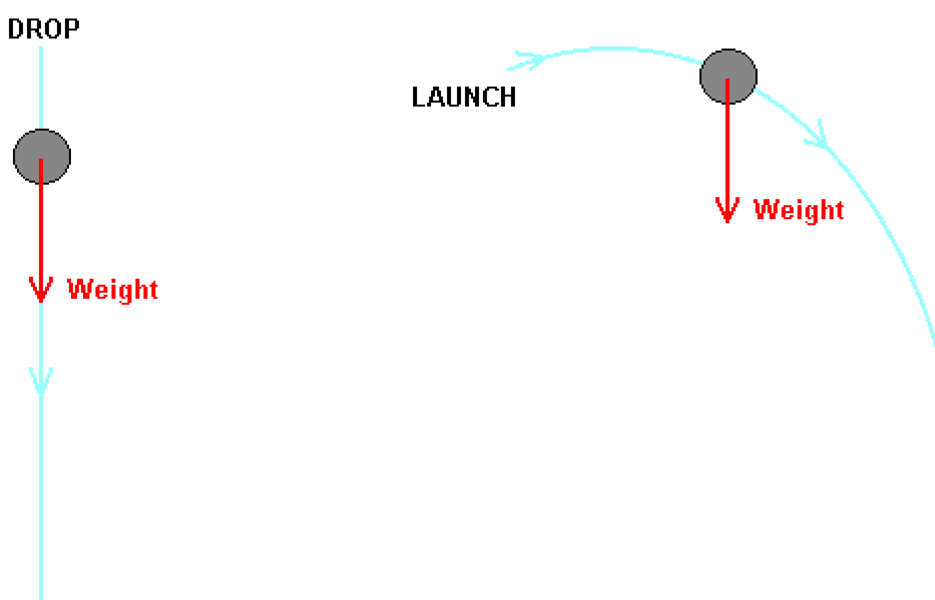
Today we worked some homework/examples in rotational motion. This wraps up our work on rotation.

Then we moved on to a new subject -- 2-dimensional (and even multi-dimensional motion)!

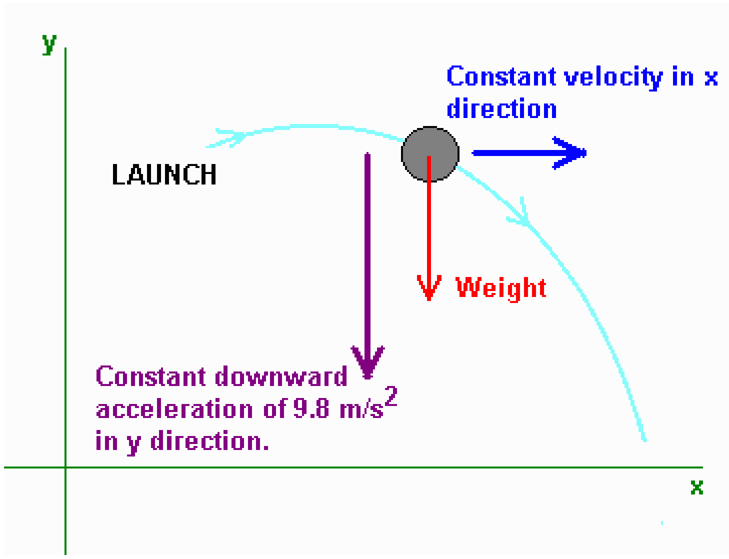
Projectile Motion

An example of motion in multiple dimensions is *projectile motion*. In projectile motion, an object is launched into the air at some angle, and air resistance is not significant.

In projectile motion, once the projectile is launched the only force that acts on the projectile is its weight. The free-body-diagram for a projectile is identical to that of a free-falling object.

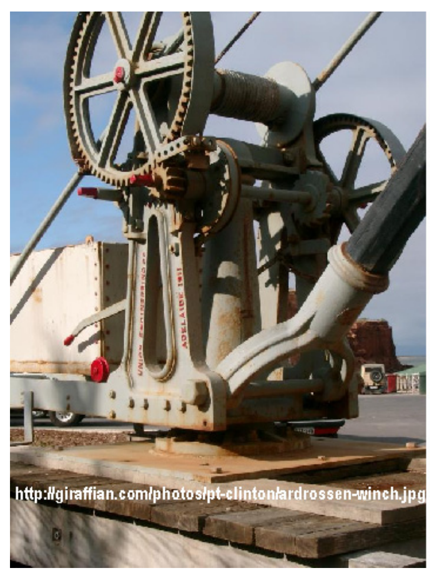


Since the vertical force on a projectile is identical to the vertical force on a free-falling object, the vertical motion of a projectile is identical to that of a free-falling object. Since there is no horizontal force on a projectile (air resistance being negligible, remember), then there is no horizontal acceleration of the projectile once it has been launched. Once launched, the projectile moves with constant horizontal velocity.



Here a problem that involves motion in plane (2 dimensions) that can solved by breaking the problem down into two <u>independent</u> motions or (*components*) along <u>perpendicular</u> <u>directions</u>. This is the hallmark a *vector* problem.

Example Problem #1 (last example for rotation):



You need a motor to run a winch. The winch is geared so that 1000 revolutions of the motor's shaft results in 1 revolution of the winch's drum (which is 50 cm in diameter).

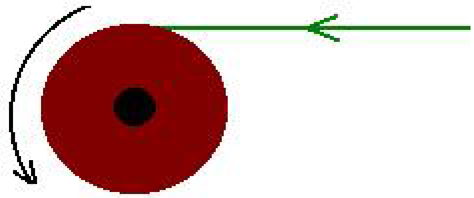
One company wants to sell you an old type of motor that they've been selling for years – The Old Chugger. Old Chugger is rated at 2.3 Hp @ 2000 RPM. The cost is $1000. Another company wants to sell you its new PowerMax motor, which is rated at 5 Hp @ 2500 RPM. The cost is $2200. Detailed power output information for both motors is given in the table.

	Power Output in HP	
RPM	**Old Chugger**	**Power Max**
0	0	0
500	1.5	0.5
1000	2.1	1.7
1500	2.25	2.5
2000	2.3	3.25
2500	2.25	5
3000	2	4.5

Your primary interest is in lifting heavy loads. All other factors are relatively unimportant.

Graph the power output of each motor vs. RPM. Put both graphs on the same axis (you may plot by hand or using EXCEL).

Graph the lifting force each motor generates in the winch vs. RPM. Which motor will lift the heavier load when attached to your winch?



Winch operation – rotation of drum winds in cable.

I'll start with old Chugger at 500 RPM.

The motor rotates the drum at a certain speed (ω). The motor also creates a torque around the drum which translates into a pull (F) via $\tau = rF$.

$\omega_{motor} = 500$ RPM $\omega_{drum} = \frac{1}{1000}(500 \text{ RPM}) = .5$ RPM

$\omega_{drum} = .05236 \ \frac{1}{s}$

$P = 1.5$ Hp $= 1119$ Watts $= 1119 \ \frac{Nm}{s}$

$P = \frac{W}{t}$ Power definition $W = \tau\theta$ Work

$P = \frac{\tau\theta}{t} = \tau\left(\frac{\theta}{t}\right) = \tau\omega$ $\omega = \frac{\theta}{t}$ speed

$P = (rF)\omega$ $\tau = rF$ torque

$F = \frac{P}{r\omega}$

$$F = \frac{1119 \ \frac{Nm}{s}}{(.25 m)(.05236 \frac{1}{s})} = 85485 \ N$$

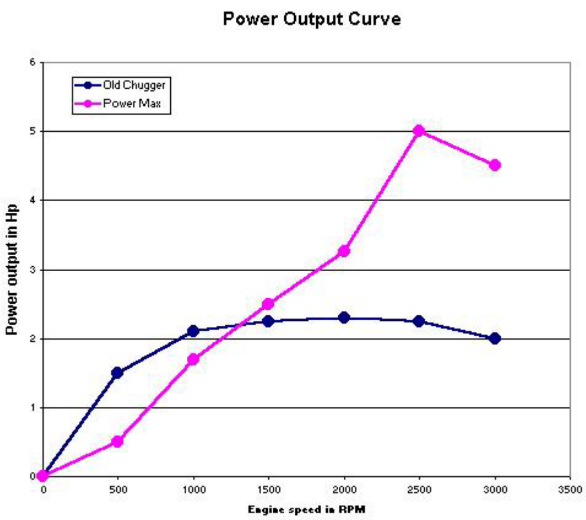
or 19220 lbs

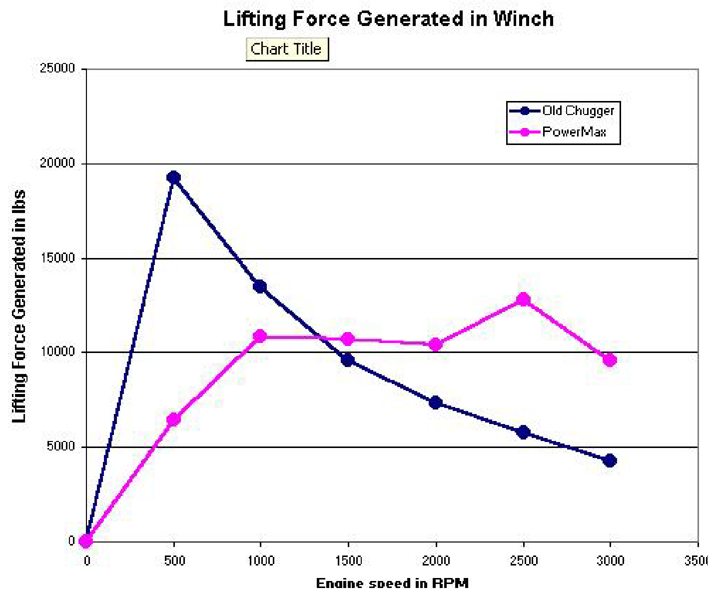
Now I'll do this for every RPM for both motors (using a spreadsheet so I don't have to repeat calculations over and over):

							Old Chugger		PowerMax	
	Power Output in HP		Power Output in Watts (Nm/s)		Motor	Drum	Winch	Winch	Winch	Winch
RPM	**Old Chugger**	**Power Max**	**Old Chugger**	**Power Max**	**ω in 1/s**	**ω in 1/s**	**F in N**	**F in lbs**	**F in N**	**F in lbs**
0	0	0	0	0	0.00	0.00	undefined	undefined	undefined	undefined
500	1.5	0.5	1119	373	52.36	0.05	85485	19219	28495	6406

1000	2.1	1.7	1566.6	1268.2	104.72	0.10	59840	13453	48442	10891
1500	2.25	2.5	1678.5	1865	157.08	0.16	42743	9609	47492	10677
2000	2.3	3.25	1715.8	2424.5	209.44	0.21	32769	7367	46305	10410
2500	2.25	5	1678.5	3730	261.80	0.26	25646	5766	56990	12813
3000	2	4.5	1492	3357	314.16	0.31	18997	4271	42743	9609

And here are my graphs:



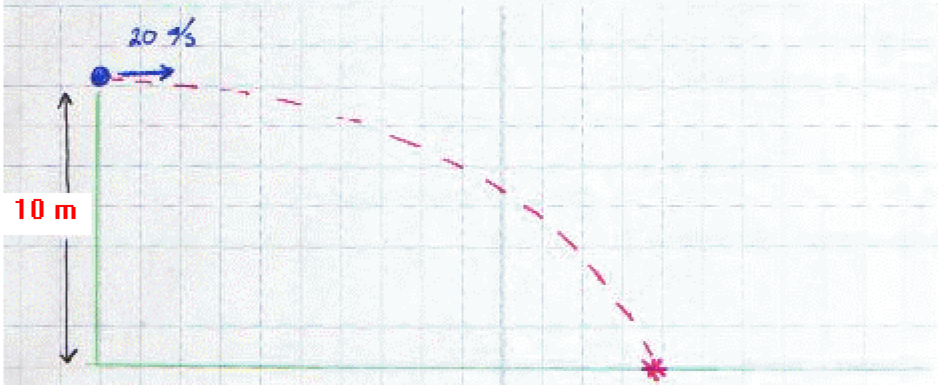


(Note – mathematically the lifting force is undefined at zero RPM. However, I plotted the force as zero since if the motor isn't running no force is generated.)

Obviously Old Chugger generates much more lifting force in the winch than Powermax. Buy Old Chugger, get the most lifting force, and save money, too!

Example Problem #2:

A heavy rock is tossed horizontally at 20 m/s into the river from the top of a 10 m bluff. How far out into the river does the rock go?



VERTICAL (Y)

$y_0 = 10m$

$y = 0m$

$a_y = -9.8 m/s^2$

$v_{0y} = 0 m/s$

Up is positive.

Up is positive

HORIZONTAL (X)

$x_0 = 0$

$a_x = 0$

$v_{0x} = 20 m/s$

Use vertical motion to find time in air:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 = 10m + 0(t) + \frac{1}{2}(-9.8 m/s^2)t^2$$

$$4.9 m/s^2 \, t^2 = 10m$$

$$t = \sqrt{\frac{10m}{4.9 m/s^2}} = 1.4286 \text{ sec}$$

Use Horizontal motion to find range:

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$x = 0 + (20 m/s)(1.4286 s) + \frac{1}{2}(0)(1.4286 s)^2$$

$$x = 28.571 m$$

The rock lands 28.6 m from the base of the bluff.