DAY 11

Summary of Primary Topics Covered

Rotational Physics –
The Father of Newton's Laws & Modern Science?

We now are going to move from studying the physics of things that move in a straight-line path (called "translational motion" in physics) to studying the physics of things that spin (called "rotational motion"). There is an interesting historical aspect to rotational physics in that a number of historians of science believe that the study of rotation led to ideas of inertia that eventually led to Newton's concept of Inertia and his Three Laws of Motion.

With rotating bodies it is easier to visualize that absent an external disturbance "an object at rest will stay at rest and an object in motion will stay in motion with constant speed". We can talk about how, if you took away enough friction, a rock pushed across a table top would travel forever at a constant rate, but it's really impossible to do – the rock falls off the table, hits a wall, etc. But with something that spins it is easy to see that if you took away enough friction a spun wheel would spin for a very, very long time. Rotating objects give us a clue that objects don't prefer to be at rest. You can set an object rotating and imagine that, with a little better lubrication, it might spin forever.

Medieval scientists in France in the 1200's and 1300's came up with a name for this – "impetus". They had wanted to figure out how God kept the objects in the universe moving, such as the Moon circling the Earth.

One philosopher, Thomas Aquinas, had basically put a religious interpretation on the ideas of Aristotle. Aristotle had said something had to power all motion, even the motion of the Moon, and Aquinas basically said that the Moon moved because God or his angels kept pushing the Moon along. William of Ockam argued that perhaps God simply set the Moon in motion and, absent friction, it just kept going. Another, Jean Buridan, argued "…since the Bible does not state that appropriate intelligences move the celestial bodies, it could be said that it does not appear necessary to posit intelligences of this kind, because it would be answered that God, when He created the world, moved each of the celestial orbs as He pleased, and in moving them He impressed in them impetuses which moved them without His having to move them any more except by the method of general influence whereby He concurs as co-agent in all things that take place.... And these impetuses which he impressed in the celestial bodies were not decreased nor corrupted afterwards, because there was no inclination of the celestial movements for other movements. Nor was there resistance which could be corruptive or repressive of that impetus [*A Source Book in Medieval Science*, 1974, E. Grant ed., p 277]." In other words, once the Moon was set in motion it tended to stay in motion.

These ideas would be developed over the centuries until Galileo and Newton built on them to develop modern physics and launch the scientific revolution that brought us the modern world. Keep that in mind next time you see something spinning!

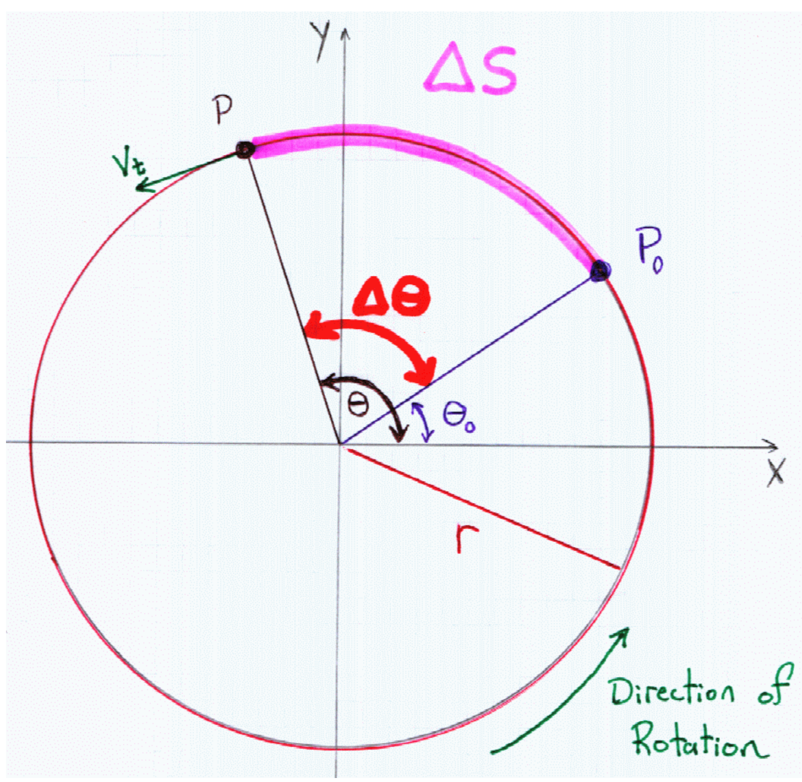
Rotational Units

The simplest unit of rotation is the REVOLUTION – one full turn. Other rotation units are

Degree: $360^\circ = 1$ REV
Radian: $2\pi = 1$ REV
Grad: $400 = 1$ REV

Rotational Position and Velocity

In the figure is shown a rotating disk of radius r. A point on the disk moves from P_0 (at angle θ_0 from the x axis) to P (at angle θ from the x axis). The distance moved along the circle's edge is Δs. The angle it moves through is $\Delta\theta = \theta - \theta_0$. These two are related via



$$\Delta s = r(\Delta\theta)$$

(For a full revolution Δs is the circumference and $\Delta\theta$ is 2π, so you get **Circumference = 2πr**). *Angles must be in radian measure for this equation – and for all equations based on this equation (which includes most equations for rotational motion).*

The tangential velocity (v_t) of the point on the edge of the disk is found by taking the distance moved (Δs) divided by time (t)

$$v_t = \frac{\Delta s}{t} = \frac{r(\Delta\theta)}{t} = r\left(\frac{\Delta\theta}{t}\right)$$

(the radius is constant)

$$v_t = r(\omega) \quad \text{where} \quad \omega = \frac{\Delta\theta}{t}.$$

This ω is the "rotational velocity" of the disk.

Since mathematically the equations for angular position and angular velocity are no different than the equations for linear position and velocity, all the kinematic equations that we derived for linear motion have corresponding rotational equivalents:

	Linear (Translational) Motion	**SI units**	**Angular (Rotational) Motion**	**SI units**
Position:	x	M	θ	1 (rad)
Velocity:	$v = \frac{\Delta x}{t}$	m/s	$\omega = \frac{\Delta\theta}{t}$	1/s (rad/s)
Acceleration:	$a = \frac{\Delta v}{t}$	m/s^2	$\alpha = \frac{\Delta\omega}{t}$	$1/s^2$ (rad/s^2)

Kinematic Equations:

$$x = x_0 + \tfrac{1}{2}(v + v_0)t \qquad \theta = \theta_0 + \tfrac{1}{2}(\omega + \omega_0)t$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$v = v_0 + at \qquad \omega = \omega_0 + \alpha t$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

A Note About Radian Measure and Greek Letters

"Radians" are not really units. They are simply a ratio between the radius of a curved path (r) and the length of the path (Δs). Let us re-arrange the equation

$$\Delta s = r(\Delta\theta) \quad \text{to read} \quad \Delta\theta = \frac{\Delta s}{r}$$

Both *r* and Δs have SI units of meters – which cancel out. Therefore there are no units for $\Delta\theta$. A radian is just the angle you get when the ratio of Δs to *r* is *1*. So a *radia*n is just "*1*". Similarly a *radian/second* is just *1*/s, and a *radian/second*^{2} is just $1/s^2$.

Radians are the PHANTOM units!

Here's how the Greek letters used to represent angular quantities are pronounced:

- **θ – "Theta"**
- **ω – "Omega"**
- **α – "Alpha"**

Example Problem #1:

A CD goes from rest to 200 RPM in 3 seconds. What is its angular acceleration? How many revolutions does it make? A CD measures 12 cm in diameter. What is the speed of the edge of the CD when it is spinning at 200 RPM?

diam - 12 cm

r = 6 cm

$\Theta_0 = 0$

$\omega_0 = 0$

$\omega = 200 \frac{Rev}{min}\left(\frac{2\pi}{Rev}\right)\left(\frac{1\,m}{60\,sec}\right) = 20.9440\ \frac{1}{sec}$

$t = 3\ sec$

* Finding α (angular accel):

$\omega = \omega_0 + \alpha t$

$20.9440\ \frac{1}{s} = 0 + \alpha(3s)$

$\alpha = 6.9813\ 1/s^2$

$\boxed{\text{Accel is } 6.98\ rad/sec^2}$

* Finding Θ (Revolutions):

$\Theta = \Theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\Theta = 0 + 0(3s) + \frac{1}{2}\left(6.9813\ \frac{1}{s^2}\right)(3s)^2$

$\Theta = 31.4159\ (1)$ (Radians)

$\Theta = 31.4159\left(\frac{1\ Rev}{2\pi}\right) = 5$

$\boxed{\text{CD makes 5 revolutions}}$

* Finding edge speed (V_t):

$V_t = r\omega$

$V_t = 6\ cm\left(20.9440\ \frac{1}{s}\right)$

$V_t = 125.664\ cm/s$

$\boxed{\text{Edge moves at } 126\ cm/s}$