DAY 10

Summary of Primary Topics Covered

Some In-Depth, Tougher, Problems

At this point we have learned a lot. We can solve many different kinds of problems. But remember the basics:

- Newton's Laws
- Energy Conservation
- Momentum Conservation

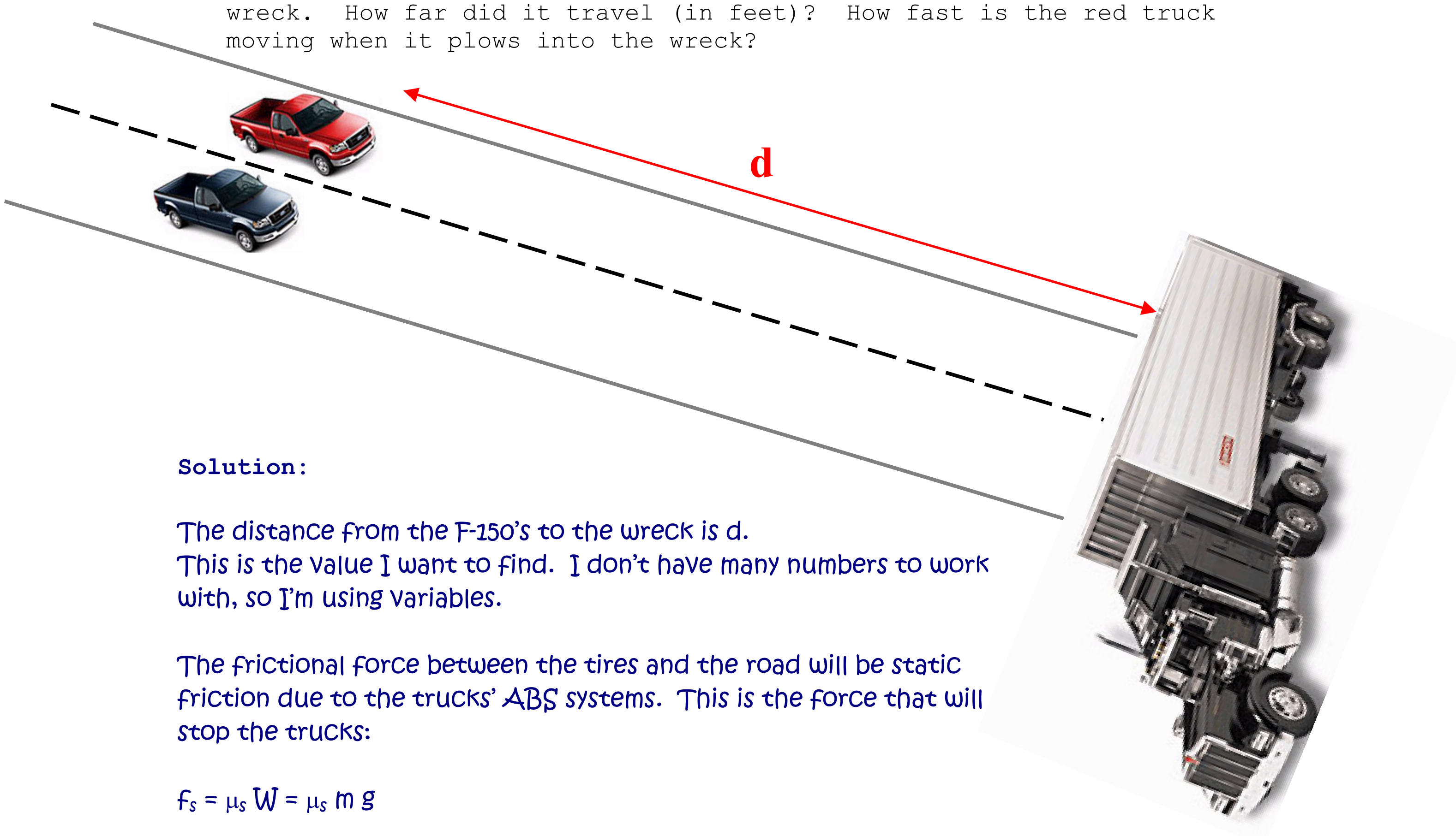
There are different kinds of energy and different kinds of forces, but the basics are always the same. And remember -- never make up your own physics! Always do things you know are physically and mathematically valid, and you can't go wrong. Eventually you will arrive at the answer you are looking for.

The example problems given here are tough problems that involve multiple concepts. However, we know enough to tackle any of them.

Tough, Multi-Concept Example Problem #1:

A blue pickup truck (Ford F-150) is driving along the interstate in the right-hand land going 60 mph. A red F-150 passes the blue truck. The red F-150 is going 75 mph. It is night and the road is damp. Both trucks have anti-lock brakes (ABS) and the μ_s for their tires on the pavement is 0.75.

Just as the red truck pulls even with the blue truck, both drivers see a semi truck jackknifed across the highway, blocking both lanes, the shoulders of the road, and more. The drivers slam on their brakes. The blue truck grinds to a halt with its bumper mere inches from the wreck. How far did it travel (in feet)? How fast is the red truck moving when it plows into the wreck?



Solution:

The distance from the F-150's to the wreck is d.
This is the value I want to find. I don't have many numbers to work with, so I'm using variables.

The frictional force between the tires and the road will be static friction due to the trucks' ABS systems. This is the force that will stop the trucks:

$f_s = \mu_s W = \mu_s m g$

The acceleration of the blue truck is

$a = F/m = -(\mu_s m g)/m = -\mu_s g$ (negative because the force is opposite the truck's motion).

The F-150 has speed v when it is moving and speed 0 when it has stopped. Stopping distance can be found using a kinematic equation:

$v^2 = v_o^2 + 2a(x-x_o)$
$0^2 = v_o^2 + 2(-\mu_s g)(d-0)$
$2\mu_s g d = v_o^2$

If solve this for d I get this equation: $$d = \frac{v^2}{2\mu_s g}$$

$v = 60$ mph = 26.82 m/s
$\mu_s = 1.1$
$g = 9.8\ m/s^2$

$$d = \frac{(26.82m/s)^2}{2(0.75)9.8m/s^2} = 48.94m = 160\,ft$$

So d = 160 ft. is the distance the blue truck traveled in coming to a halt.

Now for the red truck. Since it is identical to the blue truck, its acceleration is also going to be $a = -\mu_s g = -0.75(9.8\ m/s^2) = -7.35\ m/s^2$. The distance in which it can slow down is 48.94 m. But its starting speed is 75 mph, or 33.53 m/s. So using the same kinematic equation

$v^2 = v_o^2 + 2a(x-x_o)$
$v^2 = (33.53m/s)^2 + 2(-7.35m/s^2)(48.94m - 0)$
$v^2 = 1124.063m^2/s^2 - 719.418\ m^2/s^2$
$v^2 = 404.645\ m^2/s^2$
$v = 20.116$ m/s = 45 mph

So the red truck plows into the wrecked semi going 45 mph. Like they say – "speed kills".

Tough, Multi-Concept Example Problem #2: (not a "relates to every day life" type of problem):

A 2 kg model rocket has an engine that generates 300 N of thrust for 10 s before running out of fuel. The rocket is launched vertically on a planet where the gravitational field strength is 1/6 what it is on Earth and there is no air. How high does the rocket go? The mass of the fuel is not significant.

NOTE to purists -- it's a large planet, so the g field strength is pretty constant for at least a thousand km above the surface of the planet.

I'll break the problem into two parts – when the engine is running & when the engine is out of fuel.

Engine running

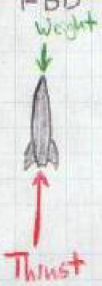
$m = 2\,kg$ $\quad V_0 = 0$

$g = \frac{1}{6}(9.8\,\frac{m}{s^2}) = 1.633\,m/s^2$

$T = 300\,N$ $\quad y_0 = 0$

$t = 10\,sec$

FBD



$\Sigma F = ma$

$T - W = ma$

$300N - (2kg)\,1.633\,m/s^2 = (2kg)\,a$

$296.73\,kg\,m/s^2 = (2\,kg)\,a$

$148.37\,m/s^2 = a$

When engine shuts off:

$V = v_0 + at = 148.37\,m/s^2\,(10\,s) = 1483.7\,m/s$

$y = y_0 + v_0 t + \frac{1}{2}at^2$

$y = 0 + 0(10s) + \frac{1}{2}(148.37\,m/s^2)(10s)^2$

$y = 7418.5\,m$

So when the engine burns out the rocket is 7418.5 m high and is moving upwards at 1483.7 m/s.

Engine not running

Rocket is in free fall. $\quad a = -g = -1.633\,m/s^2$

$V_0 = 1483.7\,m/s$

$y_0 = 7418.5\,m$

How high does the rocket go? When the rocket hits the top of its flight $v=0$. Now find y.

Use $v^2 = v_0^2 + 2a(y - y_0)$

$0^2 = (1483.7\,m/s)^2 + 2(-1.633\,m/s^2)(y - 7418.5\,m)$

$0 = 2201365.69\,m^2/s^2 - 3.266\,m/s^2\,(y - 7418.5\,m)$

$3.266\,\frac{m}{s^2}(y - 7418.5\,m) = 2201365.69\,\frac{m^2}{s^2}$

$y - 7418.5\,m = \frac{2201365.69\,m^2}{3.266\,m}$

$y - 7418.5\,m = 674025.0122\,m$

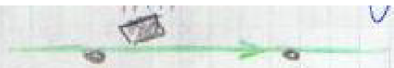
$y = 681443.5122\,m$

ANSWER: The rocket rises 681 km above the surface.

Tough, Multi-Concept Example Problem #3 (a problem that some people might relate to, depending on where they work):

A conveyor belt is moving at 400 ft/min. A package that weighs 50 lb drops vertically onto the belt. The μ_k between the package and the belt is 0.5.

a) Describe what happens in detail.
b) How far will the package travel from its starting point before it stops sliding on the belt?
c) How much energy went into moving the package? How much heat was generated?
d) How much power was required to keep the belt moving at constant speed?

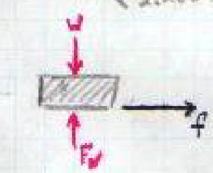


The package drops onto the belt. At first the package won't be moving as fast as the belt, and the belt will slide under the package. When the package gets up to the belt's speed, there is no more slippage and the package moves at constant speed.

$$V_{belt} = 400 \frac{ft}{min}\left(\frac{1\ m}{3.281\ ft}\right)\left(\frac{1\ min}{60\ sec}\right) = 2.0319\ m/s$$

$$m_{package} = 50\ lb\left(\frac{1\ kg}{2.203\ lb}\right) = 22.6963$$

Package FBD



friction accelerates package to right (in direction of belt's motion)

$F_N = W = mg$

$\boxed{f = \mu F_N} = \mu mg$

$\mu = .5$

$\boxed{\Sigma F = ma}$

$f = ma$

$\mu \cancel{m} g = \cancel{m} a$

$\mu g = a$

$a = 4.9\ m/s^2$

The time it takes the package to go from $V_0 = 0$ to $V = 2.0319\ m/s$ can be found via

$\boxed{V = V_0 + at}$

$2.0319\ m/s = 0 + 4.9\ m/s^2\ t$

$t = .41467\ sec$

$v = 2.0319\ m/s$

$V_0 = 0$

$a = 4.9\ m/s^2$

$X_0 = 0$

How far does it move?

$X = X_0 + \frac{1}{2}(v + v_0)t$

$x = 0 + \frac{1}{2}(2.0319\ m/s + 0)(.41467\ s)$

$x = .42128\ m$

(b) ANSWER:

The package moves

.421 m or 1.38 ft

The package had no energy to start with - it ends up with KE.

$KE = \frac{1}{2}mv^2 = \frac{1}{2}(22.6963\ kg)(2.0319\ m/s)^2$

$KE = 46.85217\ J$

The heat generated is tougher. The package moved .421 m before moving off at constant speed. But the belt moved at a constant 2.0319 m/s, so it moved

$x = x_0 + \frac{1}{2}(v + v_0)t$

$x = 0 + \frac{1}{2}(2.0319\ m/s + 2.0319\ m/s)(.41467\ s) = .84257\ m$

Belt FBD

So work friction does on belt is $W = Fd$

$W = fd$

$= \mu mg(d)$

$= .5(22.6963\ kg)(9.8\ \frac{N}{kg})(.84257\ m)$

$W = 93.7036\ J$

So, of that 93.7036 J from the belt, 46.85217 J went into moving the package. The rest 46.8514 J must have gone to heat.

These are only different due to rounding errors.

$$\boxed{\begin{array}{l}\text{(C) ANSWER:} \\ \quad 46.9\text{ J went into moving package} \\ \quad 46.9\text{ J went into heat}\end{array}}$$

93.7036 J of Work are done in a time of .41467 sec

$$P = \frac{W}{t} = \frac{93.7036\text{ J}}{.41467\text{ s}} = 225.971\text{ Watts}$$

.303 Hp

$$\boxed{\begin{array}{l}\text{(d) ANSWER} \\ \quad 226\text{ Watts or }.30\text{ Hp}\end{array}}$$

Tough, Multi-Concept Example Problem #4:

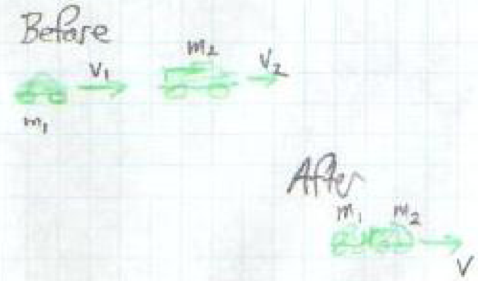
A 3000 lb car moving at 20 mph plows into the back of a 5500 lb truck moving at 10 mph in the same direction. The two stick together after the wreck. How fast will the wreckage be moving after the collision? How much KE was lost in the collision? Where did that energy go?

$$3000\text{ lb}\left(\frac{1\text{ kg}}{2.205\text{ lb}}\right) \Rightarrow \overset{m_1}{1361.78}\text{ kg}$$

$$5000\text{ lb} \Rightarrow \overset{m_2}{2269.63}\text{ kg}$$

$$20\text{ mph} = \overset{v_1}{8.9405}\text{ m/s}$$

$$10\text{ mph} = \overset{v_2}{4.4703}\text{ m/s}$$



Use momentum conservation

Before = After

$$P_{TOTAL} = P_{TOTAL}$$

$$m_1 v_1 + m_2 v_2 = m_1 V + m_2 V = (m_1 + m_2) V$$

$$1361.78\text{ kg}\,(8.9405\text{ m/s}) + (2269.63\text{ kg})(4.4703\text{ m/s}) = (1361.78\text{ kg} + 2269.63\text{ kg})\,V$$

$$22268.11\ \cancel{\text{kg}}\,\text{m/s} = (3631.41\ \cancel{\text{kg}})\ V$$

$V = 6.1321\ m/s$

ANSWER
Wreckage moves at
6.13 m/s or 13.7 mph

Figuring KE

Before collision:

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2}(1361.78\ kg)(8.9405\ m/s)^2 + \frac{1}{2}(2269.63\ kg)(4.4703\ m/s)^2$$

$$= 77102.936\ J$$

After collision

$$KE = \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}(1361.78\ kg + 2269.63\ kg)(6.1321\ m/s)^2$$

$$= 68275.320\ J$$

$$KE\ lost = 77102.936\ J - 68275.320\ J$$

$$= 8827.616\ J$$

ANSWER: 8828 J of KE were lost.
This 8828 J went into heat, sound, and mostly into work done bending up the cars (work of deformation).