DAY 12

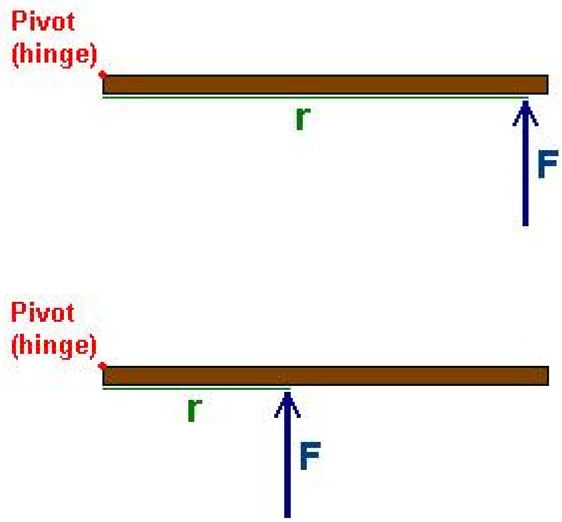
Summary of Primary Topics Covered

Torque

You know that it is easier to open a door if you push on the side opposite the door's hinges. If you push the door on the side by the hinges, chances are the door isn't going to swing open.

That which causes an object at rest to rotate is *torque* – the rotational equivalent of force. Torque (τ) is the "CROSS product" of force and distance. The force and distance are perpendicular to each other:

This will open the door easily.



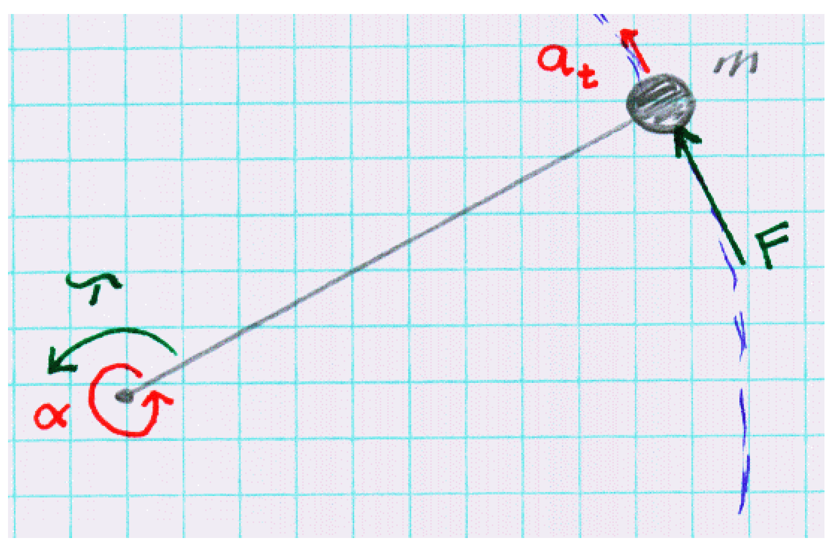
This isn't so easy.

$$\tau = r \times F$$

Torque is measured in m-N or ft-lb. A m-N is not the same as a Joule. A Joule is what you get when a Newton of force and a meter of distance are acting <u>parallel</u> to each other (a DOT product). A Joule is an energy unit. A m-N is what you get when a Newton of force and a meter of distance are acting <u>perpendicular</u> to each other (a CROSS product). A m-N is a torque unit.

Rotational Version of Newton's 2nd Law

If a tangential force is acting on a mass that swings around a pivot point, the force causes a tangential acceleration (a_t) and an angular acceleration (α).



$F = m\, a_t$	Newton's 2nd Law
$F = m\, r\, \alpha$	$a_t = r\,\alpha$ just like $v_t = r\omega$
$rF = r\,(m\, r\, \alpha)$	Multiply both sides by r because rF will give torque
$\tau = m\, r^2\, \alpha$	Torque
$\tau = I\, \alpha$	$I = mr^2$ I is rotational inertia (also called "moment of inertia")

$\tau = I\ \alpha$ is the rotational form of Newton's 2nd law of motion. Moment of Inertia is the rotational equivalent of mass. For rotation, inertia doesn't just depend on mass – inertia also depends on how far the mass is from the point of rotation. The moment of inertia formula for a single mass is $I = mr^2$ but for extended objects the formulas get more complex and either have to be looked up in a table or calculated using integrals in calculus. A table of moments of inertia for various types of objects has been added to the class web page.

We now have more things to add to our linear/angular table:

	Linear (Translational) Motion	**SI units**	**Angular (Rotational) Motion**	**SI units**
"Force":	F	N	τ (torque)	mN
"mass":	m	Kg	I (moment of inertia)	kg m^2
Newton's 2nd Law:	$F = ma$		$\tau = I\alpha$	
KE:	$KE = \frac{1}{2}mv^2$	Joules	$KE = \frac{1}{2}I\omega^2$	Joules
Momentum:	$p = mv$	kgm/s	$L = I\omega$	kgm^2/s
Work:	$W = Fd$	Joules	$W = \tau\theta$	Joules

The bottom line is that all the concepts that we've learned so far in class for motion in a straight line apply to rotational motion, too -- with a few relatively minor changes.

Example Problem #1:

Show that the formula for rotational KE results in the same units (Joules) as the formula for translational KE.

Solution:

$KE = \frac{1}{2} mv^2$
m has units of kg, v has units of m/s,
so KE units are $[kg][m/s]^2$
KE units are kgm^2/s^2

$KE = \frac{1}{2} I\omega^2$
I has units of kgm^2, ω has units of 1/s,
so KE units are $[kgm^2][1/s]^2$
KE units are kgm^2/s^2

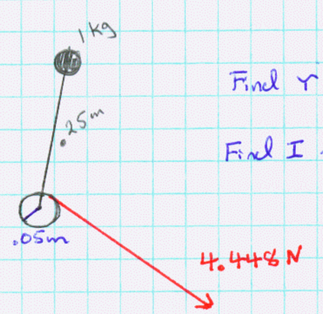
1 kgm^2/s^2 is a Joule

Example Problem #2:

A 1 kg mass is attached to an arm that is 25 cm long. The arm in turn is attached to a pulley that is 5 cm in radius. A string was wrapped around the pulley. If a force of 4.448 N is applied to the string, what will the angular acceleration of the system be? If the string is pulled for 1 m what will be the angular speed of the system (in RPM). The system is well-lubricated so friction is negligible and the pulley and arm are lightweight.

Solution:

The last sentence tells me I don't worry about friction, or about the mass of the pulley and the arm. I only worry about the 1 kg mass.



$\tau = I\alpha$

Find τ → $\tau = rF = (.05m)\,4.448N = .2224\ mN$

Find I → $I = mr^2 = (1kg)(.25m)^2 = .0625\ kgm^2$

So now

$.2224\ mN = .0625\ kgm^2\ \alpha$

$.2224\ \cancel{m}\left(\frac{\cancel{kgm}}{s^2}\right) = .0625\ \cancel{kgm^2}\ \alpha$

$3.5584\ \frac{1}{s^2} = \alpha$

Use energy — the work done pulling the string goes into the KE of the rotating object

$$W = KE$$

$$F \cdot d = \frac{1}{2} I \omega^2$$

$$4.448\ Nm = \frac{1}{2}(0.0625\ kg\, m^2)\ \omega^2 \qquad Nm = \frac{kg\, m}{s^2}\, m = \frac{kg\, m^2}{s^2}$$

$$4.448\ \frac{\cancel{kg}\,\cancel{m^2}}{s^2} = 0.03125\ \cancel{kg}\,\cancel{m^2}\ \omega^2$$

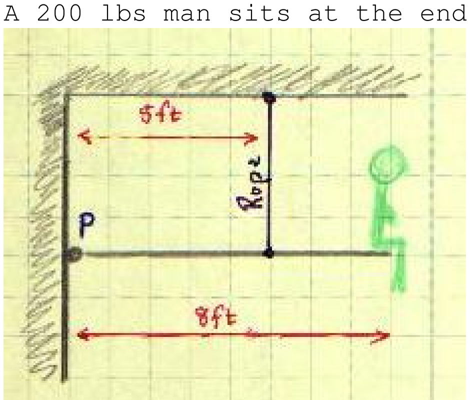
$$11.93\ \tfrac{1}{s} = \omega$$

$$11.93\ \tfrac{1}{s}\left(\frac{1\ Rev}{2\pi}\right)\left(\frac{60\ sec}{1\ min}\right) = 113.93\ \frac{Rev}{min}$$

$$\boxed{\text{So } \alpha = 3.56\ \tfrac{1}{sec^2} \quad \text{and } \omega = 114\ RPM}$$

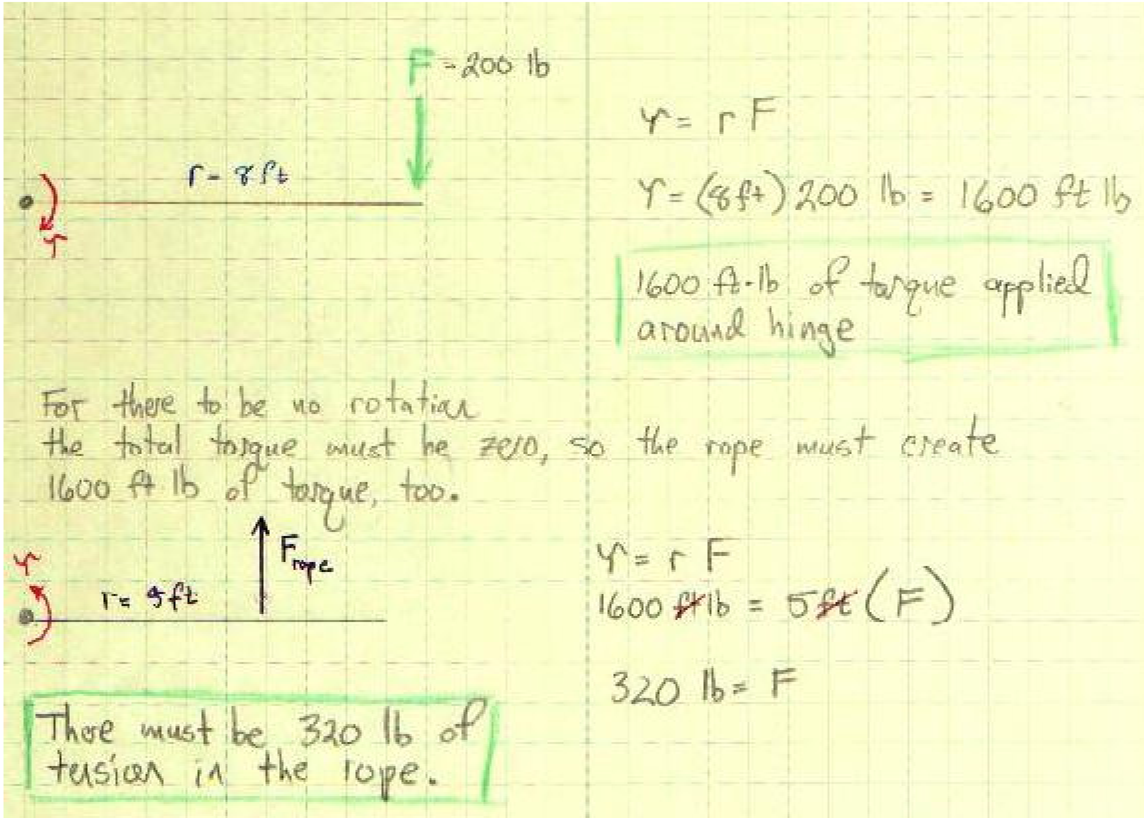
Example Problem #3:

A 200 lbs man sits at the end of an 8 ft board as shown.



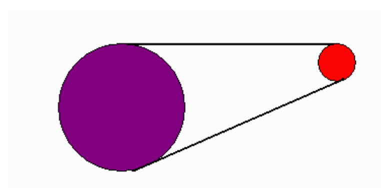
What torque is applied around the hinge at point P? What tension force must be in the rope if there is to be no rotation?

Solution:



Example Problem #4:

A pulley with diameter 20 cm is connected by a belt to a pulley with diameter 5 cm. The larger pulley is turning at a speed of 20 rad/s and accelerating at 0.25 rad/sec^2. Find the rotational speed and acceleration of the smaller pulley.



Solution:

Because the edges of the pulleys are physically connected together by a belt they must move together. The edges of the pulleys must therefore have the same speeds and accelerations (v_t and a_t).

For the large pulley v_t can be found using $v_t = r\,\omega$. r = 10 cm = .1 m

$v_t = r\,\omega$ = .1 m (20 1/s) = 2 m/s

The small pulley has the same v_t but has r = 2.5 cm = .025 m

$v_t = r\,\omega$

2 m/s = .025 m (ω)

ω = 80 1/sec or rad/sec for the small pulley

Now I'll do the same thing for acceleration:

Big pulley: $a_t = r\,\alpha = .1\ m\,(0.25\ 1/s^2) = 0.025\ m/s^2$

Small pulley: $a_t = r\,\alpha$

$0.025\ m/s^2 = .025\ m\,(\alpha)$

$1\ 1/s^2$ or $rad/s^2 = \alpha$ for the small pulley

Note how the factor of 4 relationship between the sizes of the pulleys carries through to their rotational speeds and accelerations.

Because the edges of the pulleys are physically connected together by a belt they must move together. The edges of the pulleys must therefore have the same speeds and accelerations (v_t and a_t).

Example Problem #5:

A motor generates ¼ Hp and rotates at a speed of 750 RPM. What torque does it produce? Give your answer in ft×lbs.

Solution:

P = ¼ Hp

¼ Hp × (746 W/1 Hp) = 186.5 W

Power = Work/time .... (P= W/t)

Work = Torque × Angle .... ($W=\tau\theta$)

If the motor runs for 1 minute it does work of 186.5 J/s × 60 s = 11190 J.

During that time it rotates 750 times.

$\theta = 750\ Rev \times (2\pi/Rev) = 1500\pi$ (radians – the phantom unit).

$W=\tau\theta$

$11190\ Nm = \tau\,(1500\pi)$

$2.375\ mN = \tau$

$2.375\ mN \times (3.281\ ft/1\ m) \times (1\ lb/4.448\ N) =$ 1.75 ft×lb